

## OPTIMAL KERNEL ESTIMATION OF DENSITIES\*

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**Abstract.** Precise asymptotic behavior for mean integrated squared error (MISE) is determined for sequences of kernel estimators of a density in a broad class, including discontinuous and possibly unbounded densities. The paper shows that the sequence using the kernel optimal at each fixed sample size is asymptotically more efficient than a sequence generated by changing the bandwidth of a fixed kernel shape, regardless of the kernel shape. The class of densities considered are those whose characteristic functions behave at large arguments like the product of a Fourier series and a regularly varying function. This condition may be related to the smoothness of an  $m$ -th derivative of the density.

*Key words and phrases:* Kernel density estimation, mean integrated squared error, optimal kernel, regular variation.