IEEE-ICCSE 2016

Information Reduction for Chaotic Patterns

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Universal Property of Chaos

Edward N. Lorenz

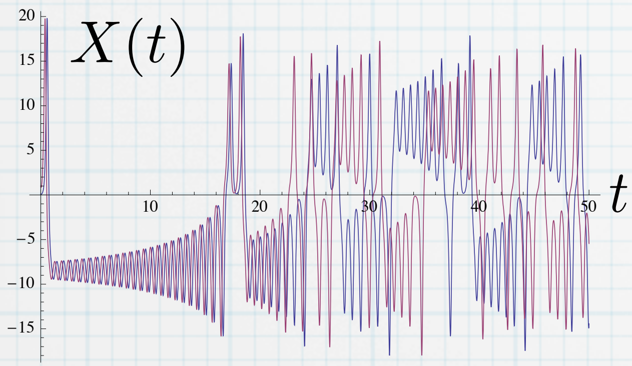
Lorenz Model

Discover of Chaos

$$\frac{\mathrm{d}X}{\mathrm{d}t} = p(Y - X)$$

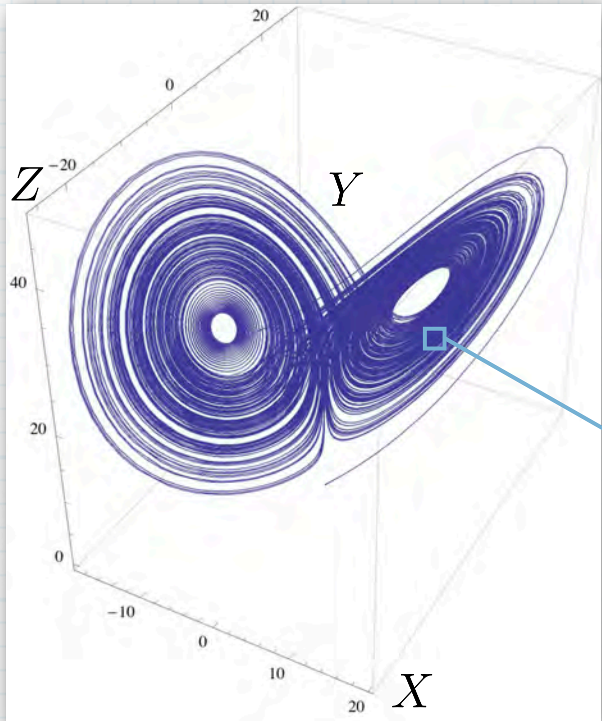
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -XZ + rX - Y$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = XY - bZ$$



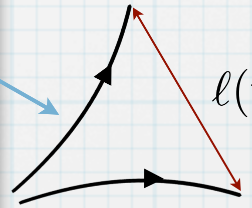
Non-periodic oscillation
= Turbulence

Butterfly Effect



Strange Attractor

Non-Integer Fractal Dimension



$\ell(t) \sim \exp(\lambda t)$

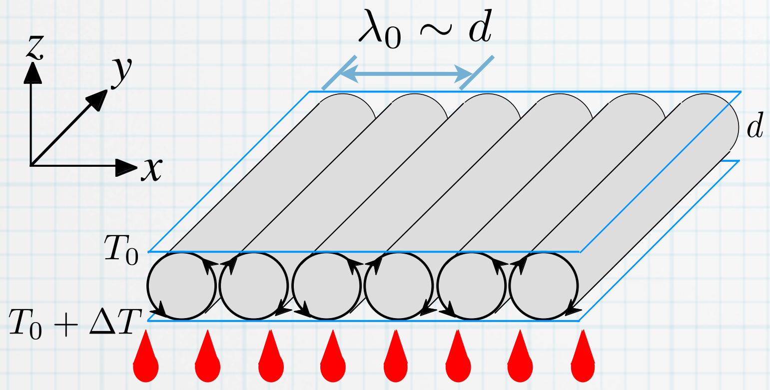
Positive Lyapunov Exponent $\lambda > 0$

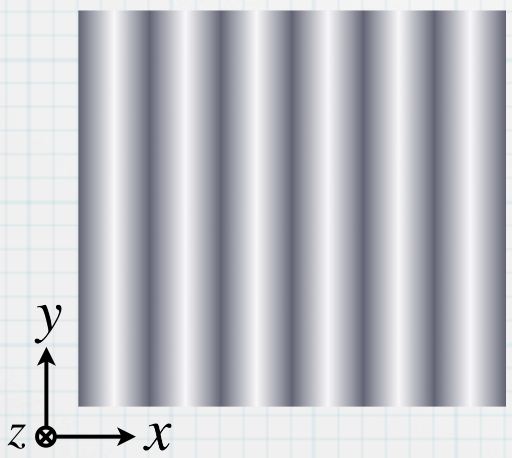
The non-periodic oscillation is regarded as chaos.

Derivation of Lorenz Model

Lorenz modeled weather as **atmospheric convection.**

Periodic Structure of Convection





$\mathbf{v}(\mathbf{r},t) = (u, 0, w)$

$\theta(\mathbf{r},t) = T(\mathbf{r},t) - T_0 - \Delta T/2 + (\Delta T/d)z$

↑

Deviation From the Thermal Conduction State

Periodic Structure with Time-Dependent Amplitude

$$u = u_0 X(t)\,\sin(\pi/d)z \sin qx$$

$$w = w_0 X(t)\,\cos(\pi/d)z \cos qx$$

$$\theta = \theta_1 Y(t)\,\cos(\pi/d)z \cos qx + \theta_2 Z(t)\,\sin(2\pi/d)z$$

↓

Fluid Dynamics with Temperature Field

$$p^{-1}\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla P + \theta\,\mathbf{g} + \nabla^2\mathbf{v}$$

$$\nabla\cdot\mathbf{v} = 0$$

$$\frac{\partial\theta}{\partial t} + \mathbf{v}\cdot\nabla\theta = R\,\mathbf{g}\cdot\mathbf{v} + \nabla^2\theta$$

Partial Differential Equation of $(\mathbf{r}, t)$

Ignoring Spatial Variation $(\mathbf{r})$

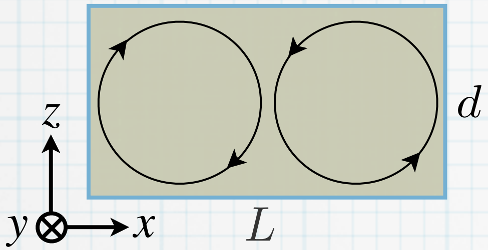
$$\frac{\mathrm{d}X}{\mathrm{d}t} = p(Y - X)$$

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -XZ + rX - Y$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = XY - bZ$$

Ordinary Differential Equations of the Amplitudes

Chaos in Real Convective Systems

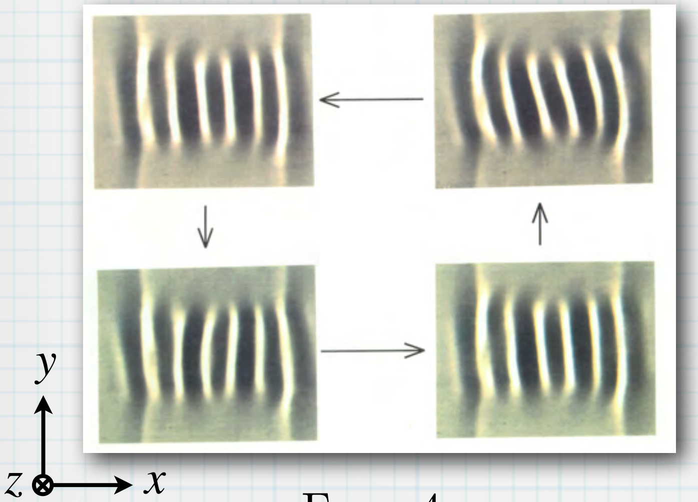


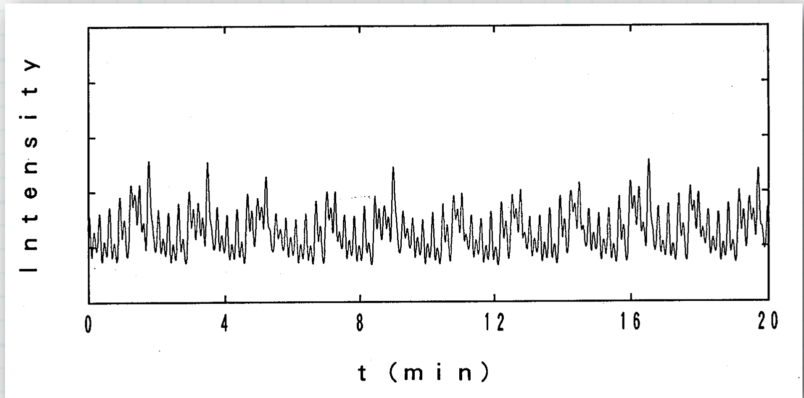
$$\Gamma = \frac{L}{d} \sim \mathcal{O}(1)$$

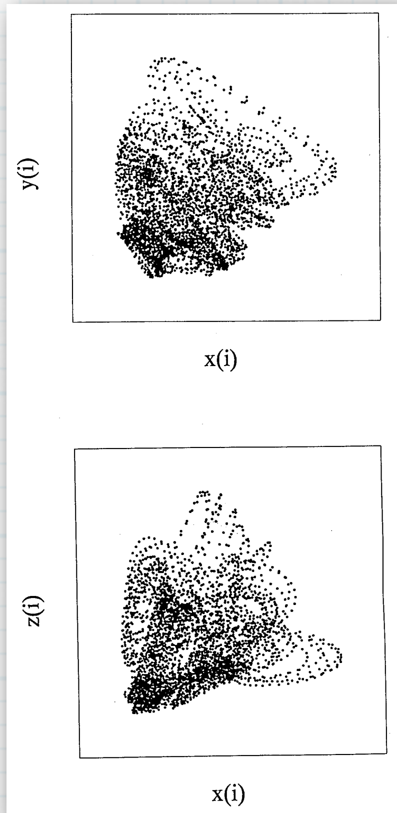
Spatial coherence is kept
in such **small system**
even in weak turbulence.
↓
Spatial variation can be ignored.

<e.g.> Chaos in an Electroconvective System of Liquid Crystals

Y. Hidaka *et al.*: J. Phys. Soc. Jpn. 61, 3950 (1992)







$\Gamma_x = 4$

$\Gamma_y = 2$

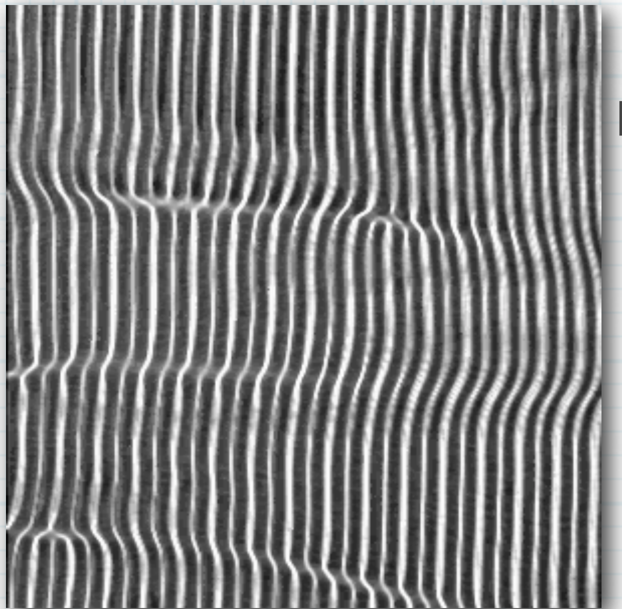
Weak Turbulence

$D = 2.6$

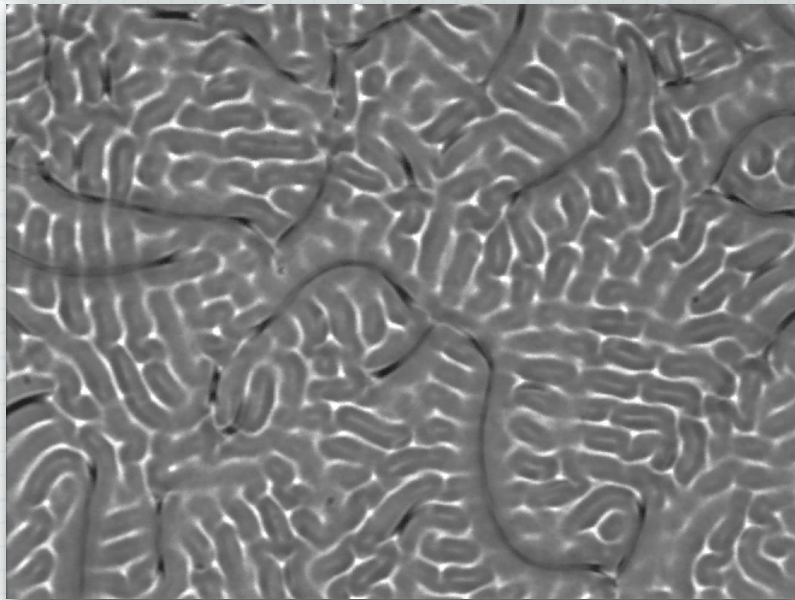
$\lambda > 0$

Weak Turbulence in Spatially-Extended Convective Systems

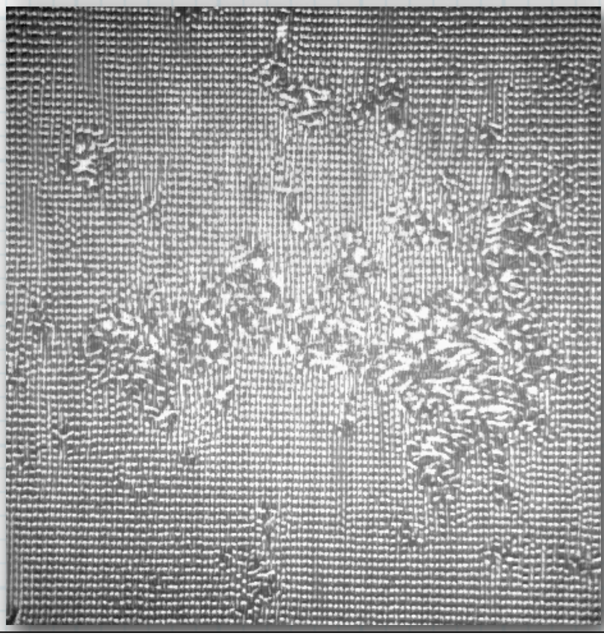
$\Gamma \gg 1$ → Spatiotemporal Chaos



Defect Turbulence

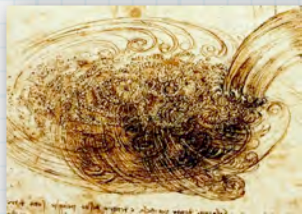


Soft-Mode Turbulence



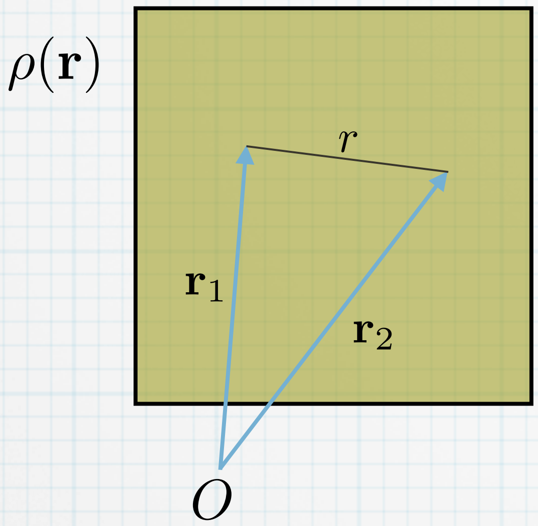
Spatiotemporal Intermittency

- What is the universal property for spatiotemporal chaos?
- How is spatiotemporal chaos distinguished from fully-developed turbulence?



Leonardo da Vinci

Universal Property for Spatiotemporal Chaos

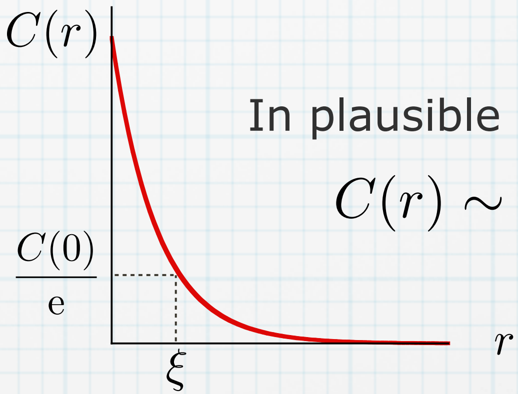


Two-Point Correlation Function for a Spatial Pattern

$$C(\mathbf{r}_1, \mathbf{r}_2) = \langle \rho(\mathbf{r}_1) \cdot \rho(\mathbf{r}_2) \rangle_{\mathbf{r}_1}$$

Homogeneous and Isotropic

$$C(\mathbf{r}_1, \mathbf{r}_2) = C(r), \quad r = |\mathbf{r}_1 - \mathbf{r}_2|$$



In plausible cases,

$$C(r) \sim \exp(-r/\xi)$$

Monotonous Decay

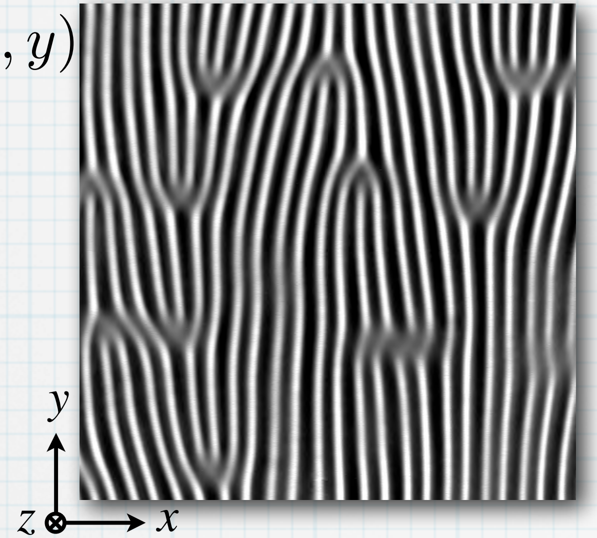
Correlation Length ξ : Roughness of the Pattern

What is adopted as $\rho(\mathbf{r})$?

Information Reduction

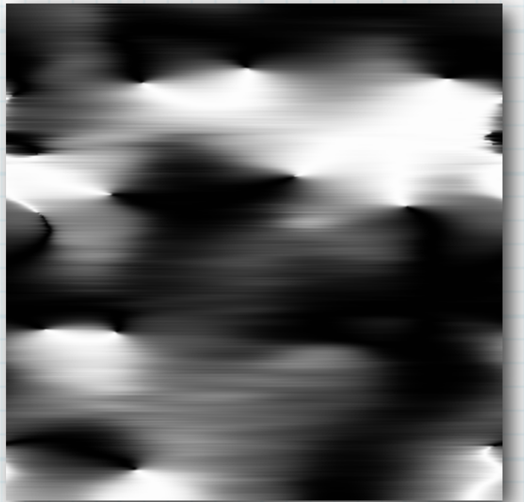
Defect Turbulence

$u(x,y)$



Complex Demodulation Technique

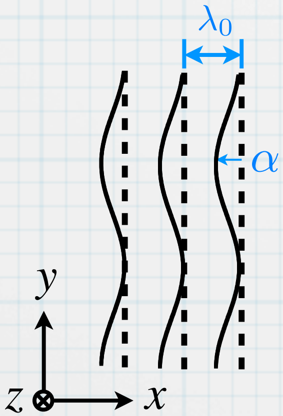
$\sin\alpha(x,y)$



$$u(x,y) = A_0 \exp\left[\mathrm{i}\left(q_0 x + \alpha(x,y)\right)\right] + \text{c.c.}$$

Wavelength of Original Stripe $\lambda_0 = q_0/2\pi$

$\alpha(x,y)$ Phase



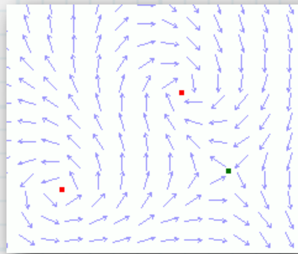
Two-Point Correlation Function of Phase

$$C(r) = \langle \cos\Delta\alpha(r)\rangle$$

$$\Delta\alpha(r) = \alpha(\mathbf{r}_1) - \alpha(\mathbf{r}_2), \quad r = |\mathbf{r}_1 - \mathbf{r}_2|$$

By analogy with 2D-XY spin model:

$$C(r) \sim \exp(-r/\xi)$$

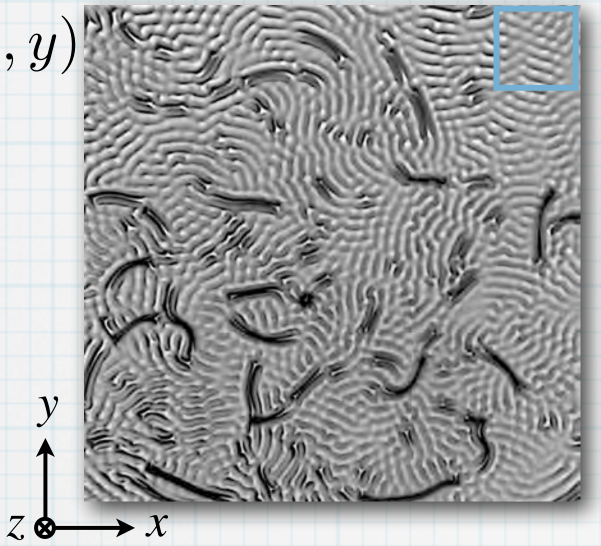


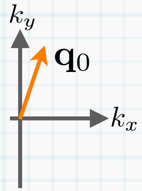
ξ corresponds to the average distance between neighboring defects.

$$\xi > \lambda_0$$

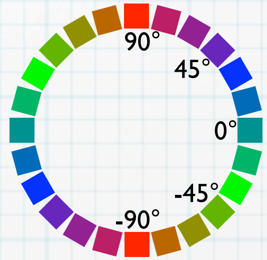
Soft-Mode Turbulence

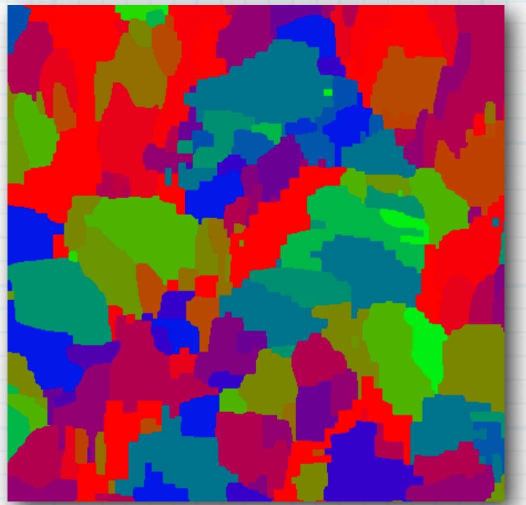
$u(x,y)$





$\psi(x,y)$

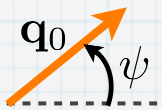


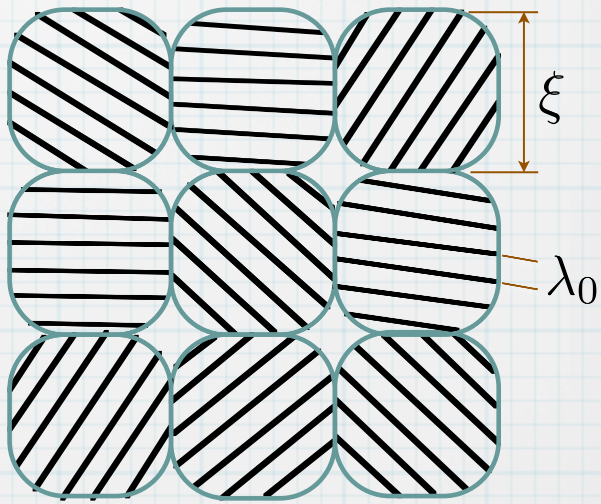


$$u(x,y) = A_0 \exp(\mathrm{i}\mathbf{q}_0 \cdot \mathbf{r}) + \mathrm{c.c.}$$

$$\mathbf{r} = (x,y)$$

$$\mathbf{q}_0(\mathbf{r}) = (q_0 \cos\psi(\mathbf{r}), q_0 \sin\psi(\mathbf{r}))$$





Two-Point Correlation Function of Roll Direction

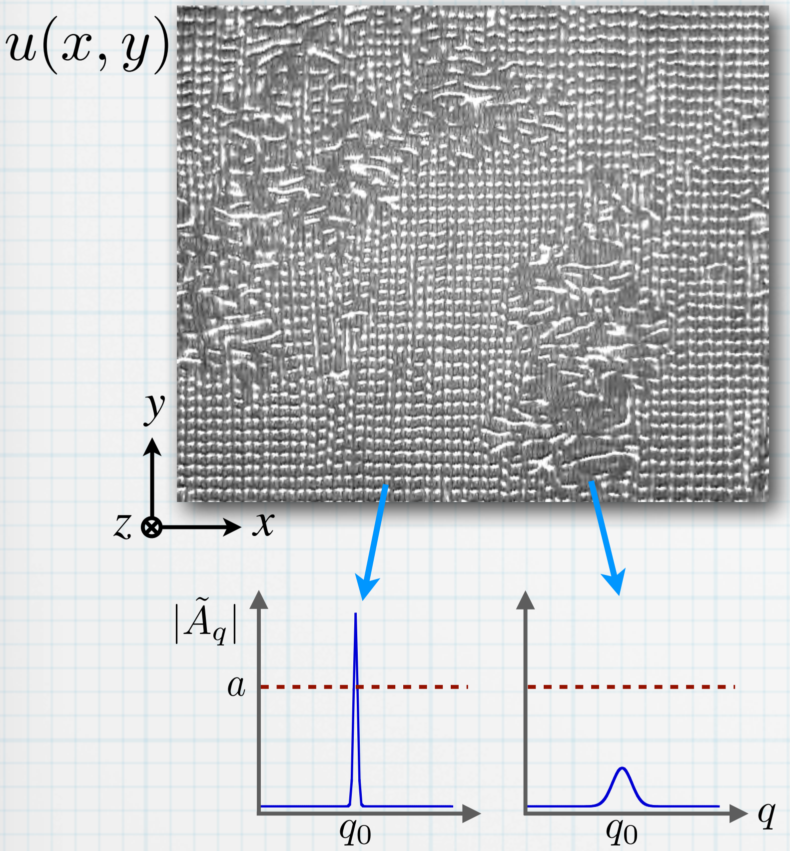
$$C(r) = \langle \cos 2\Delta\psi(r) \rangle \sim \exp(-r/\xi)$$

$$\Delta\psi(r) = \psi(\mathbf{r}_1) - \psi(\mathbf{r}_2), \quad r = |\mathbf{r}_1 - \mathbf{r}_2|$$

ξ corresponds to the average size of the domain.

$$\xi > \lambda_0$$

Spatiotemporal Intermittency



$$u(x,y) = \sum_q \tilde{A}_q(x,y)\exp(iqx) + \text{c.c.}$$

$$\delta(x,y) = \begin{cases} 1 & (|\tilde{A}_{q_0}| > a) \ \text{Order} \\ 0 & (|\tilde{A}_{q_0}| < a) \ \text{Disorder} \end{cases}$$

Two-Point Cluster Function $C(r), \quad r = |\mathbf{r}_1 - \mathbf{r}_2|$

Probability of finding both points $\mathbf{r}_1$ and $\mathbf{r}_2$ in the same cluster of one phase.

Analogy with site percolation model

One phase is put at each site with probability p.

$$C(r) = p^r = \exp(-r/\xi)$$

$$\xi \equiv -1/\ln p$$

ξ corresponds to the average size of cluster of the phase.

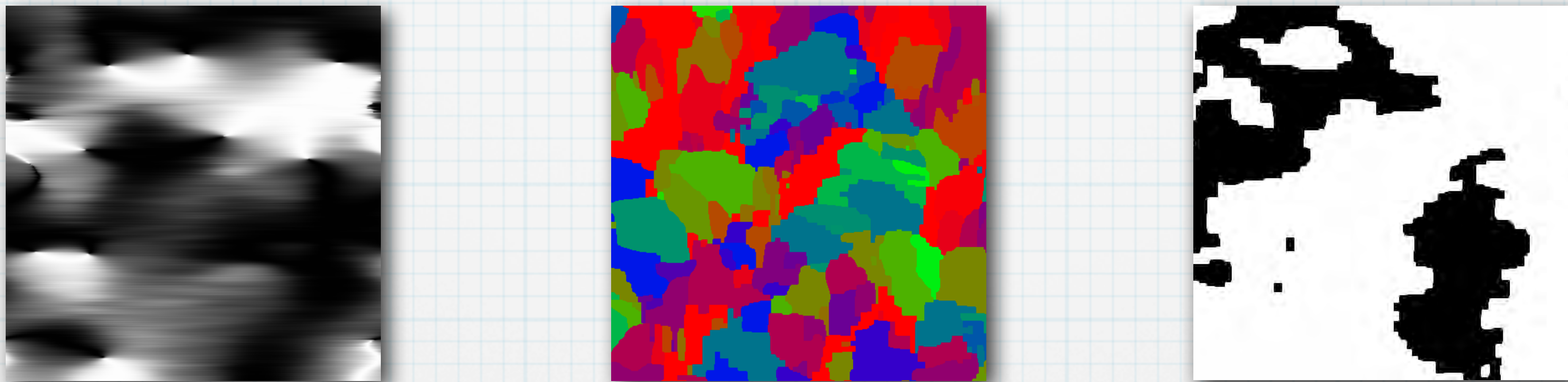
$$\xi > \lambda_0$$

Summary

$u(x, y)$

Information Reduction

$\rho(x, y)$



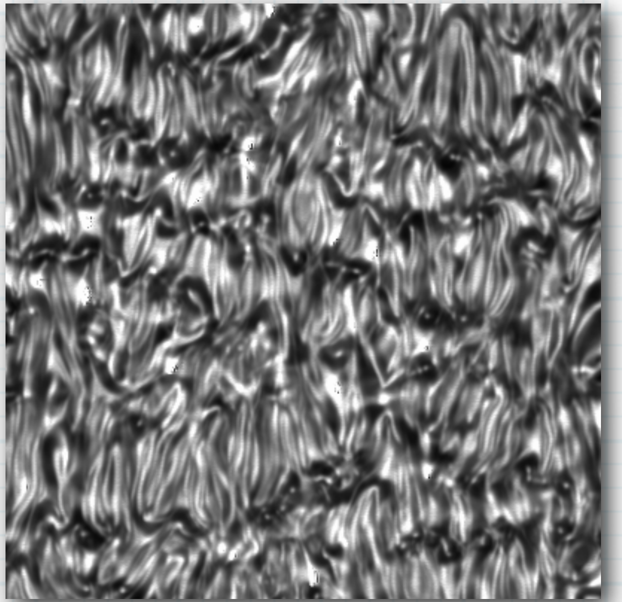
$\sin \alpha(x, y)$ $\psi(x, y)$ $\delta(x, y)$

Correlation Length $\xi > \lambda_0$ Local Order Size

Universal Property of Spatiotemporal Chaos

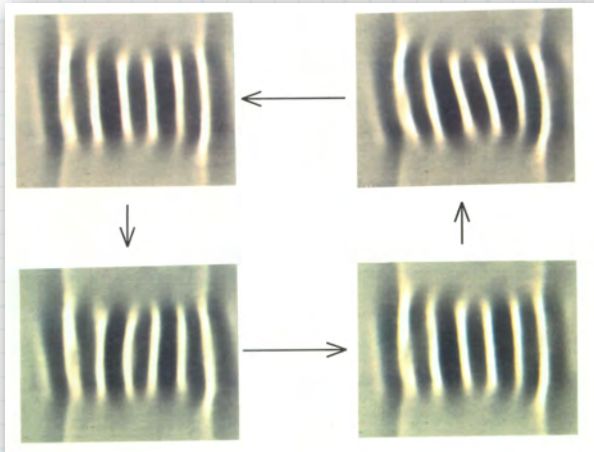
Summary & Future Plan

Fully-Developed Turbulence



$$\xi \ll \lambda_0$$

Chaos



$$\Gamma = \frac{L}{d} \sim \frac{L}{\lambda_0} \sim \mathcal{O}(1)$$

$$\xi > \lambda_0$$

$\therefore \;\; L < \xi$ → Spatial Coherence

Is it true $C(r) \sim \exp(-r/\xi)$?

Modeling by $\rho(\mathbf{r})$