STATISTICAL MORPHISMS AND RELATED INVARIANCE PROPERTIES

DOMINIQUE B. PICARD

Université Paris VII, U.F.R. de Mathematiques, Tour 45-55, 5eme etage. 2, Place Jussieu 75251 Paris, France
CNRS Unites associées 1321 et 1323 Statistique des Processus en Milieu Médical Université Paris V, 45 Rue des Saints Pères, 75005 Paris, France

(Received December 12, 1988; revised September 25, 1990)

Abstract. Our aim is to investigate a way to characterize the elements of a statistical manifold (the metric and the family of connections) using invariance properties suggested by Le Cam's theory of experiments. We distinguish the case where the statistical manifold is flat. Then, there naturally exists an entropy and it is proven that experiment invariance is equivalent to entropy invariance. If the statistical manifold is not flat, we introduce a notion of local invariance of selected order associated to the asymptotic (on n observations, n tending to infinity) expansion of the power of the Neymann Pearson test in a contiguous neighborough of some point. This invariance provides a substantial number of morphisms. This was not always true for the entropy invariance: particularly, the case of Gaussian experiments is investigated where it can be proven that entropy invariance does not characterize a metric or a family of connections.

Key words and phrases: Statistical manifold, Amari connections, comparison of experiments, likelihood expansions, asymptotic properties of tests.