H28.8．22.　数学協働プログラムWorkshop (ICCSE2016 Satellite)

A method to construct Mathematical models of time series data in dual space -Expression of balance function during Galvanic Vestibular Stimulation-

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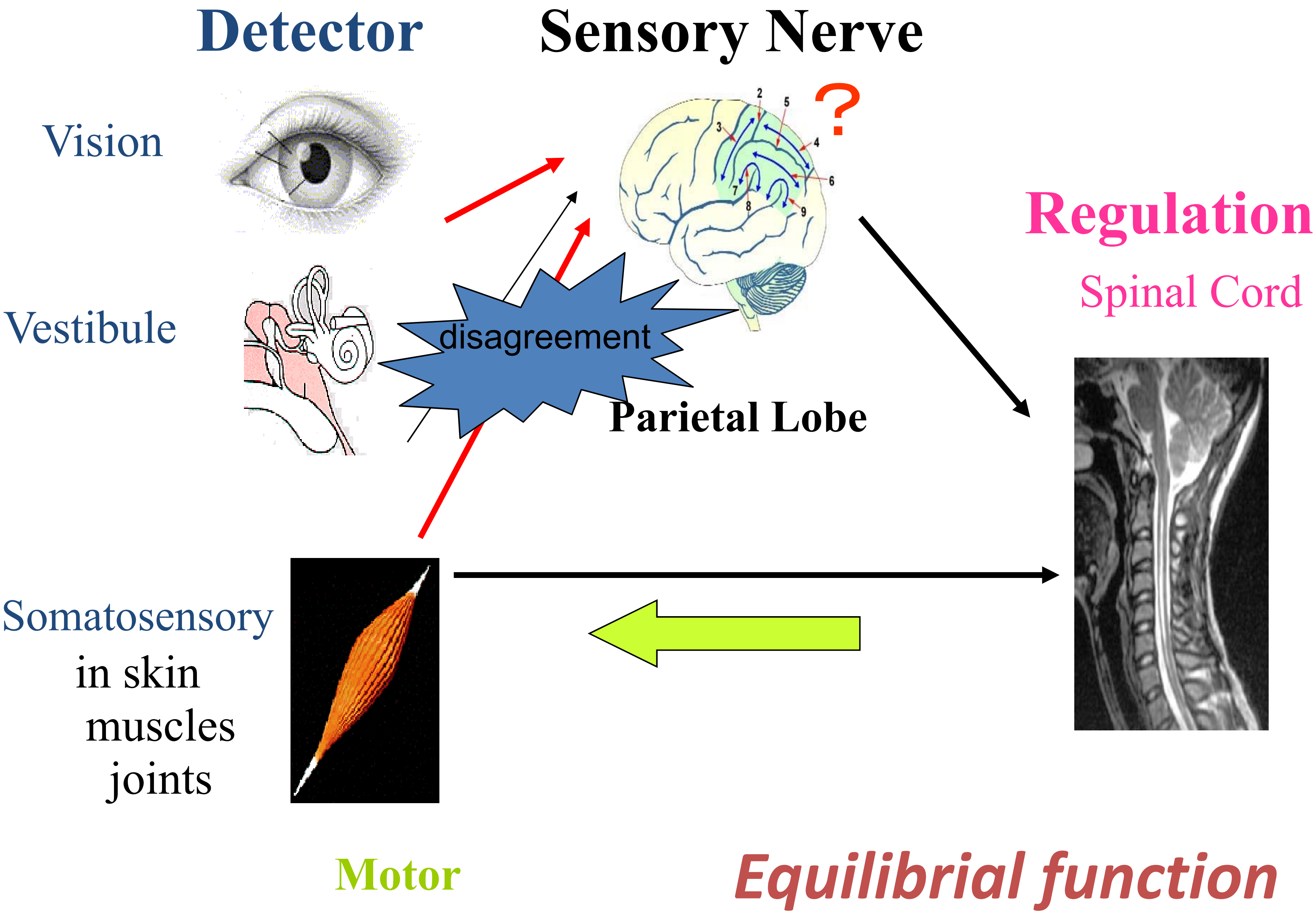
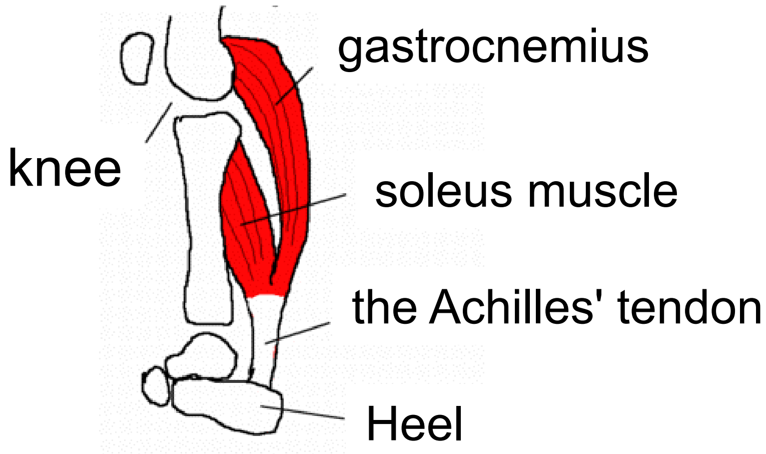
Neural System to Control Static Standing Posture

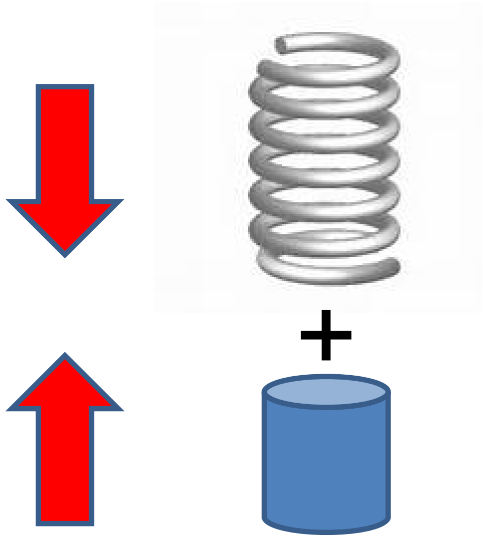
- Feedback system
- Ankle Stiffness
- Stretch Reflex


Detector
Sensory Nerve
Vision
Vestibule
disagreement
?
Parietal Lobe
Regulation
Spinal Cord
Somatosensory
in skin
muscles
joints
Motor
Equilibrial function

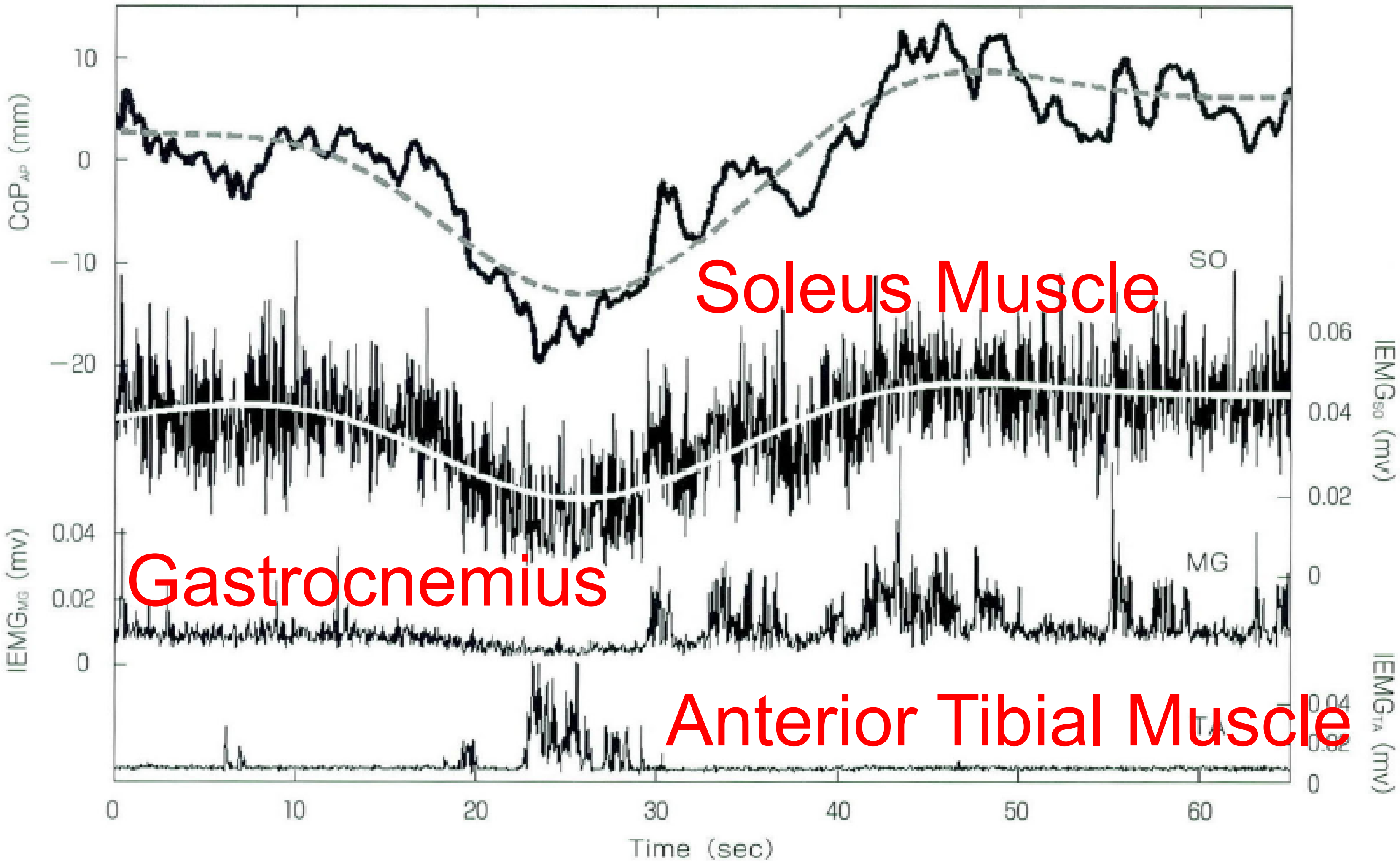
Inverted Pendulum

- Feedback system
- Ankle Stiffness
- Stretch Reflex

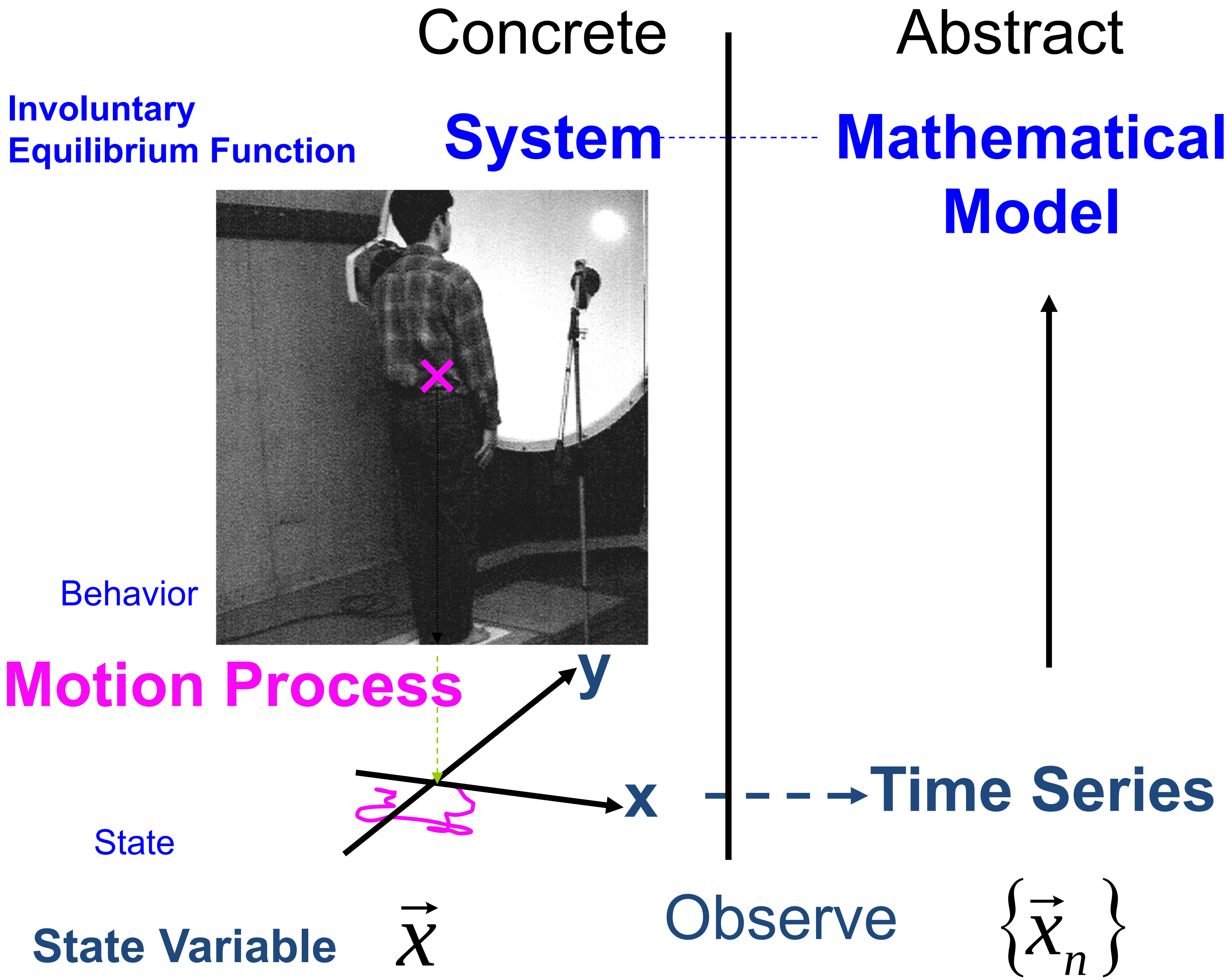




Body Sway & EMG

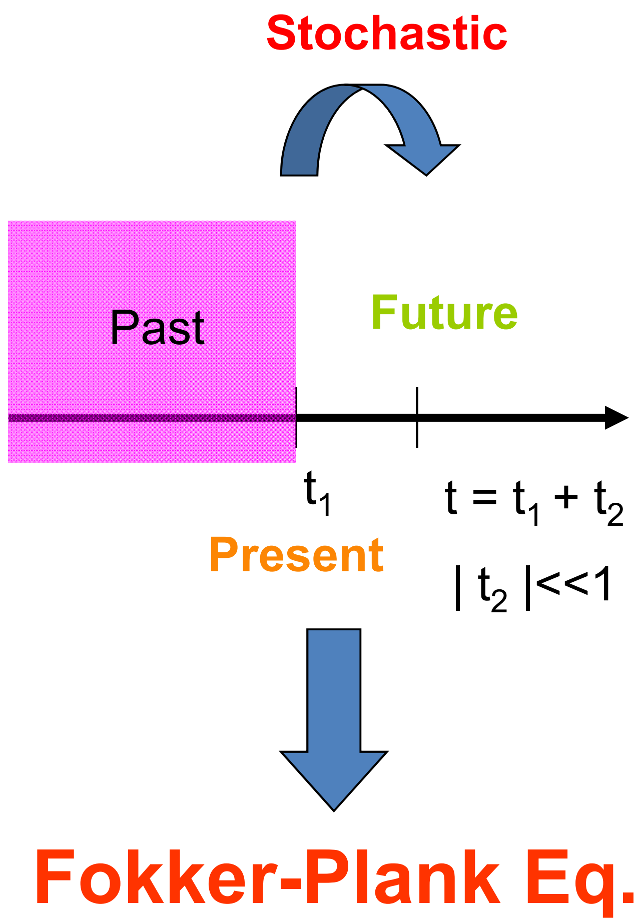


Nomura(2011)

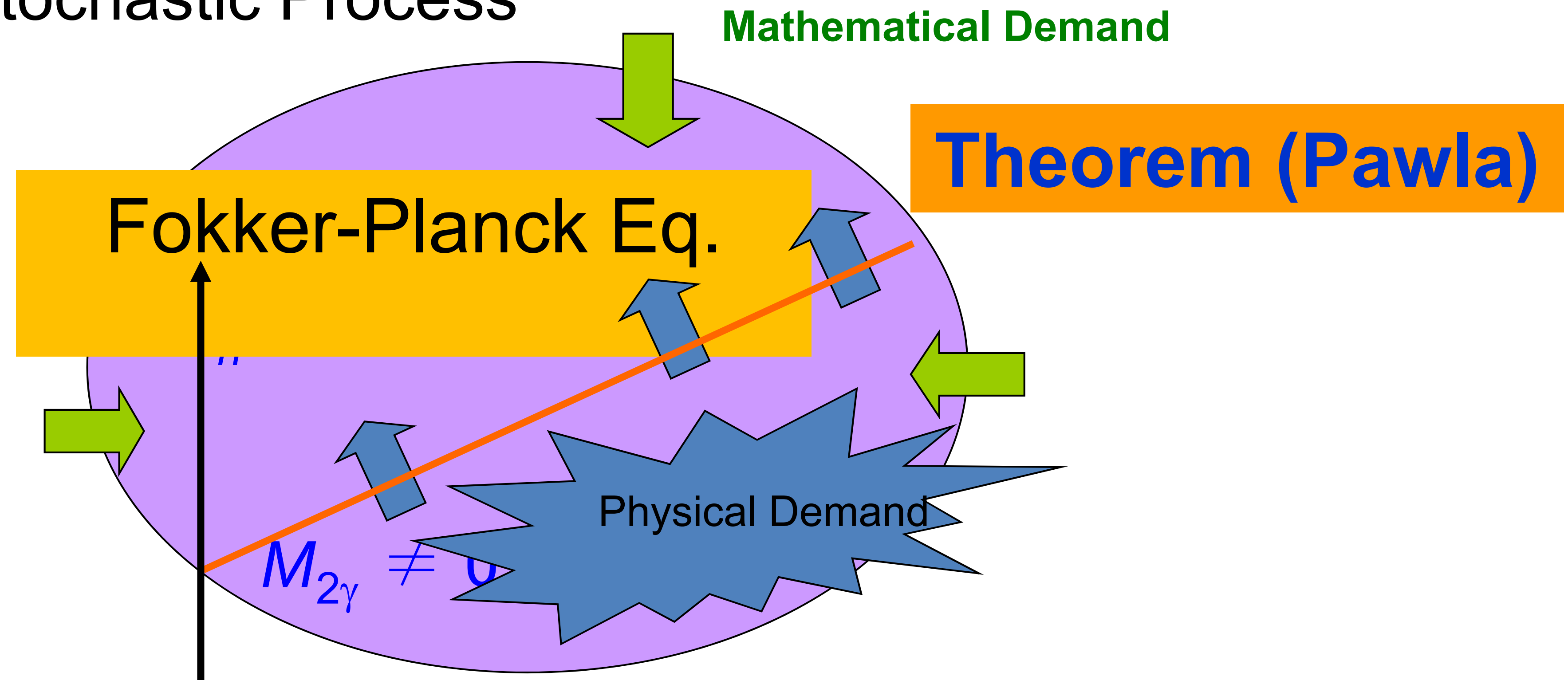

Concrete
Abstract
Involuntary
Equilibrium Function
System
Mathematical
Model
Behavior
Motion Process
y
x
Time Series
State
State Variable $\vec{x}$
Observe $\{\vec{x}_n\}$

Mathematical Modeling for motion processes

- Stationary,
  National Boundary Condition
- Markov Property
- Non-Anomalous Diffusion



Stochastic Process



Moment of Transition Probability

$$M_n(z) = \frac{\langle (\Delta z)^n \rangle}{\tau}$$

SDE: $\dfrac{dz}{dt} = F(z) + 1 w(t) = -grad V(z) + \underline{\underline{w(t)}}$

White Noise

Problem

- [Preprocessing]

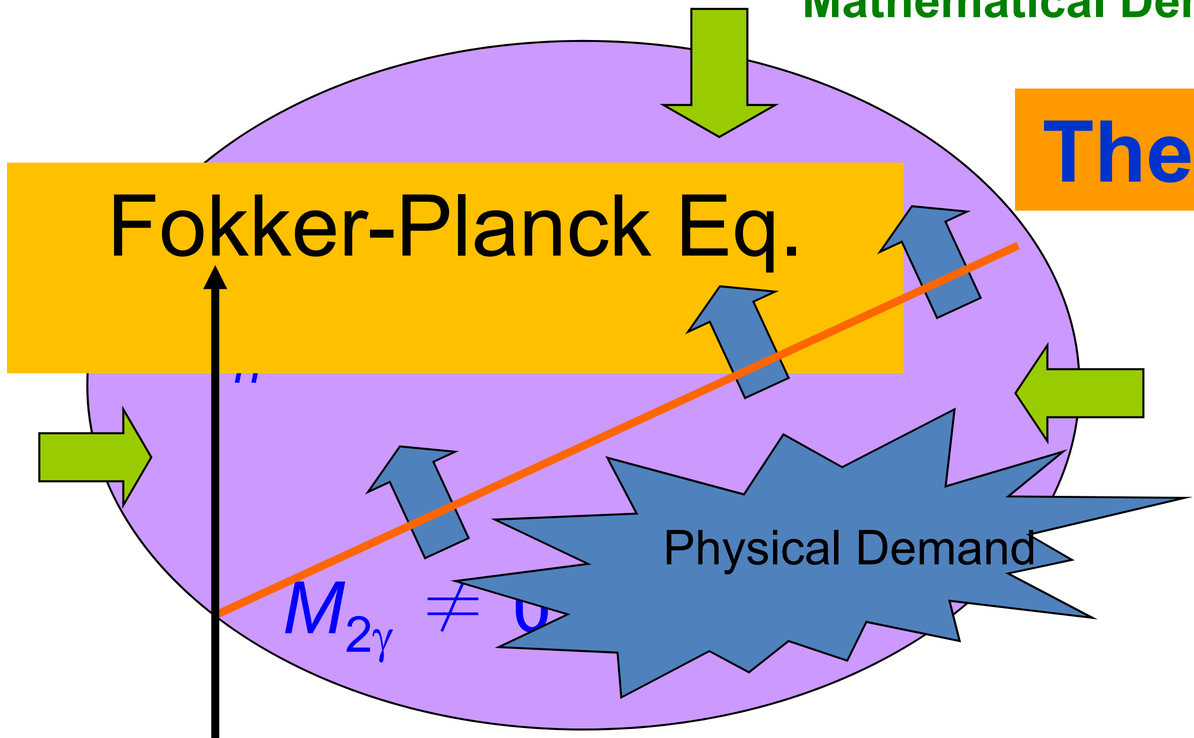
  Normalization for Time Series Data
- It is NOT appropriate to normalize time series data

as the following case…

1) To compare components in more than two-dim time series

2) To compare biological loads

Stochastic Process



Moment of
Transition Probability (TP)

$$M_n(z) = \frac{\langle (\Delta z)^n \rangle}{\tau}$$

SDE: $\frac{dz}{dt} = F(z) + \beta w(t) = -grad V(z) + \beta \underline{w(t)}$

ホワイトノイズ

1対1

Stochastic Differential Equation —— *Moment of TP* ——→ **Fokker-Plank Eq.**

① $SDE_z(\hat{a}(z),1) \longrightarrow FP_z(\hat{a}(z),1)$

Variable Transformation

$$z \mapsto x; \quad dz = \beta^{-1}dx, \quad g = \beta f$$

$SDE_x(a(x),\beta(x))$;

$$\frac{dx}{dt} = a(x) + \beta(x)w(t)$$

② $\longrightarrow FP_x(a(x),b(x))$

$FP_x(a,b)$;

$$\frac{\partial f(x|y,t)}{\partial t} = -\frac{\partial}{\partial x}\left[a(x)f(x|y,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[b(x)f(x|y,t)\right]$$

Scheme②

Stochastic Differential Equation

Moment of TP →

Fokker-Plank Eq.

One to One

$$W(z) = -\frac{\beta^2}{2}\ln\left(g(z)/C\right)$$

<Equilibrium Space>

Correspondence

Stationary Sol. (Prob. Dist.)

Observe

Sample Function

Polynomial Regression

Estimate <Potential>

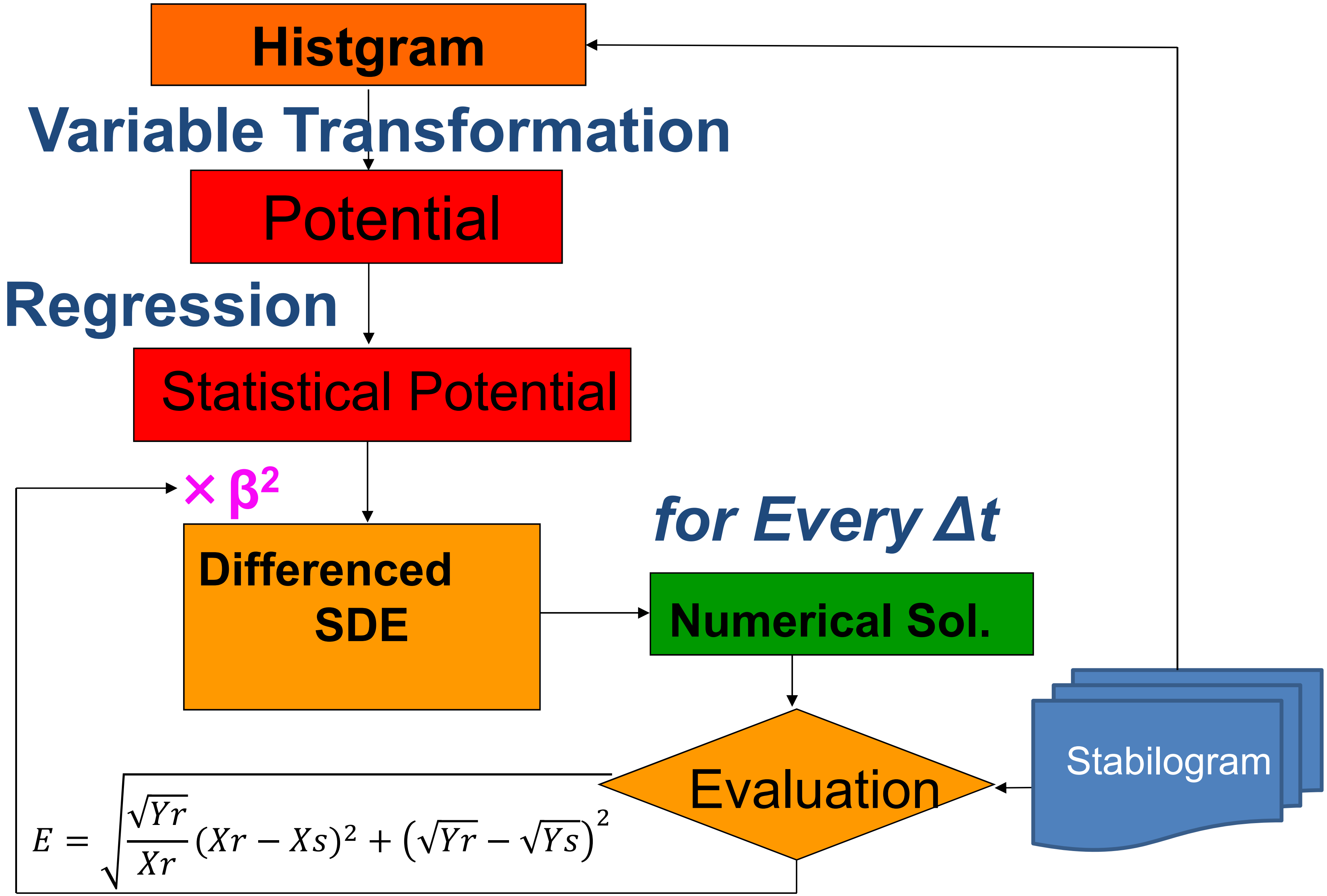
Parameter Estimation in Numerical Analysis

	Model	Potential	Numerical
①	$SDE_z(\hat{a}(z),1)$	$-\frac{1}{2}\ln g(z)$	Δt
①'	$SDE_z(a(z),\beta)$	$-\frac{1}{2}\ln g(z)$	$\beta\Delta t$
②	$SDE_x(a(x),\beta)$	$-\frac{\beta^2}{2}\ln f(x)$	$\beta,\quad \Delta t$

Polynomial Regression (Model, Potential)

Evaluation (Numerical)

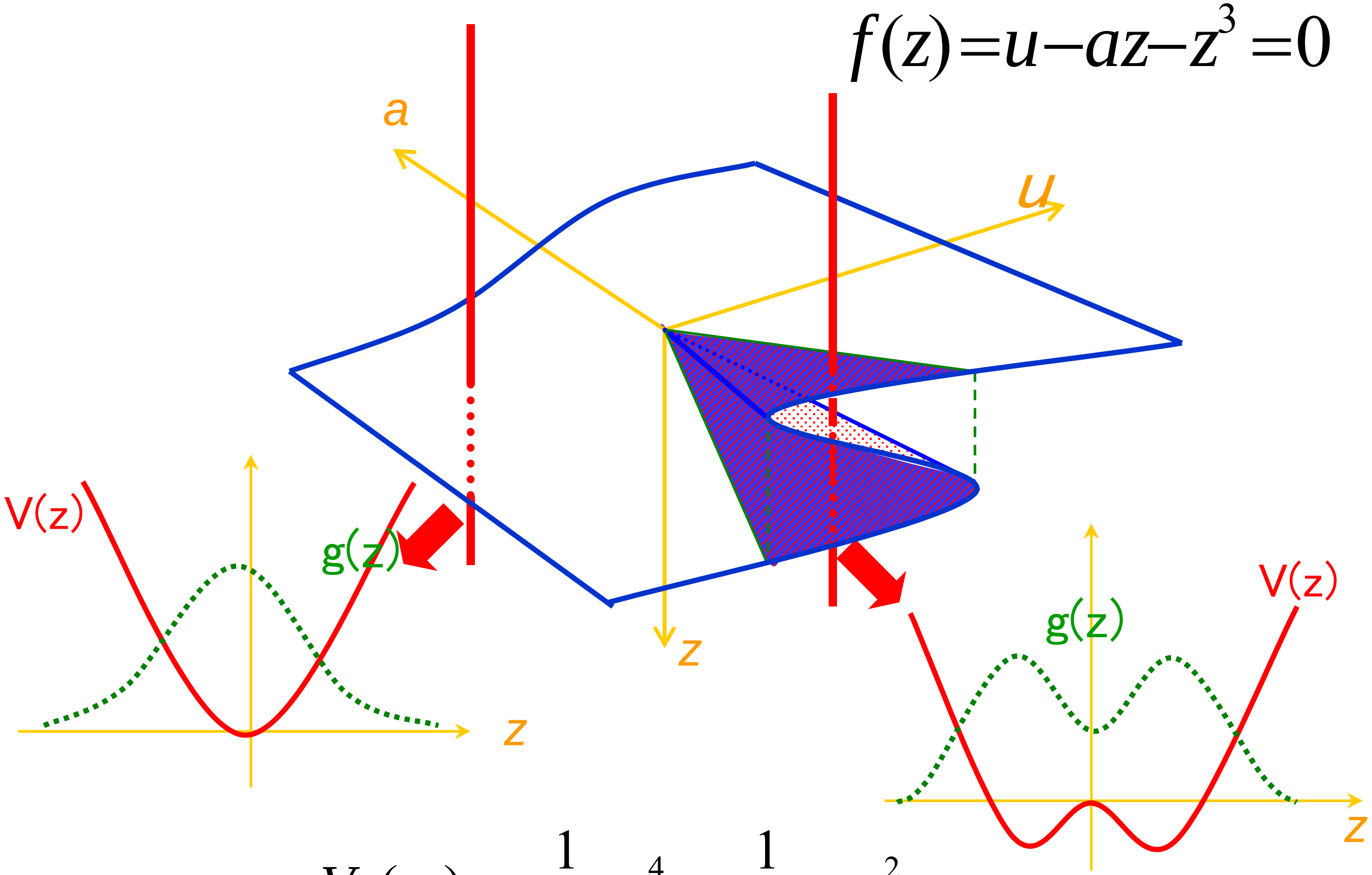
Proposed Scheme



$$V(z) = -\frac{1}{2}\ln(g(z)/C)$$

Ex. Riemann-Hugoniot Manifold: V'(z)=0

$$f(z) = u - az - z^3 = 0$$



$$V(z) = \frac{1}{4}z^4 + \frac{1}{2}az^2 - uz$$

Potential Function → One point in Dual Space

- Ex :

<Potential Function> ≒ Polynomial for degree 4

$$V(z) = a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4$$

VT : $z \mapsto z - \dfrac{a_3}{4a_4}$ Translation

$$= A_1 z + A_2 z^2 \qquad + a_4 z^4$$

$$(z \quad z^2 \quad z^3 \quad z^4)$$ **Linear Space**

$$(A_1 \quad A_2 \quad a_4) \in U^\times$$ **: Dual Space**

Structure of Dual Space