ON THE COMPOUNDED BIVARIATE POISSON DISTRIBUTION: A UNIFIED TREATMENT

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Abstract. A unified treatment is presented here of compounding with the bivariate Poisson distribution. Exploiting the exponential nature of its probability generating function, it is shown that the pgf of the compound distribution is the moment generating function of the compounding random variable. This relationship leads to rather interesting general results. Particularly, the development of the conditional distribution is simplified. Four cases are presented in detail.

Key words and phrases: Bivariate Poisson distribution, probability generating function, bivariate-Hermite, -negative binomial, -Poisson-Inverse Gaussian, -Neyman Type A, conditional distributions.

1. Introduction

The bivariate Poisson distribution is defined by the probability generating function

$$\Pi(z_1, z_2) = \exp\{w_1(z_1 - 1) + w_2(z_2 - 1) + w_3(z_1z_2 - 1)\},$$

where $w_1, w_2, w_3$ are all positive. (See Feller (1957), p. 261.) This distribution has been widely applied in the literature to describe several real life models. On the other hand, several authors have examined the problem where $w_i = \tau \lambda_i$, $i = 1, 2, 3$ with $\lambda_1, \lambda_2, \lambda_3$ being constants and $\tau$ a random variable characterizing an 'individual' in the population. Such models have been considered among others by Holgate (1966), Subrahmaniam (1966), Gillings (1974) and Kemp and Papageorgiou (1982). In each case $\tau$ is assumed to have a distribution of the discrete or continuous type.

In the present paper, we present a unified development of the distributions associated with such compounding.