Final Presentation of Study Group Workshop

March 4, 2016

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General backgrounds of the proposed various subjects by Professor Yuko Hatano and Professor Takuya Kawanishi on environmental engineering

Our main working concept is

- Understanding environmental serious issues
- Mathematical models and feasibility
- Numerical simulations
- Fitting with real or experimental data

Outline

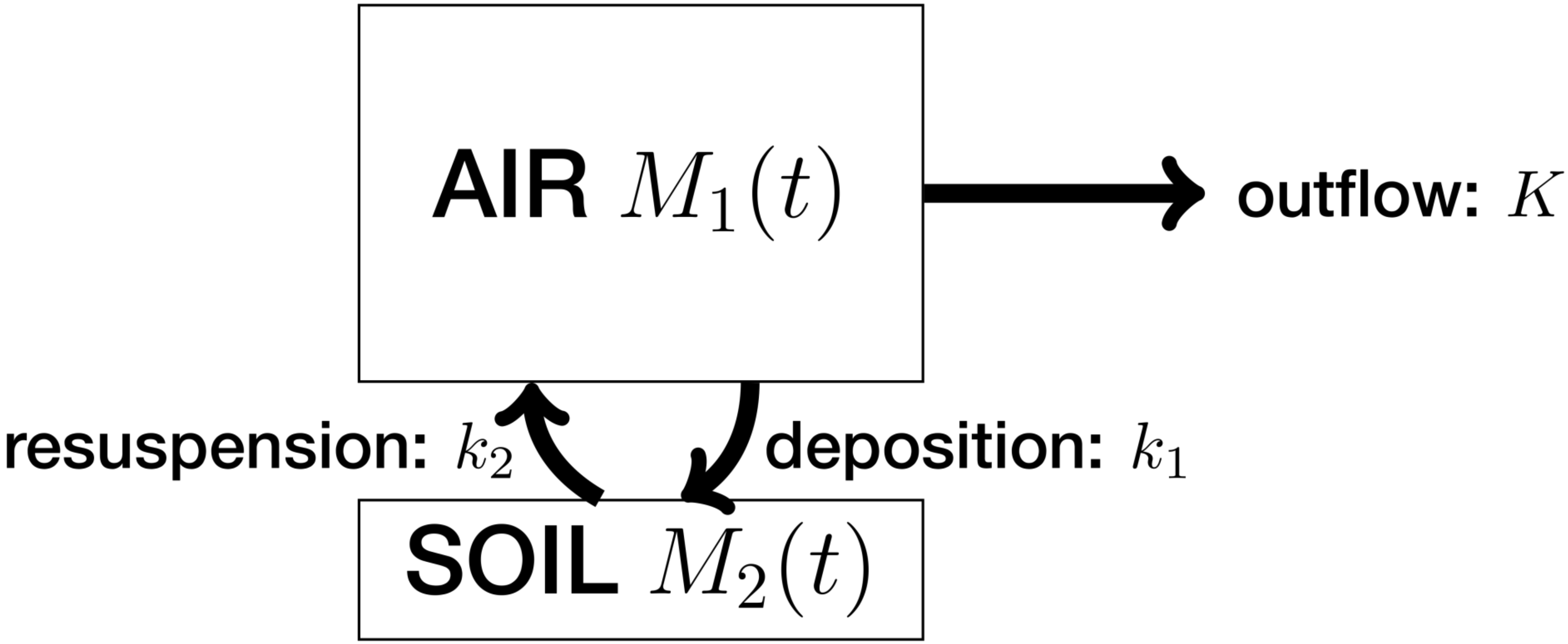
- Topic 1: Improvement of a box model and estimation of the initial explosion amounts (from Prof. Hatano)
- Topic 2: Advection equation modeling of radioactive material in the air (from Prof. Hatano)
- Topic 3: Satellite image analysis for effective afforestation (from Prof. Kawanishi)

TOPIC 1: IMPROVEMENT OF THE BOX MODEL AND ESTIMATION OF THE INITIAL EXPLOSION AMOUNTS

Box Model

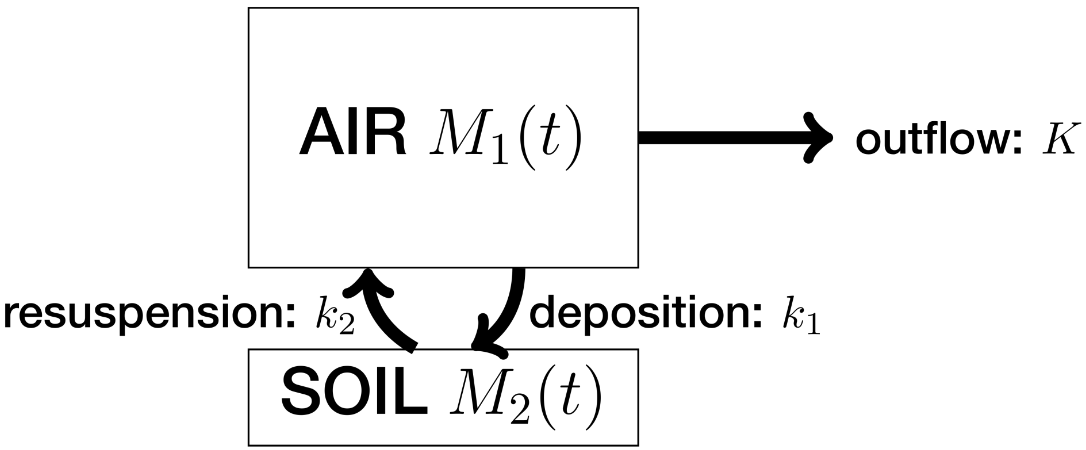
Regard air, soil, ...etc as boxes

Example(1) resuspension and deposition of radioactive material



Box Model

Example(1) resuspension and deposition of radioactive material

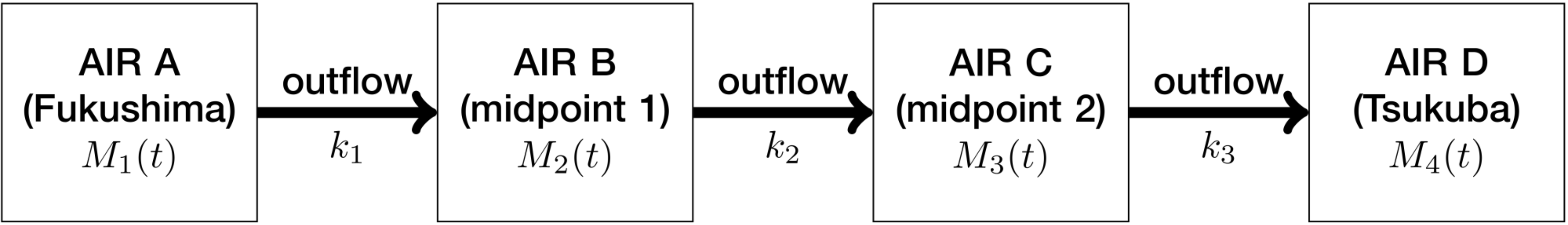


$$\begin{cases} M_1'(t) = -KM_1(t) + k_2 M_2(t) \\ M_2'(t) = -k_1 M_1(t) - k_2 M_2(t) \end{cases}$$

Box Model

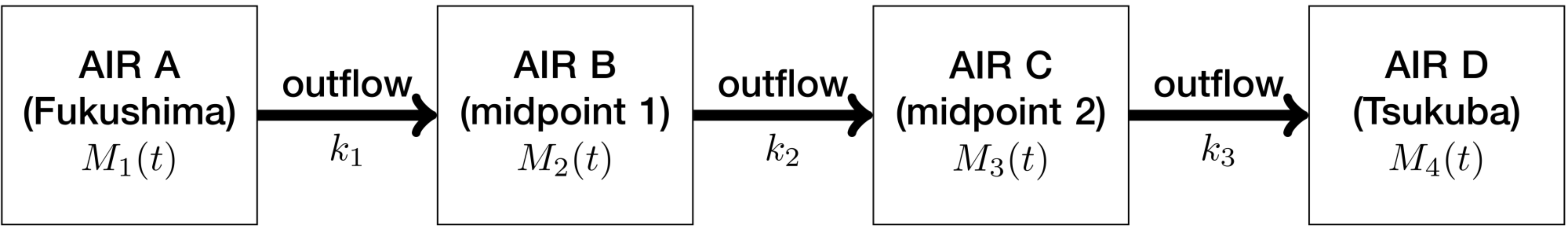
Regard air, soil, …etc as boxes

Example(2) Incoming radioactive materials from Fukushima to Tsukuba



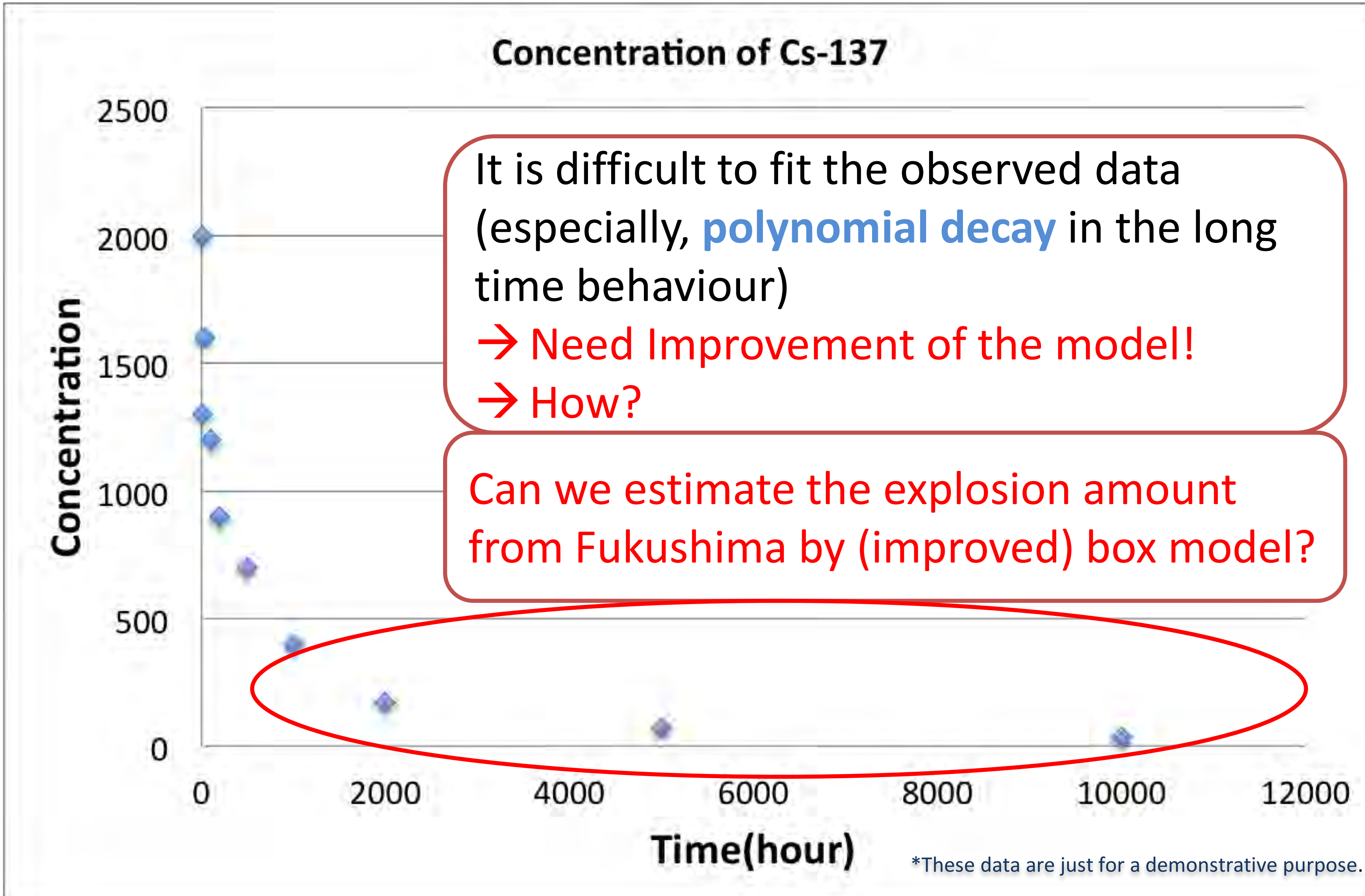
Box Model

Example(2) Incoming radioactive materials from Fukushima to Tsukuba



$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \\ M_4(t) \end{pmatrix} = \begin{pmatrix} -k_1 & 0 & 0 & 0 \\ k_1 & -k_2 & 0 & 0 \\ 0 & k_2 & -k_3 & 0 \\ 0 & 0 & k_3 & 0 \end{pmatrix} \begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \\ M_4(t) \end{pmatrix}$$

Problem on box model



It is difficult to fit the observed data (especially, **polynomial decay** in the long time behaviour)
→ Need Improvement of the model!
→ How?

Can we estimate the explosion amount from Fukushima by (improved) box model?

Basic Strategy

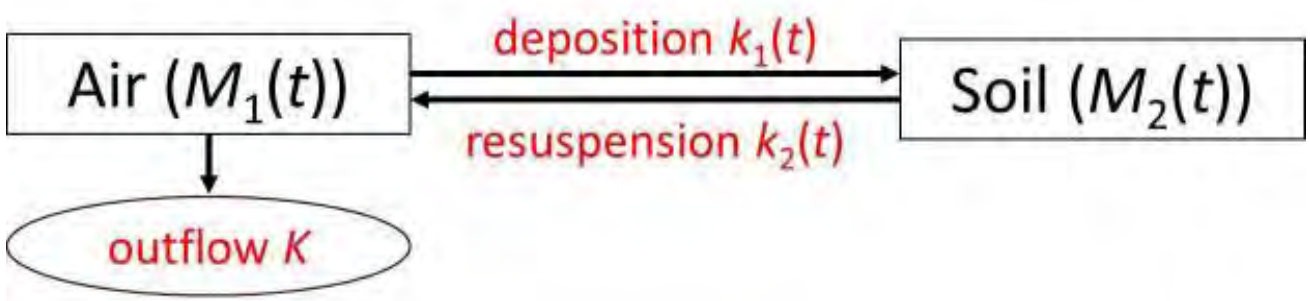
$$\begin{cases} M_1'(t) = -KM_1(t) + k_2M_2(t) \\ M_2'(t) = -k_1M_1(t) - k_2M_2(t) \end{cases}$$

Coefficients are assumed to be constants
→ Description power is limited.
(Only exponential decay can be expected.)

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}M_1(t)\\M_2(t)\\M_3(t)\\M_4(t)\end{pmatrix} = \begin{pmatrix}-k_1 & 0 & 0 & 0\\ k_1 & -k_2 & 0 & 0\\ 0 & & & \\ 0 & & & \end{pmatrix}\begin{pmatrix}M_1(t)\\M_2(t)\\ \\ \end{pmatrix}$$

We introduce the time dependent coefficients.

Air & soil model



$M_1(t)$: mass radioactive materials in air.

$M_2(t)$: mass radioactive materials in soil.

Governing equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} M_1(t) \\ M_2(t) \end{pmatrix} = \begin{pmatrix} -K & k_2(t) \\ k_1(t) & -k_2(t) \end{pmatrix}\begin{pmatrix} M_1(t) \\ M_2(t) \end{pmatrix} + \begin{pmatrix} f_1(t) \\ 0 \end{pmatrix}.$$

Coefficient settings

Assumptions

- $k_1(t)$: deposition effect decreases rapidly, e.g.,

$$k_1(t) = \mathrm{e}^{-t}.$$

- $k_2(t)$: resuspension effect decrease moderately, e.g.,

$$k_2(t) = \frac{\alpha}{t + \tau}.$$

- K: small ($10^{-3} \sim 10^{-2}$).
- τ: large, e.g., $\tau > K^{-1}$.
- $f_1(t)$: newly introduced (artificial) source term.

Numerical simulation

We choose $K = 10^{-2}$, $\tau = 10^3$, $\alpha = 1$ and

$$f_1(t) = A\exp(-t^{\beta}), \quad \beta = 0.2.$$

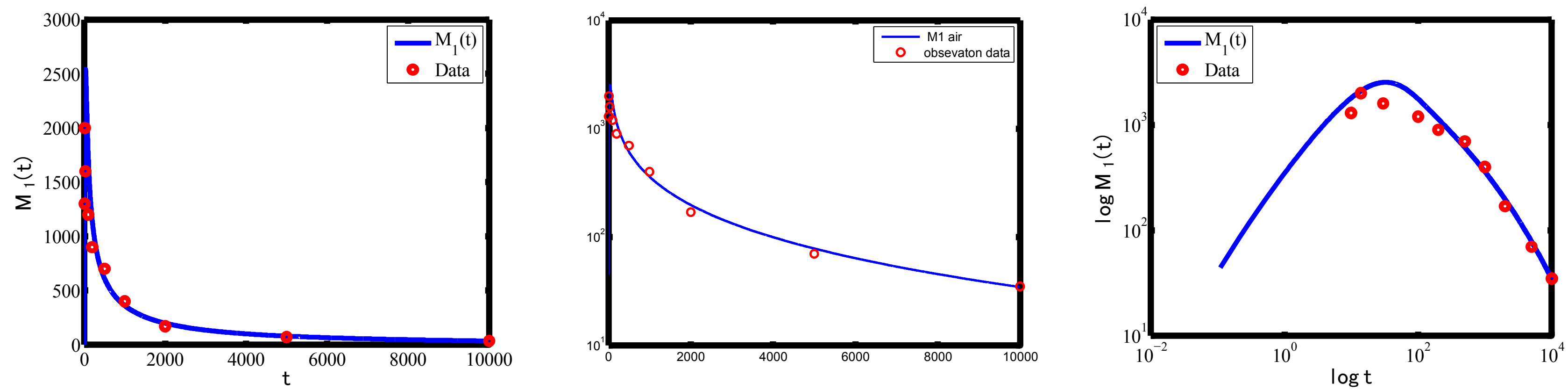
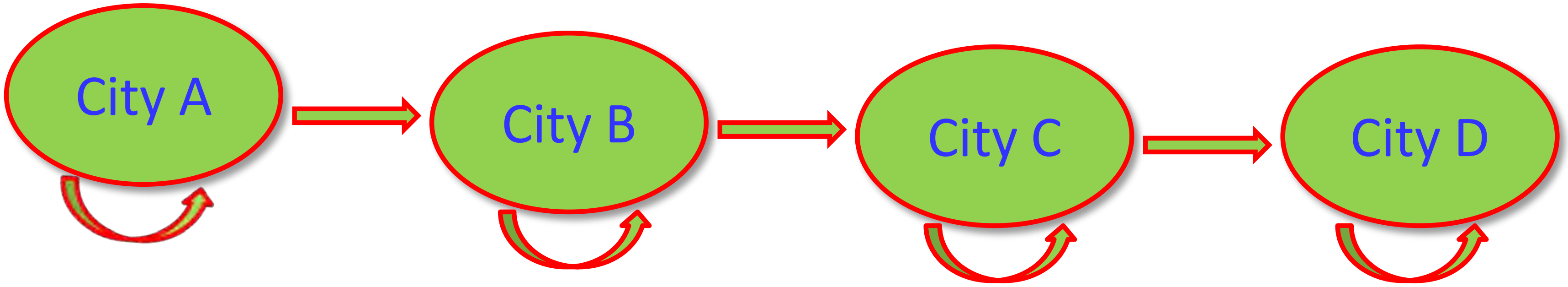


Figure: Left: Plot of $M_1(t)$ and the data. Middle: Semilog plot. Right: log-log plot.

Improvement of 4 Boxes Model



- One way propagation
- Every city could be viewed as a point
- Every city has a known absorption coefficient
- Propagation needs time

Improvement of Four Boxes Model

$$\frac{dC_1(t)}{dt} = -a_1(t)C_1(t) + f(t) \qquad - k_1(t)C_1(t), t > 0,, \quad (1\text{-a})$$

$$\frac{dC_2(t)}{dt} = -a_2(t)C_2(t) + k_1(t)C_1(t) - k_2(t)C_2(t), t > T_1,, \quad (1\text{-b})$$

$$\frac{dC_3(t)}{dt} = -a_3(t)C_3(t) + k_2(t)C_2(t) - k_3(t)C_3(t), t > T_2,, \quad (1\text{-c})$$

$$\frac{dC_4(t)}{dt} = -a_4(t)C_4(t) + k_3(t)C_3(t) - k_4C_4(t), t > T_3, \quad (1\text{-d})$$

where $a_i(t) > 0$ are known absorption coefficient. $k_i(t) > 0$ are the propagation ability from one city to the next one.

Assumption: We know the observation data $Z(t)$ for $t \in [T_3, T_n]$.

Inverse Problem: Determine $\{k_i(t)\}_{i=1}^{3}$ such that $C_4(t) = Z(t)$.

Improvement of Four Boxes Model

Procedure:

- $C_1(T_3)$ + Eq.(1-a):
  Solving $C_1(t)$ in $t \in [0, T_1]$ to get $C_1(T_1)$

Improvement of Four Boxes Model

Procedure:

- $C_1(T_3)$ + Eq.(1-a):
  Solving $C_1(t)$ in $t \in [0, T_1]$ to get $C_1(T_1)$
- $C_2(T_1) = C_1(T_1)$ and Eq.(1-b):
  Solving $C_2(t)$ in $t \in [T_1, T_2]$ to get $C_2(T_2)$

Improvement of Four Boxes Model

Procedure:

- $C_1(T_3)$ + Eq.(1-a):
  Solving $C_1(t)$ in $t \in [0, T_1]$ to get $C_1(T_1)$
- $C_2(T_1) = C_1(T_1)$ and Eq.(1-b):
  Solving $C_2(t)$ in $t \in [T_1, T_2]$ to get $C_2(T_2)$
- $C_3(T_2) = C_2(T_2)$ and Eq.(1-c):
  Solving $C_3(t)$ in $t \in [T_2, T_3]$ to get $C_2(T_3)$
- $C_4(T_3) = C_3(T_3)$ and Eq.(1-d):
  Solving $C_4(t)$ in $t \in [T_3, T_n]$ to get $C_4(t)$ for $t \in [T_3, T_n]$

Improvement of Four Boxes Model

Procedure:

- $C_1(T_3)$ + Eq.(1-a):
  Solving $C_1(t)$ in $t \in [0, T_1]$ to get $C_1(T_1)$
- $C_2(T_1) = C_1(T_1)$ and Eq.(1-b):
  Solving $C_2(t)$ in $t \in [T_1, T_2]$ to get $C_2(T_2)$
- $C_3(T_2) = C_2(T_2)$ and Eq.(1-c):
  Solving $C_3(t)$ in $t \in [T_2, T_3]$ to get $C_2(T_3)$
- $C_4(T_3) = C_3(T_3)$ and Eq.(1-d):
  Solving $C_4(t)$ in $t \in [T_3, T_n]$ to get $C_4(t)$ for $t \in [T_3, T_n]$

Optimization:

Assume $k_i(t)$ have special form with unknown parameters, we can solve the following problem to get $k_i(t)$

$$\min \int_{T_3}^{T_n} |C_4(t) - Z(t)|^2 dt.$$

Numerical Simulations

Set $a_i(t) = \exp(-K * t_i^{\beta})$, $f_1(t) = A\exp(-t^{\gamma})$, $k_i(t) = \frac{\alpha_i}{t + T_i}$,
$T_i = 2$,

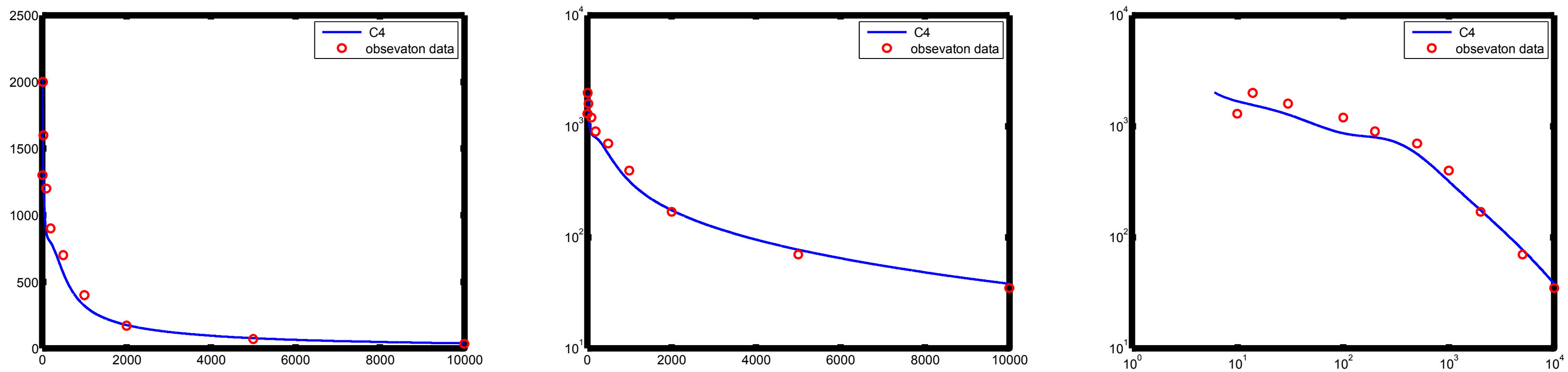


Figure: Left: Plot of $C_4(t)$ and the data. Middle: Semilog plot. Right: log-log plot.

Parameter Identification by Using the One-box Model

One-box Model

$$\begin{cases} M'(t) + p(t)M(t) = q(t), & t > 0 \\ M(0) = 0. \end{cases}$$

where $p(t) = \frac{\alpha}{t+☒} + k$ and $q(t) = C\exp(-(t+☒)^{\beta})$. Here, the parameters α, $☒$, k, C, β are all undetermined.

The explicit solution is as follows

$$M(t) = \frac{C\int_0^t \exp(ks-(s+☒)^{\beta})(s+☒)^{\alpha}\,ds}{\exp(kt)(t+☒)^{\alpha}}.$$

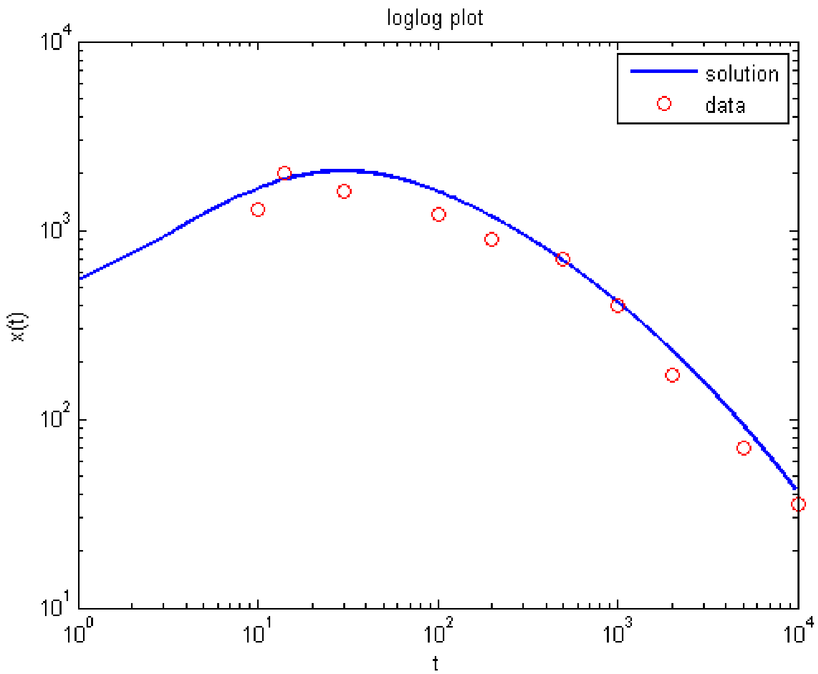
Parameter Identification: $☒ = t_e$, $k = 1/☒$, $C \approx x(t_e)$ and $\alpha \approx 1$.

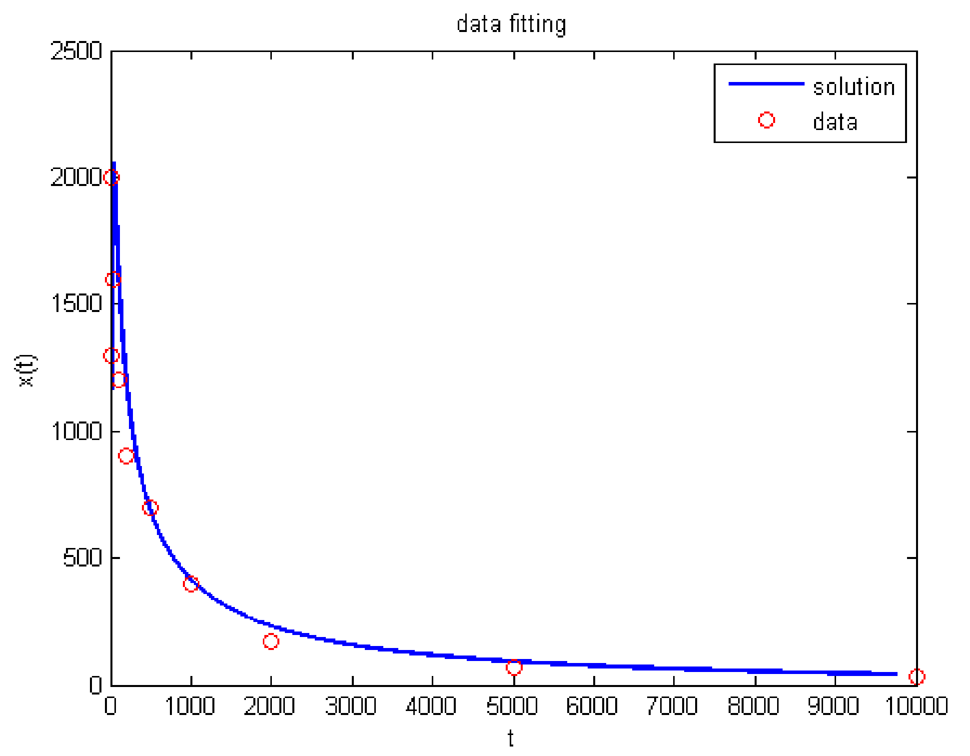
Asymptotic Expansion for determining β, when *t* is large

$$x(t) \approx C\left(\frac{1}{k} + \frac{\beta}{k^2}(t+☒)^{\beta-1}\right)\exp(-(t+☒)^{\beta}),\quad \beta < 1.$$

Numerical method: Bisection of $\beta \Rightarrow$ $\beta = 0.2$.

In this example, $\alpha = 1$, $\beta = 0.2$, $☒ = 14$, $C = 1500$.


loglog plot
solution
data
x(t)
t


data fitting
solution
data
x(t)
t

One Box Model

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -Ku.$$

Here we assume

$$K(t) = \frac{\alpha}{t+\tau} \; (\alpha, \tau > 0).$$

$$K(t) \approx \begin{cases} \dfrac{\alpha}{\tau}, & t \ll \tau, \\ \dfrac{\alpha}{t}, & t \gg \tau. \end{cases}$$

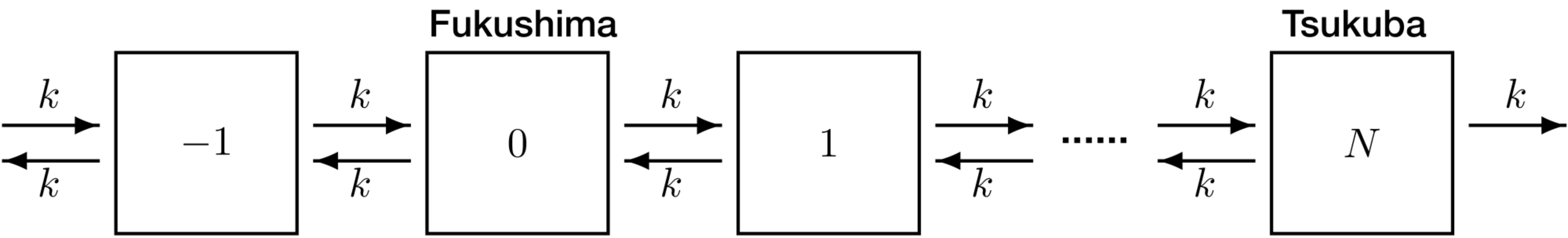
One Box Model

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -Ku. \quad K(t) = \frac{\alpha}{t+\tau} \ (\alpha, \tau > 0).$$

$$u(t) = u(0)\tau^{\alpha}(t+\tau)^{-\alpha}$$

$$u(t) \approx u(0) \times \begin{cases} e^{-\alpha t/\tau}, & t \ll \tau, \\ \left(\frac{\tau}{t}\right)^{\alpha}, & t \gg \tau. \end{cases}$$

Multi-box model



$$
\begin{cases}
\dfrac{du_{-N}}{dt} = -2ku_{-N} + ku_{-N+1}, \\
\dfrac{du_j}{dt} = -2ku_j + k(u_{j-1} + u_{j+1}), \ (j = -N+1, \dots, N-1) \\
\dfrac{du_N}{dt} = -2ku_N + ku_{N-1}.
\end{cases}
$$

Multi-box model

$$\begin{cases} \dfrac{du_{-N}}{dt} = -2ku_{-N} + ku_{-N+1}, \\ \dfrac{du_j}{dt} = -2ku_j + k(u_{j-1} + u_{j+1}), \ (j = -N+1, \ldots, N-1) \\ \dfrac{du_N}{dt} = -2ku_N + ku_{N-1}. \end{cases}$$

We assume that

$$k = \frac{K(t)}{(\Delta x)^2}$$

where Δx is a distance between two boxes:

$$\Delta x = \frac{\text{distance between Fukushima and Tsukuba}}{N}.$$

Multi-box model

In the limit of N to infty, our box model is expressed by

$$\begin{cases} \partial_t u = K(t)\partial_x^2 u, & x \in \mathbb{R}, \quad t > 0, \\ u = a(x), & x \in \mathbb{R}, \quad t = 0. \end{cases}$$

Assuming that $a(x) = \delta(x)$, we have

$$u(x,t) = \frac{1}{\sqrt{4\pi\alpha}} \left[\log\left(1 + \frac{t}{\tau}\right)\right]^{-1/2} \exp\left[\frac{-x^2}{4\alpha\log\left(1+\frac{t}{\tau}\right)}\right].$$

Multi-box model

$$u(x,t) = \frac{1}{\sqrt{4\pi\alpha}}\left[\log\left(1+\frac{t}{\tau}\right)\right]^{-1/2}\exp\left[\frac{-x^2}{4\alpha\log\left(1+\frac{t}{\tau}\right)}\right].$$

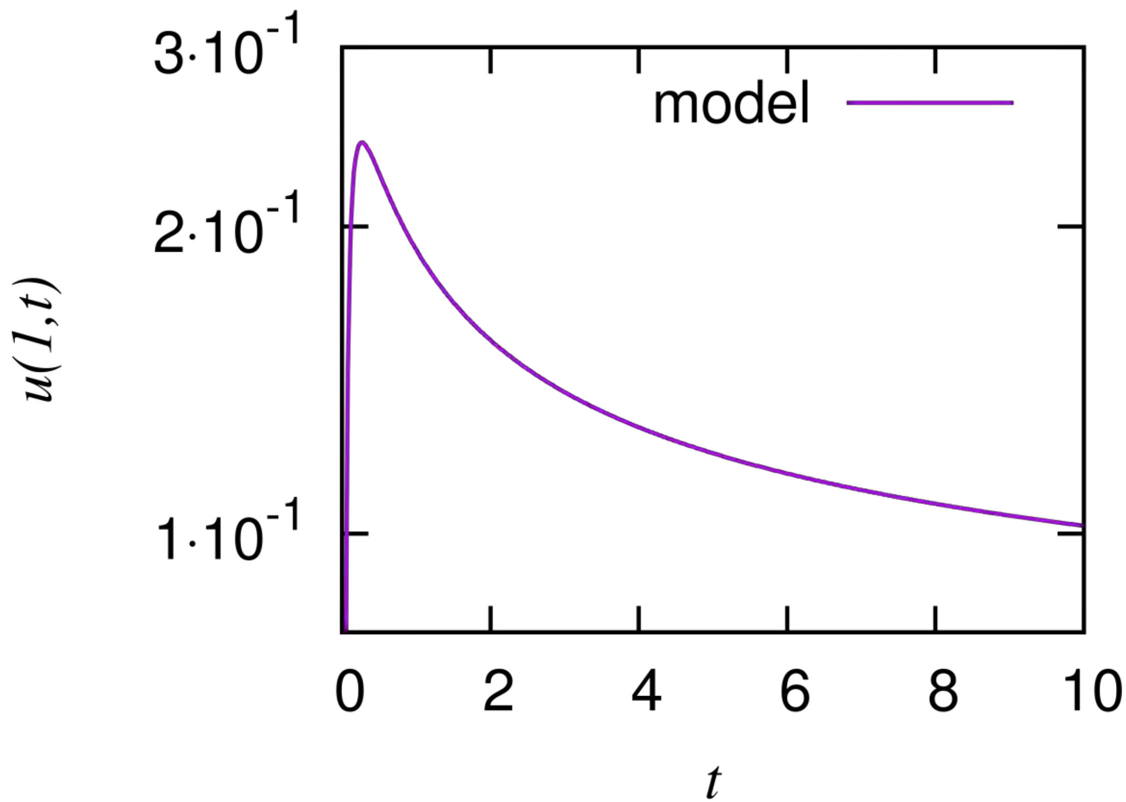
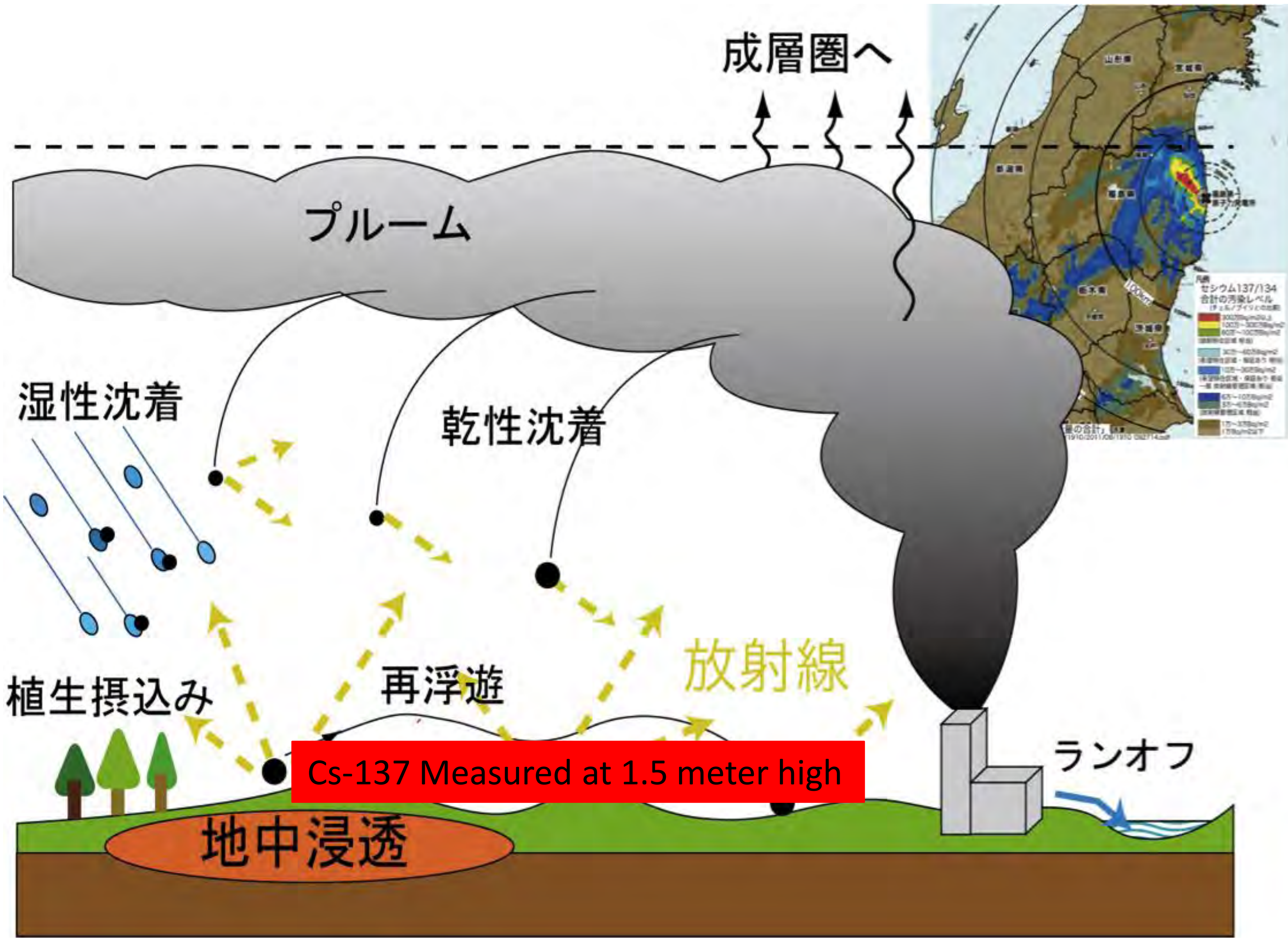


Figure 1: the graph of $u(t)$ when $\alpha = 4$ and $\tau = 2$.

TOPIC 2: ADVECTION EQUATION MODELING OF RADIOACTIVE MATERIAL IN THE AIR


成層圏へ
プルーム
湿性沈着
乾性沈着
放射線
再浮遊
植生摂込み
ランオフ
地中浸透
Cs-137 Measured at 1.5 meter high

(x_0, y_0): Location of Fukushima Daiichi Nuclear Power Plant
t: Elapsed day after the accident
$C(x, y, t)$: Concentration of Cs-137 in the air at $z = 1.5$ meter high
$v_1(t)$, $v_2(t)$: Effective wind velocity
Initial Condition: $C(x, y, 0) = \delta(x - x_0)\delta(y - y_0)$
Boundary Condition: $C(\pm\infty, \pm\infty, t) = 0$

==============================

We treat the 2-D problem because

$$C(x, y, z, t) \simeq C(x, y, t) \times const. \qquad (Gavrilov)$$

Advection Equation of Cs-137:

$$\frac{\partial C(x,y,t)}{\partial t} + v_i(t)\frac{\partial C(x,y,t)}{\partial x_i} + \lambda_{env}(t)C(x,y,t) + \lambda_{dec}C(x,y,t) = 0. \quad (1)$$

===== CONSTANT REMOVAL PROCESSES =====

$$\lambda_{dec} = \lambda_{rad} + \lambda_{inf} + \lambda_{runoff} + \ldots$$

λ_{rad}: radioactive decay rate
λ_{inf} : soil-infiltration rate
λ_{runoff}: runoff rate
(all the removal processes with the first-order kinetics)

Can we generate the following decay by choosing appropriate coefficients?

$$Ae^{-\lambda t}t^{-\gamma}$$

Discussions on model equations describing specified decay rates

Backgrounds

In order to quantitatively interpret observed data of cesium and predict, we need mathematical model equations. Here we summarize them among linear equations and test feasibility.

(I) Box model (Subject I)

(II) Advection model (Subject 2)

(III) Diffusion equation

(IV) Fractional diffusion equation

(I) Box model

$$\frac{du}{dt}(t) = A(t)u(t), \quad t > 0.$$

Here $u = (u_1, \cdots, u_n)^T$, $A(t)$: $N \times N$ symmetric matrix.

Given $\mu_1(t), ..., \mu_N(t)$ (e.g., $\mu_k(t) = c_k t^{-\gamma_k}$), determine $A(t)$ such that $u_k(t) \sim \mu_k(t)$ as $t \to \infty$, $k = 1, 2, ..., N$.

(II): Advection equation

$$\begin{cases} \partial_t C(x,t) + v_1(t)\partial_1 C + v_2(t)C + q(x,t)C = 0, \\ C(x,0) = \delta(x) : \quad \text{Dirac delta} \end{cases}$$

Here $x = (x_1, x_2) \in \mathbb{R}^2$

Remark. $q(x,t) = \lambda_{env}(x,t) + \lambda_{dec}$.

Determine $q(x,t)$ such that

$$\int_{\mathbb{R}^2} C(x,t)dx \sim Ae^{-\lambda t}t^{-\gamma} \quad \text{for large } t > 0.$$

(III): Diffusion equation with time dependent diffusivity

$$
\begin{cases}
\partial_t u = p(t)\Delta u, \ t > 0, \ x \in \Omega, \\
u = 0 \quad \text{on } \partial\Omega, \quad u(x,0) = a(x), \quad x \in \Omega.
\end{cases}
$$

Ω: bounded domain.

Given $\mu(t)$, determine $p(t)$ such that $u(x,t) \sim \mu(t)$ as $t \to \infty$.

(IV) Fractional diffusion equation

- multiple orders

$$\sum_{k=1}^{N} q_k \partial_t^{\alpha_k} = u = \Delta u$$

- distributed orders

$$\int_0^1 \mu(\xi) \partial_t^{\xi} u d\xi = \Delta u$$

(II) Advection equation

By method of characteristics:

$$\begin{cases} \partial_t C(x,t) + v_1(t)\partial_1 C + v_2(t)C + q(x,t)C = 0, \quad x = (x_1, x_2) \in \mathbb{R}^2,\ t > 0, \\ C(x,0) = \delta(x) : \text{ Dirac delta.} \end{cases}$$

Then:

$$\int_{\mathbb{R}^2} C(x,t)dx = \exp\left(-\int_0^t q\left(\int_0^s v_1(\xi)d\xi, \int_0^s v_2(\xi)d\xi, s\right) ds\right).$$

Example. Let $v_1(t) = 2t$, $\mu_2 = 1$ and $q(x,t) = \frac{x_1}{1+t} + r_3$. Then

$\int_{\mathbb{R}^2} C(x,t)dx = \frac{1}{1+t} e^{-\frac{t^2}{2} + (1-r_3)t}$.

(III) Diffusion equation

$$\begin{cases} \partial_t u = p(t)\Delta u, \ t > 0, \ x \in \Omega \text{bounded domain}, \\ u = 0 \quad \text{on } \partial\Omega, \quad u(x,0) = a(x), \quad x \in \Omega. \end{cases}$$

Then

$$\|u(\cdot,t)\|^2_{L^2(\Omega)} = \sum_{k=1}^{\infty} (a, \varphi_k)^2 \exp\left(-2\lambda_k \int_0^t p(\xi)d\xi\right).$$

Here $\Delta\varphi_k = -\lambda_k \varphi_k$, $\lambda_k > 0$, $\varphi_k = 0$ on $\partial\Omega$, $\|\varphi_k\| = 1$ and $(\varphi_k, \varphi_\ell) = 0$ if $k \neq \ell$.

Example: Let $p(t) = \frac{\alpha}{1+\beta t}$, $\alpha, \beta > 0$. **Then** $\|u(\cdot,t)\|_{L^2(\Omega)} \le \|a\|(1+\beta t)^{-\frac{\lambda_1 \alpha}{\beta}}$

(IV) Fractional diffusion equation

Fractional diffusion equation can simulate slow diffusion? $\Longrightarrow$ **YES**

$$\|u(\cdot,t)\|_{L^2(\Omega)} = \begin{cases} O(t^{-\alpha}), & \text{single term,} \\ O(t^{-\alpha_1}), & \text{multi-term,} \\ O((\log t)^{-1}), & \text{distributed order.} \end{cases}$$

α_1: minimum order of derivatives

Fractional diffusion equation can simulate slow diffusion from the polynomial to logarithmic decay.

- Distributed order case. We assume $\mu(\alpha) = \mu(0) + o(\alpha^\delta)$ with some $\delta > 0$ as $\alpha \to 0$, $\mu \in C[0,1]$, ≥ 0, $\mu(0) > 0$. Then

  $$\|u(\cdot,t)\|_{L^2(\Omega)} = o((\log t)^{-1}) \Rightarrow u \equiv 0$$

  (Li-Luchko-Yamamoto 2014): Log. decay is the best possible

- Single case.

  $$\|u(\cdot,t)\|_{L^2(\Omega)} = o(t^{-\alpha}) \Longrightarrow u \equiv 0.$$

Summary

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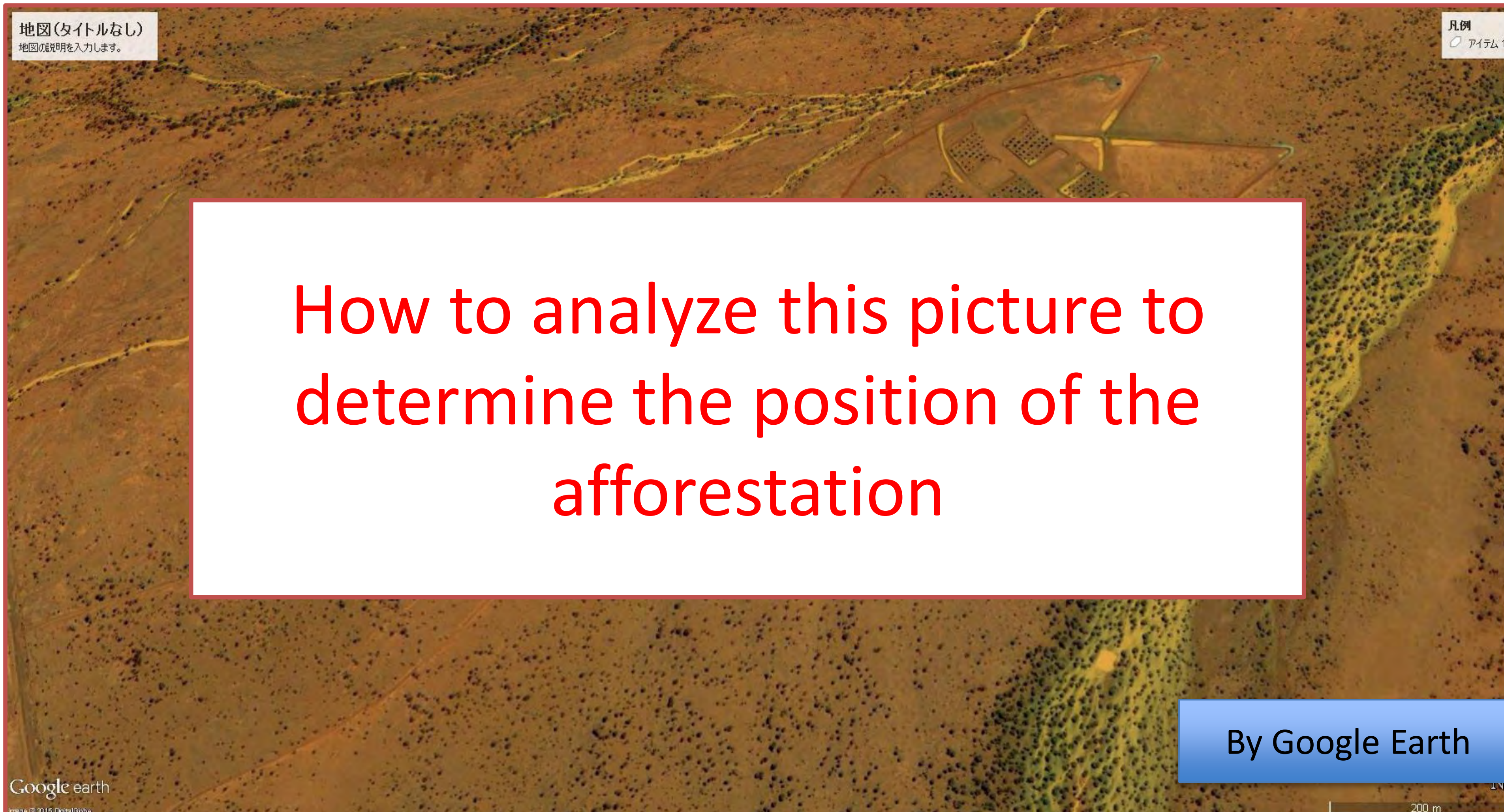
- (I): Input-out linear system
- (II) - (III): Assuming some physical equations, we make data fitting for coefficients.
- (IV): Fractional diffusion equation is based on continuous time random walk.

Physical backgtounds: (I) < (II), (III) < (IV)

TOPIC 3: SATELLITE IMAGE ANALYSIS FOR EFFECTIVE AFFORESTATION

Problem



How to analyze this picture to determine the position of the afforestation

By Google Earth

Proposals

- Application of **generalized polarization tensor**

→Cluster the trees in several group

- Blurring using **time-cone model**

→Find a void area

Our Problem

Let Ω be a one of the grid in the region of interest. Let $T = \bigcup_{k=1}^{N} T_k$, where T_k is well seperated small domain (Tree or Trees). We want to approximate T by relatively large domains $\tilde{T} = \bigcup_{k=1}^{M} \tilde{T}_k$ $(M << N)$ in a 'good' way.

Suggestion : Use the Generalized Polariztion Tensors

Properties of GPT

- The full set of GPTs determines T uniquely.
- Finite number of GPTs can give the good approximation of T. For example, if we use GPT $m_{\alpha\beta}$ for $|\alpha| = |\beta| = 1$ (Polarization Tensor), then we can approximate T by a ellipse.

Generalized Polarization Tensors (GPT)

Ω : electric conductive medium.

T_k : an inclusion with conductivity σ.

$$T = \bigcup_{k=1}^{N} T_k, \ \gamma = \chi(\Omega \backslash T) + (\sigma - 1)\chi(T).$$

Consider the following transmission problem:

$$\begin{cases} \nabla \cdot \gamma \nabla u = 0 & \text{in } \mathbb{R}^2, \\ u(x) - h(x) = O(|x|^{1-d}) & \text{as } |x| \to \infty. \end{cases}$$

Then solution u can be represented by a single layer potentials

$$u(x) = h(x) + \sum_{k=1}^{N} \mathcal{S}_{\partial T_k}[\phi^{(k)}](x),$$

where $\phi^{(k)}$ satisfies

$$(\lambda I - \mathcal{K}^*_{\partial T_k})[\phi^{(k)}] - \sum_{s \neq k} \frac{\partial \mathcal{S}_{\partial T_k}[\phi^{(s)}]}{\partial \nu^{(k)}}\Big|_{\partial T_k} = \frac{\partial h}{\partial \nu^{(k)}}\Big|_{\partial T_k} \text{ on } \partial T_k.$$

Here we denote

$$\lambda = \frac{\sigma + 1}{2(\sigma - 1)}.$$

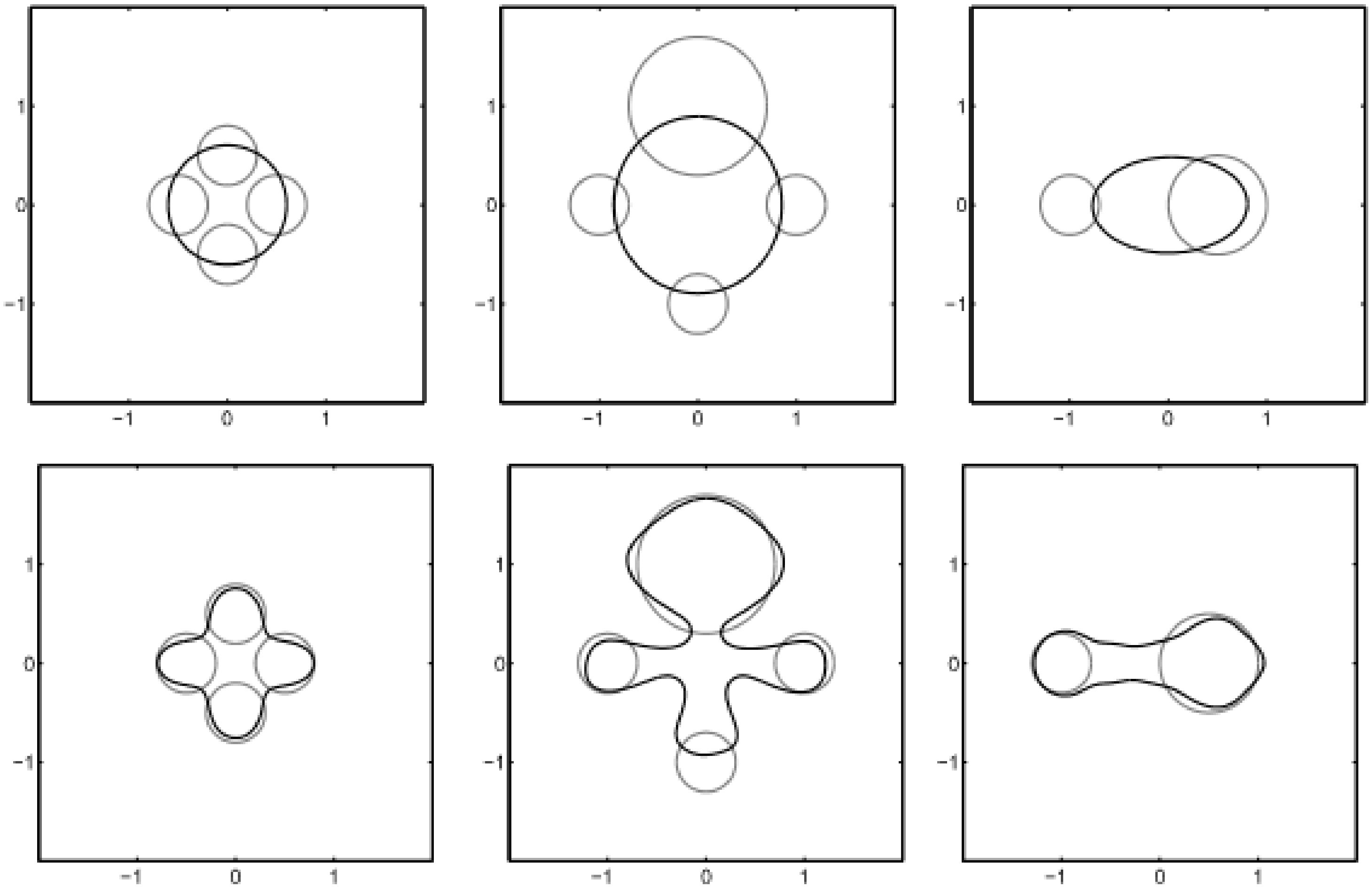
For the multi-index $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2) \in \mathbb{Z}^2_+$, let $\phi^{(k)}_\alpha$ be the solution of

$$(\lambda I - \mathcal{K}^*_{\partial T_k})[\phi^{(k)}_\alpha] - \sum_{s \neq k} \frac{\partial \mathcal{S}_{\partial T_k}[\phi^{(s)}_\alpha]}{\partial \nu^{(k)}} \Big|_{\partial T_k} = \frac{\partial x^\alpha}{\partial \nu^{(k)}} \Big|_{\partial T_k} \text{ on } \partial T_k.$$

Then the generalized polarization tensor defined $m_{\alpha\beta}$ to be

$$m_{\alpha\beta} = \sum_{k=1}^N \int_{\partial T_k} x^\beta \phi^{(k)}_\alpha(x) d\sigma(x).$$

Some numerical results of the GPT



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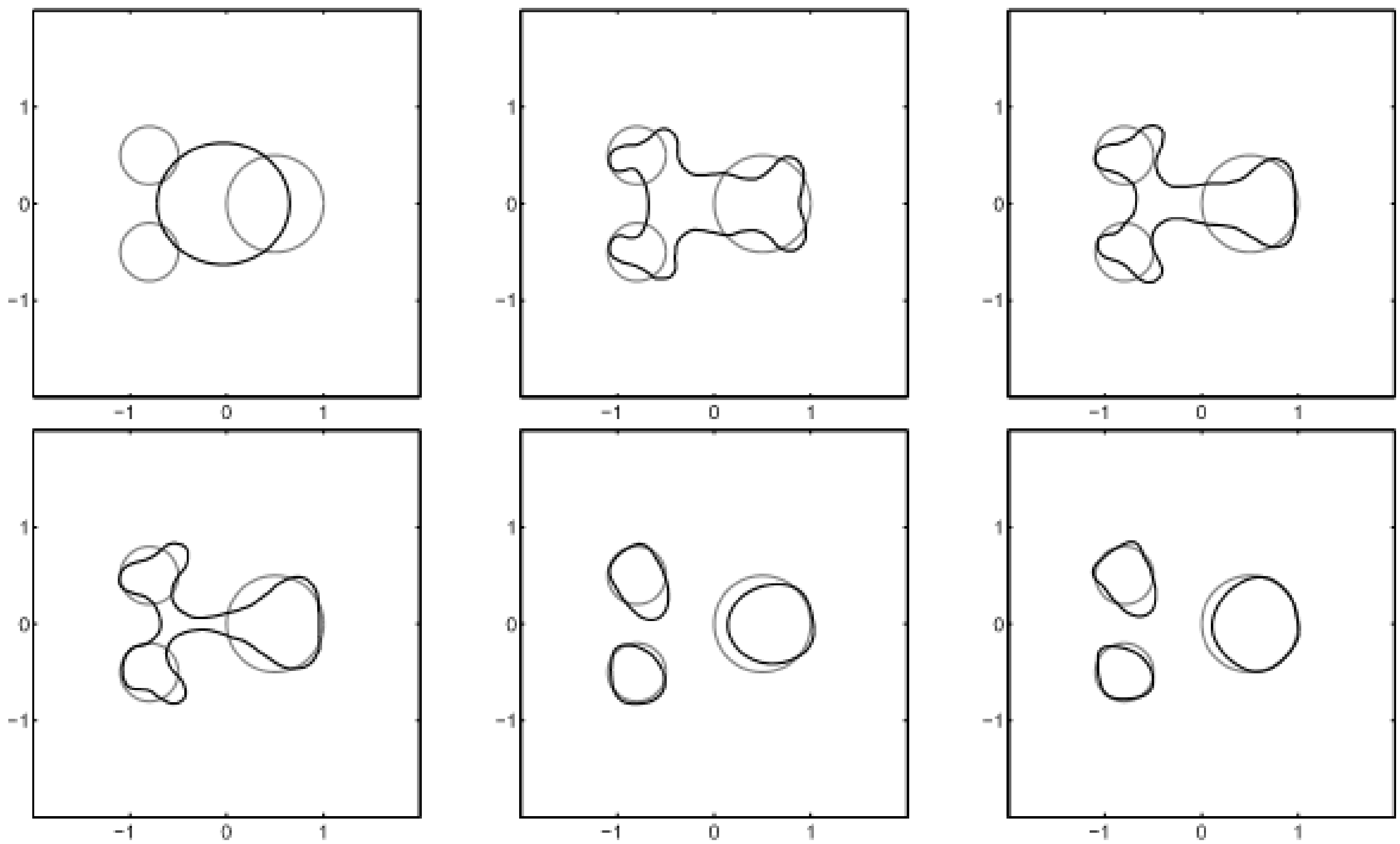

Time cone model

Idea Stationary scatter diagram $\Longrightarrow$ Time-evolution system describing density distribution (blurring/smoothing effect).

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One candidate Cahn's time cone model (1995) for crystallization.

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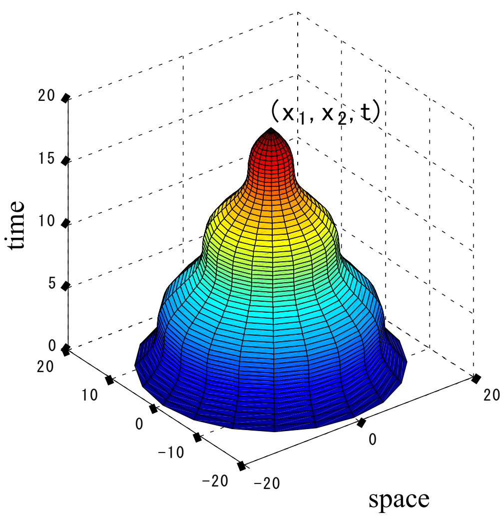
Time cone (domain of dependence):

$$\Omega_\rho(x,t) := \left\{(y,s);\ 0 < s < t,\ |y-x| < \int_s^t \rho(\tau)\,d\tau\right\}.$$



Equivalent wave-type equation[1]

Essence $u(x,t)$ is determined by the integrated effect of $\Psi(x,t)$ in the time cone $\Omega_\rho(x,t)$:

$$u(x,t) = \int_{\Omega_\rho(x,t)} \Psi(y,s)\,dyds \quad (x \in \mathbb{R}^3,\ t \geq 0).$$

[1] Y. Liu and M. Yamamoto, Appl. Anal., 93, 2014, 1297–1318.

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Lemma (Double wave equation)

Let $\rho(t) \equiv 1$. Then u satisfies

$$\begin{cases} (\partial_t^2 - \Delta)^2 u(x,t) = 8\pi \Psi(x,t) & (x \in \mathbb{R}^3,\ t > 0), \\ \partial_t^j u(x,0) = 0 & (x \in \mathbb{R}^3,\ j = 0,1,2,3). \end{cases}$$

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In our problem, we can choose

$$\Psi(x,t) = \chi_T(x), \quad T = \bigcup_{k=1}^{K} \{y \in \mathbb{R}^3;\ |y - x_k| < \varepsilon\}.$$

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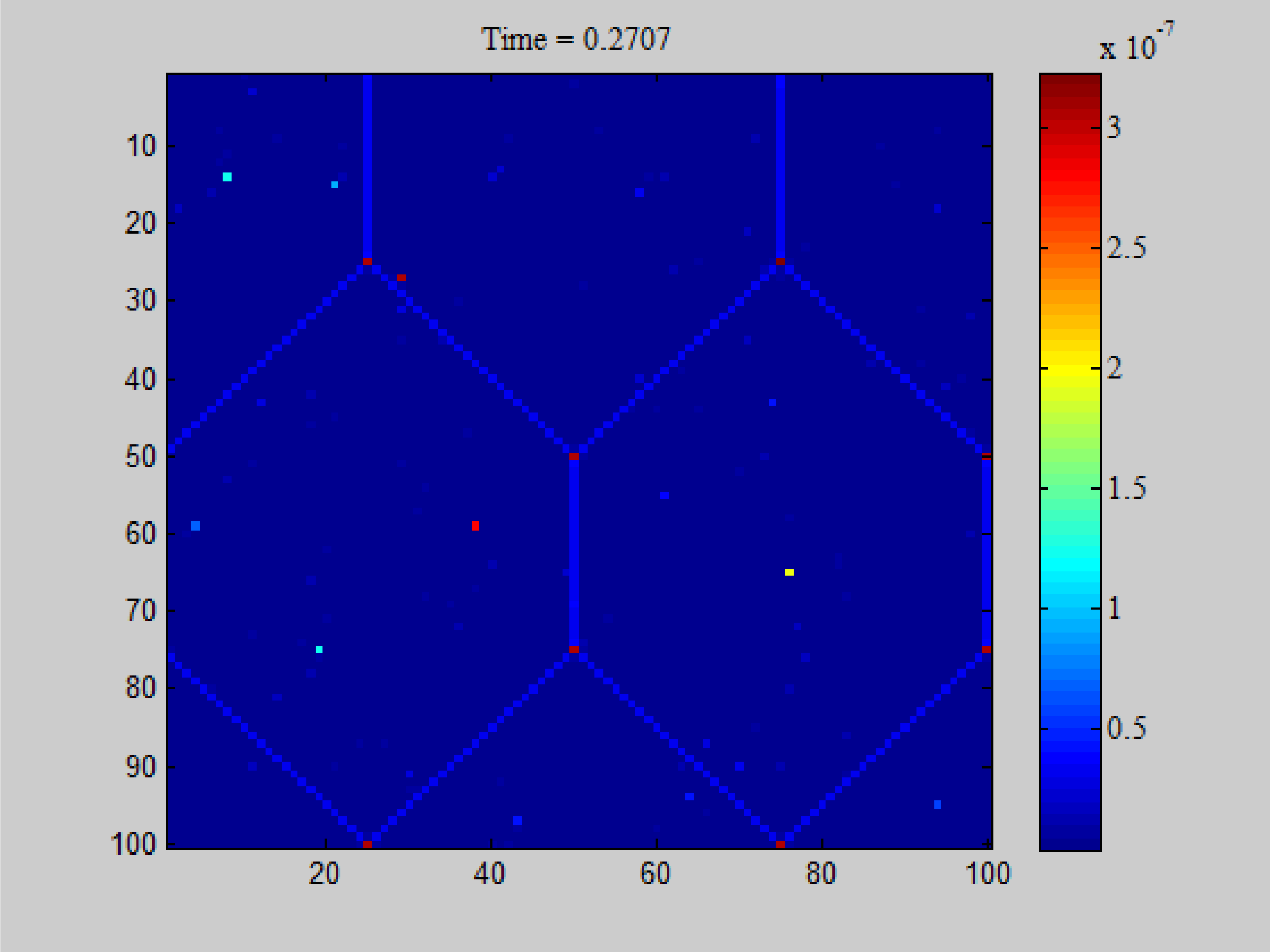
Numerical simulation

Fact We have a fast solver for 3-dim. double wave equations.

Example

$\chi_D(x)$: Hexagonal shape with random noise. ▶

(2-D Forward Problem)



Time = 0.2707
x 10^-7
3
2.5
2
1.5
1
0.5
10
20
30
40
50
60
70
80
90
100
20
40
60
80
100

Summary and Future works

Topic 1: Improvement of a box model and estimation of the initial explosion amounts

- We improve the box model introducing time-dependent coefficients.
- We do the data fitting.
- (Future work) Estimation of the initial explosion amounts
  - Inverse problem
  - Data assimilation

Summary and Future works

Topic 2:Advection equation modeling of radioactive material in the air

- We summarize and compare several PDEs which describe a polynomial decay phenomena.
- (Future works) More mathematical Analysis and simulation

Summaries and Future works

Topic 3: Satellite image analysis for effective afforestation

- We propose two approaches
  - Generalized polarization tensors
  - Time cone model
- (Future works) Simulation