

ASYMPTOTIC THEOREMS FOR ESTIMATING THE DISTRIBUTION FUNCTION UNDER RANDOM TRUNCATION

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Abstract. Representation theorem and local asymptotic minimax theorem are derived for nonparametric estimators of the distribution function on the basis of randomly truncated data. The convolution-type representation theorem asserts that the limiting process of any regular estimator of the distribution function is at least as dispersed as the limiting process of the product-limit estimator. The theorems are similar to those results for the complete data case due to Beran (1977, *Ann. Statist.*, **5**, 400-404) and for the censored data case due to Wellner (1982, *Ann. Statist.*, **10**, 595-602). Both likelihood and functional approaches are considered and the proofs rely on the method of Begun *et al.* (1983, *Ann. Statist.*, **11**, 432-452) with slight modifications.

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