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RECURSIVE KERNEL DENSITY ESTIMATORS UNDER A WEAK DEPENDENCE CONDITION

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Abstract. Let X_t, $t = \ldots, -1, 0, 1, \ldots$ be a strictly stationary sequence of random variables (r.v.'s) defined on a probability space $(\Omega, \mathcal{F}, P)$ and taking values in R^d. Let $X_1, \ldots, X_n$ be n consecutive observations of X_t. Let f be the density of X_1. As an estimator of $f(x)$, we shall consider $\hat{f}_n(x) = n^{-1} \sum_{j=1}^{n} b_j^{-d} K((x - X_j)/b_j)$. Here K is a kernel function and b_n is a sequence of bandwidths tending to zero as $n \to \infty$. The asymptotic distribution and uniform convergence of $\hat{f}_n$ are obtained under general conditions. Appropriate bandwidths are given explicitly. The process X_t is assumed to satisfy a weak dependence condition defined in terms of joint densities. The results are applicable to a large class of time series models.

Key words and phrases: Asymptotic normality, uniform convergence, absolute regularity, density estimation, kernel, bandwidth.