STATISTICAL PROCEDURES BASED ON SIGNED RANKS IN k SAMPLES WITH UNEQUAL VARIANCES

TAKA-AKI SHIRAISHI

Department of Mathematics, Yokohama City University, Yokohama 236, Japan

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Abstract. In k samples with unequal variances, test procedures based on signed ranks for the homogeneity of k location parameters are proposed. The asymptotic χ^2-distribution of the test statistics is shown. It is found that the asymptotic relative efficiency of the rank tests relative to Welch's test (1951, *Biometrika*, **38**, 330–336) under local alternatives agrees with that of the one-sample signed rank tests relative to the t-test. A simulation study for the goodness of the χ^2-approximate of significance points is done. Then, surprisingly it can be seen that the χ^2-approximate for the critical points of the proposed tests is better than that of Kruskal-Wallis test and the Welch-type test. Next R-estimators and weighted least squares estimators for common mean of k samples under the homogeneity of k location parameters are compared in the same way as the test case. Furthermore, positive-part shrinkage versions of R-estimators for the k location parameters are considered along with a modified James-Stein estimation rule. The asymptotic distributional risks of the usual R-estimators, the positive-part shrinkage R-estimators (PSRE's), and the preliminary test and shrinkage R-versions under an arbitrary quadratic loss are derived. Under Mahalanobis loss, it is shown that the PSRE's dominate the other R-estimators for $k \geq 4$. A simulation study leads strong support to the claims that the PSRE's dominate the other type R-estimators and they are robust about outliers.

Key words and phrases: Hypothesis-testing, parameter estimation, Behrens-Fisher's problem, asymptotic relative efficiency, asymptotic distributional risk, modified James-Stein rule, simulation study.