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A MINIMUM AVERAGE RISK APPROACH TO SHRINKAGE ESTIMATORS OF THE NORMAL MEAN

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Abstract. For the problem of estimating the normal mean μ based on a random sample $X_1,\dots,X_n$ when a prior value μ_0 is available, a class of shrinkage estimators $\hat{\mu}_n(k) = k(T_n)\overline{X}_n + (1 - k(T_n))\mu_0$ is considered, where $T_n = n^{1/2}(\overline{X}_n - \mu_0)/\sigma$ and k is a weight function. For certain choices of k, $\hat{\mu}_n(k)$ coincides with previously studied preliminary test and shrinkage estimators. We consider choosing k from a natural non-parametric family of weight functions so as to minimize average risk relative to a specified prior p. We study how, by varying p, the MSE efficiency (relative to $\overline{X}$) properties of $\hat{\mu}_n(k)$ can be controlled. In the process, a certain robustness property of the usual family of posterior mean estimators, corresponding to the conjugate normal priors, is observed.

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