OPTIMAL CONVERGENCE PROPERTIES OF KERNEL DENSITY ESTIMATORS WITHOUT DIFFERENTIABILITY CONDITIONS

R. J. KARUNAMUNI AND K. L. MEHRA

Department of Statistics and Applied Probability, University of Alberta, Edmonton, Alberta, Canada T6G 2G1

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Abstract. Let $X_1, X_2, \ldots, X_n$ be independent observations from an (unknown) absolutely continuous univariate distribution with density f and let $\hat{f}(x) = (nh)^{-1} \sum_{i=1}^{n} K[(x - X_i)/h]$ be a kernel estimator of $f(x)$ at the point x, $-\infty < x < \infty$, with $h = h_n$ ($h_n \to 0$ and $nh_n \to \infty$, as $n \to \infty$) the bandwidth and K a kernel function of order r. "Optimal" rates of convergence to zero for the bias and mean square error of such estimators have been studied and established by several authors under varying conditions on K and f. These conditions, however, have invariably included the assumption of existence of the r-th order derivative for f at the point x. It is shown in this paper that these rates of convergence remain valid without any differentiability assumptions on f at x. Instead some simple regularity conditions are imposed on the density f at the point of interest. Our methods are based on certain results in the theory of semi-groups of linear operators and the notions and relations of calculus of "finite differences".

Key words and phrases: Kernel density estimation, bias, mean square error, finite differences, semi-groups, linear operators.