

SOME GEOMETRIC APPLICATIONS OF THE BETA DISTRIBUTION

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Abstract. Let θ be the angle between a line and a “random” k -space in Euclidean n -space R^n . Then the random variable $\cos^2 \theta$ has the beta distribution. This result is applied to show (1) in R^n there are exponentially many (in n) lines going through the origin so that any two of them are “nearly” perpendicular, (2) any N -point set of diameter d in R^n lies between two parallel hyperplanes distance $2d\{(\log N)/(n-1)\}^{1/2}$ apart and (3) an improved version of a lemma of Johnson and Lindenstrauss (1984, *Contemp. Math.*, 26, 189-206). A simple estimate of the area of a spherical cap, and an area-formula for a neighborhood of a great circle on a sphere are also given.

Key words and phrases: Beta distribution, Spherical cap, Johnson-Lindenstrauss Lemma.