METHODOLOGY FOR THE INVARIANT ESTIMATION OF A CONTINUOUS DISTRIBUTION FUNCTION

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Abstract. Consider both the classical and some more general invariant decision problems of estimating a continuous distribution function, with the loss function

\[ L(F, a) = \int (F(t) - a(t))^2 h(F(t)) dF(t) \]

and a sample of size \( n \) from \( F \). It is proved that any nonrandomized estimator can be approximated in Lebesgue measure by the more general invariant estimators. Some methods for investigating the finite sample problem are discussed. As an application, a proof that the best invariant estimator is minimax when the sample size is 1 is given.

Key words and phrases: Admissibility, admissibility within \( U_1 \), invariant estimator, minimaxity.

1. Introduction

Since Aggarwal (1955) found the best invariant estimator of an unknown continuous distribution function \( F(t) \), under the loss

\[ L(F, a) = \int (F(t) - a(t))^2 h(F(t)) dF(t) , \]

different methods have been used in investigating the decision theoretic properties of the best invariant estimator \( d_0 \). One interesting fact is that when \( h(t) = t^{-1}(1 - t)^{-1} \), the best invariant estimator is the same as the empirical distribution function \( \hat{F}(t) \).

The asymptotical method has been used in approaching the problem. For instance, Dvoretzky et al. (1956) studied the asymptotical minimaxity property of the best invariant estimator for some loss function; and Read (1972) considered the asymptotical admissibility property of the best invariant estimator. However, this method does not describe the decision theoretic properties when the sample size \( n \) is finite.