EMPIRICAL BAYES WITH RATES AND BEST RATES OF CONVERGENCE IN $u(x)C(\theta)\exp(-x/\theta)$-FAMILY: ESTIMATION CASE

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Abstract. Let $\{(X_i, \theta_i)\}$ be a sequence of independent random vectors where X_i, conditional on θ_i, has the probability density of the form $f(x \mid \theta_i) = u(x)C(\theta_i)\exp(-x/\theta_i)$ and the unobservable θ_i are i.i.d. according to an unknown G in some class $\mathcal{G}$ of prior distributions on Θ, a subset of $\{\theta > 0 \mid C(\theta) = \left(\int u(x)\exp(-x/\theta)dx\right)^{-1} > 0\}$. For a $\mathcal{S}(X_1, \ldots, X_n, X_{n+1})$-measurable function ϕ_n, let $R_n = E(\phi_n - \theta_{n+1})^2$ denote the Bayes risk of ϕ_n and let $R(G)$ denote the infimum Bayes risk with respect to G. For each integer $s > 1$ we exhibit a class of $\mathcal{S}(X_1, \ldots, X_n, X_{n+1})$-measurable functions ϕ_n such that for δ in $[s^{-1}, 1]$, $c_0 n^{-2s/(1+2s)} \leq R_n(\phi_n, G) - R(G) \leq c_1 n^{-2(s\delta-1)/(1+2s)}$ under certain conditions on u and G. No assumptions on the form or smoothness of u is made, however. Examples of functions u, including one with infinitely many discontinuities, are given for which our conditions reduce to some moment conditions on G. When Θ is bounded, for each integer $s > 1$ $\mathcal{S}(X_1, \ldots, X_n, X_{n+1})$-measurable functions ϕ_n are exhibited such that for δ in $[2/s, 1]$, $c_0' n^{-2s/(1+2s)} \leq R(\phi_n, G) - R(G) \leq c_1' n^{-2s\delta/(1+2s)}$. Examples of functions u and class $\mathcal{G}$ are given where the above lower and upper bounds are achieved.

Key words and phrases: Exponential family, empirical Bayes, estimation, asymptotic optimality, rates and best rates.