LARGE SAMPLE PROPERTIES OF ESTIMATES OF A DISCRETE GRADE OF MEMBERSHIP MODEL

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(Received December 28, 1989; revised June 11, 1990)

Abstract. Increasingly, fuzzy partitions are being used in multivariate classification problems as an alternative to the crisp classification procedures commonly used. One such fuzzy partition, the grade of membership model, partitions individuals into fuzzy sets using multivariate categorical data. Although the statistical methods used to estimate fuzzy membership for this model are based on maximum likelihood methods, large sample properties of the estimation procedure are problematic for two reasons. First, the number of incidental parameters increases with the size of the sample. Second, estimated parameters fall on the boundary of the parameter space with non-zero probability. This paper examines the consistency of the likelihood approach when estimating the components of a particular probability model that gives rise to a fuzzy partition. The results of the consistency proof are used to determine the large sample distribution of the estimates. Common methods of classifying individuals based on multivariate observations attempt to place each individual into crisply defined sets. The fuzzy partition allows for individual to individual heterogeneity, beyond simply errors in measurement, by defining a set of pure type characteristics and determining each individual's distance from these pure types. Both the profiles of the pure types and the heterogeneity of the individuals must be estimated from data. These estimates empirically define the fuzzy partition. In the current paper, this data is assumed to be categorical data. Because of the large number of parameters to be estimated and the limitations of categorical data, one may be concerned about whether or not the fuzzy partition can be estimated consistently. This paper shows that if heterogeneity is measured with respect to a fixed number of moments of the grade of membership scores of each individual, the estimated fuzzy partition is consistent.

Key words and phrases: Consistency, fuzzy partition, grade of membership.