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CONTIGUOUS ALTERNATIVES WHICH PRESERVE CRAMÉR-TYPE LARGE DEVIATIONS FOR A GENERAL CLASS OF STATISTICS*

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Abstract. Let P_N and Q_N, $N \geq 1$, be two possible probability distributions of a random vector $\boldsymbol{X}_N = (X_{N1}, \ldots, X_{NN})$, whose components are independent. Suppose P_N and Q_N have respective densities $p_N = \prod_{i=1}^{N} f(x_{Ni} - \bar{\theta}_N)$ and $q_N = \prod_{i=1}^{N} f(x_{Ni} - \theta_{Ni})$, where $\bar{\theta}_N = N^{-1} \sum_{i=1}^{N} \theta_{Ni}$, such that $\max_{1 \leq i \leq N} |\theta_{Ni} - \bar{\theta}_N| = O(N^{-1/2})$, $f(x) > 0$ for almost every real x, f is absolutely continuous, and $\sup_{-\theta_0 \leq \theta \leq \theta_0} \int_{-\infty}^{\infty} [f'(x-\theta)]^2 / f(x) dx < \infty$ for some $\theta_0 > 0$. The contiguity of $\{q_N\}$ to $\{p_N\}$ is well known. In this paper it is proven that under these conditions $\{Q_N\}$ preserves C.-T.L.D. (Cramér-type large deviation) from $\{P_N\}$ for a general class of statistics $\mathscr{T}$ which includes R-, U- and L-statistics as members. That means, for any $\{S_N = S_N(\boldsymbol{X}_N)\}$ from $\mathscr{T}$, a C.-T.L.D. theorem with range $C \leq x \leq o(N^{\delta})$ (any $C \leq 0$), $0 < \delta \leq 4^{-1}$, holds for $\{S_N\}$ under $\{P_N\}$, implying that the same theorem holds for $\{S_N\}$ under $\{Q_N\}$. It also provides a quick and simple way to establish C.-T.L.D. results for statistics under $\{Q_N\}$.

Key words and phrases: Contiguous alternatives, Cramér-type large deviations, linear rank statistics, U-statistics, linear combinations of order statistics.