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ON A ZERO-CROSSING PROBABILITY

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Abstract. Let $\{X(t), 0 < t < \infty\}$ be a compound Poisson process so that $E\{\exp(-sX(t))\} = \exp(-t\Phi(s))$, where $\Phi(s) = \lambda(1 - \phi(s))$, λ is the intensity of the Poisson process, and $\varphi(s)$ is the Laplace transform of the distribution of nonnegative jumps. Consider the zero-crossing probability $\theta = P\{X(t) - t = 0 \text{ for some } t, 0 < t < \infty\}$. We show that $\theta = \Phi'(\omega)$ where ω is the largest nonnegative root of the equation $\Phi(s) = s$. It is conjectured that this result holds more generally for any stochastic process with stationary independent increments and with sample paths that are nondecreasing step functions vanishing at 0.

Key words and phrases: Ballot theorem, compound Poisson process.