A CHARACTERIZATION OF DISCRETE UNIMODALITY WITH APPLICATIONS TO VARIANCE UPPER BOUNDS

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Abstract. Bertin and Theodorescu (1984, *Statist. Probab. Lett.*, **2**, 23–30) developed a characterization of discrete unimodality based on convexity properties of a discretization of distribution functions. We offer a new characterization of discrete unimodality based on convexity properties of a piecewise linear extension of distribution functions. This reliance on functional convexity, as in Khintchine's classic definition, leads to variance dilations and upper bounds on variance for a large class of discrete unimodal distributions. These bounds are compared to existing inequalities due to Muilwijk (1966, *Sankhyā, Ser. B*, **28**, p. 183), Moors and Muilwijk (1971, *Sankhyā, Ser. B*, **33**, 385–388), and Rayner (1975, *Sankhyā, Ser. B*, **37**, 135–138), and are found to be generally tighter, thus illustrating the power of unimodality assumptions.

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