MINIMAXITY AND ADMISSIBILITY OF THE PRODUCT LIMIT ESTIMATOR

E. G. PHADIA[1] AND QIQING YU[2]

[1]*Department of Mathematics, William Paterson College of New Jersey, Wayne, NJ 07470, U.S.A.*
[2]*Department of Applied Mathematics and Statistics, State University of New York at Stony Brook, Stony Brook, NY 11794, U.S.A.*

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Abstract. In this article we examine the minimaxity and admissibility of the product limit (PL) estimator under the loss function

$$L(F, \hat{F}) = \int (F(t) - \hat{F}(t))^2 F^{\alpha}(t)(1 - F(t))^{\beta} dW(t).$$

To avoid some pathological and uninteresting cases, we restrict the parameter space to $\Theta = \{F\colon F(y_{\min}) \geq \epsilon\}$, where $\epsilon \in (0, 1)$ and $y_1, \ldots, y_n$ are the censoring times. Under this set up, we obtain several interesting results. When $y_1 = \cdots = y_n$, we prove the following results: the PL estimator is admissible under the above loss function for $\alpha, \beta \in \{-1, 0\}$; if $n = 1$, $\alpha = \beta = -1$, the PL estimator is minimax iff $dW(\{y\}) = 0$; and if $n \geq 2$, $\alpha, \beta \in \{-1, 0\}$, the PL estimator is not minimax for certain ranges of ϵ. For the general case of a random right censorship model it is shown that the PL estimator is neither admissible nor minimax. Some additional results are also indicated.

Key words and phrases: Minimaxity, censored data, admissibility, nonparametric estimation, product limit estimator.