Ann. Inst. Statist. Math. Vol. 41, No. 4, 725-752 (1989)

## HIGHER ORDER ASYMPTOTICS IN ESTIMATION FOR TWO-SIDED WEIBULL TYPE DISTRIBUTIONS

MASAFUMI AKAHIRA<sup>1</sup> AND KEI TAKEUCHI<sup>2</sup>

<sup>1</sup>Institute of Mathematics, University of Tsukuba, Ibaraki 305, Japan <sup>2</sup>Research Center for Advanced Science and Technology, University of Tokyo, Komaba, Meguro-ku, Tokyo 156, Japan

(Received May 18, 1988; revised January 25, 1989)

Abstract. We consider the estimation problem of a location parameter  $\theta$ on a sample of size *n* from a two-sided Weibull type density  $f(x - \theta) = C(\alpha) \exp((-|x - \theta|^{\alpha}))$  for  $-\infty < x < \infty$ ,  $-\infty < \theta < \infty$  and  $1 < \alpha < 3/2$ , where  $C(\alpha) = \alpha/\{2\Gamma(1/\alpha)\}$ . Then the bound for the distribution of asymptotically median unbiased estimators is obtained up to the  $2\alpha$ -th order, i.e., the order  $n^{-(2\alpha-1)/2}$ . The asymptotic distribution of a maximum likelihood estimator (MLE) is also calculated up to the  $2\alpha$ -th order. It is shown that the MLE is not  $2\alpha$ -th order asymptotically efficient. The amount of the loss of asymptotic information of the MLE is given.

Key words and phrases:  $2\alpha$ -th order asymptotically median unbiased estimator,  $2\alpha$ -th order asymptotic distribution,  $2\alpha$ -th order asymptotic efficiency, Edgeworth expansion, maximum likelihood estimator.