ON THE RATE OF CONVERGENCE OF SPATIAL BIRTH-AND-DEATH PROCESSES

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Abstract. Sufficient conditions for geometrical fast convergence of general spatial birth-and-death processes to equilibrium are established.


1. Introduction

Let $S$ be an arbitrary set (e.g., a subset of $\mathbb{R}^d$). A spatial birth-and-death process on $S$ is, loosely speaking, a continuous time Markov chain with states in the space of all finite point configurations in $S$, and so that a transition can only be a birth of a new point or a death of an existing point (for a formal and more general description and definition, see Preston (1977) and Section 2 in the present paper). The object of this paper is to study the rate of convergence to equilibrium for such processes.

Spatial birth-and-death processes are interesting for many reasons. Obviously, they might be used as models for many dynamic spatial phenomena (cf. Møller and Sørensen (1989)). Their relevance in spatial statistics lies in their close relationship to Gibbs processes (Preston (1977)) and their use in simulation of spatial point patterns (Kelly and Ripley (1976) and Ripley (1977)). Simulated realizations can be seen in Ripley (1977, 1981), Diggle (1983) and Baddeley and Møller (1989).

The present paper is organized as follows. Unique existence and convergence of spatial birth-and-death processes are discussed in Section 2. In Section 3 we give sufficient conditions for geometrical convergence of a general spatial birth-and-death process to equilibrium, to the best of my knowledge, similar results have been given only in the special case of a hard core birth-and-death process (see Lotwick and Silverman (1981)). The results in Sections 2 and 3 are related to well-known results for simple