The Longitude Problem as the Unification of Space and Time

With Special Application to the Island of St Helena

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Outline

- Introduction
- Establishment of local time
- Edmond Halley’s observations in St Helena (1677-8)
- Nevil Maskelyne’s observations in St Helena (1761-2)
- Manuel J. Johnson’s observations in St Helena (1829-33)
- Conclusion

Introduction

- understanding the longitude problem within the framework of 16-8 Centuries

-> Establishment of local time

-> Comparison of observations about the same astronomical event at different places

- Exercises to determine the longitude of St Helena

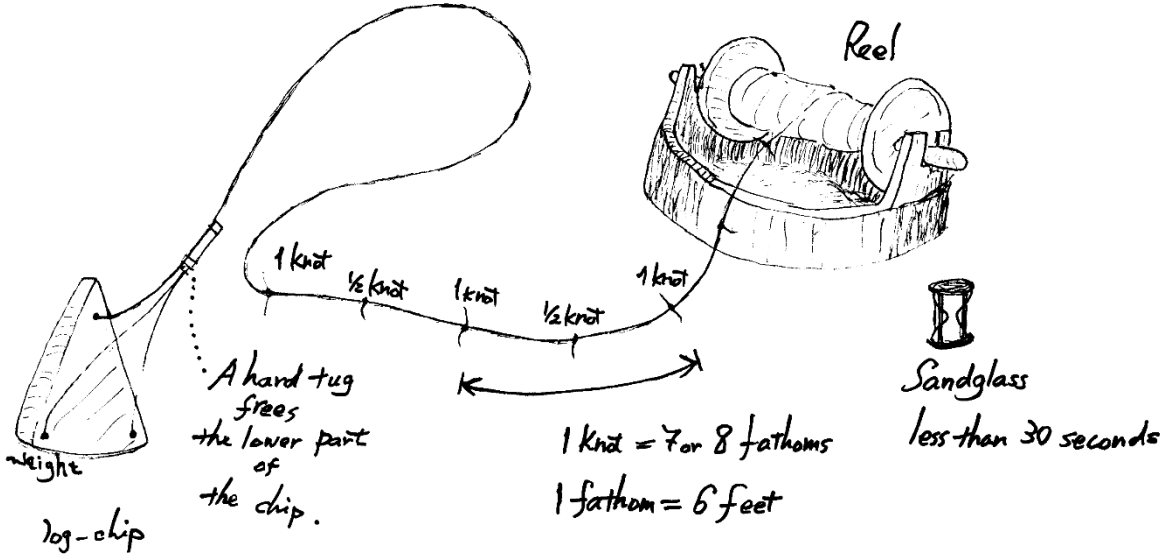
-> the Transit of Mercury on 7 Nov 1677 by Halley…

-> the lunar eclipse on 18 May 1761 by Maskelyne…

-> the solar eclipse on 27 July 1832 by Johnson…

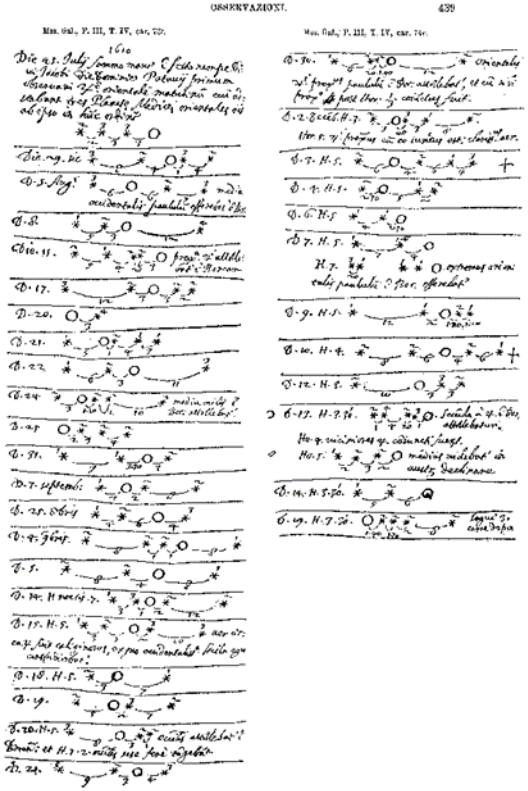
Common Reckoning

- Drop the log-chip into the wake of the ship, and a pilot and his assistants measure the length (knots) of the rope tied to the log-chip in given seconds (sandglass); one more thing to do is to know the direction of the tight rope. After simple algebra the pilot knows the velocity vector of the ship.



Eclipses of Jupiter's Satellites

- Periods of satellites vary from a couple of days to half a month. The method of Jupiter's satellites had been refined by many astronomers in 17 Century. The use 'on land' became practical in the end.



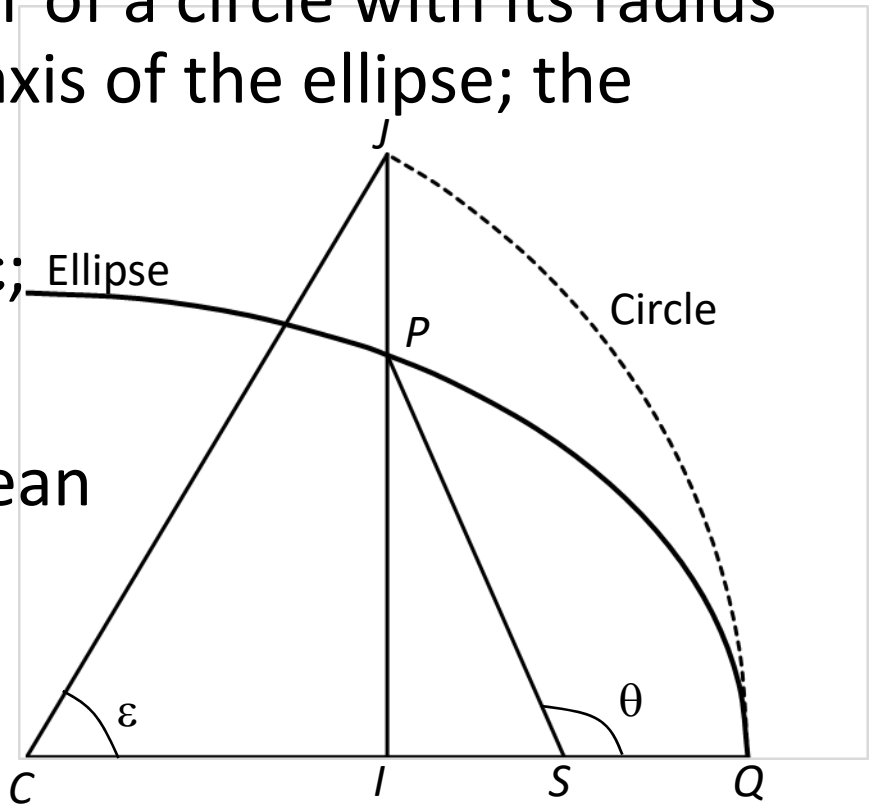
Three Anomalies

- The orbit of a planet is an ellipse; *C* is its center; the Sun sits at *S* (focus); *Q* is the perihelion; the planet is now at *P*; *CQJ* is a sector of a circle with its radius equal to the semi-major axis of the ellipse; the segment *IPJ* ⊥ *QSC*.

  the eccentric anomaly = ε;

  the true anomaly = θ;

  the area *SQP* is *M*, the mean anomaly.



Relations amongst anomalies

- Kepler's eqn: *M* is given by subtracting the area of D*IJC* from the area of the circular sector *QJC*, multiplying by (minor axis)/(major axis), and then subtracting the area of *SIP*.

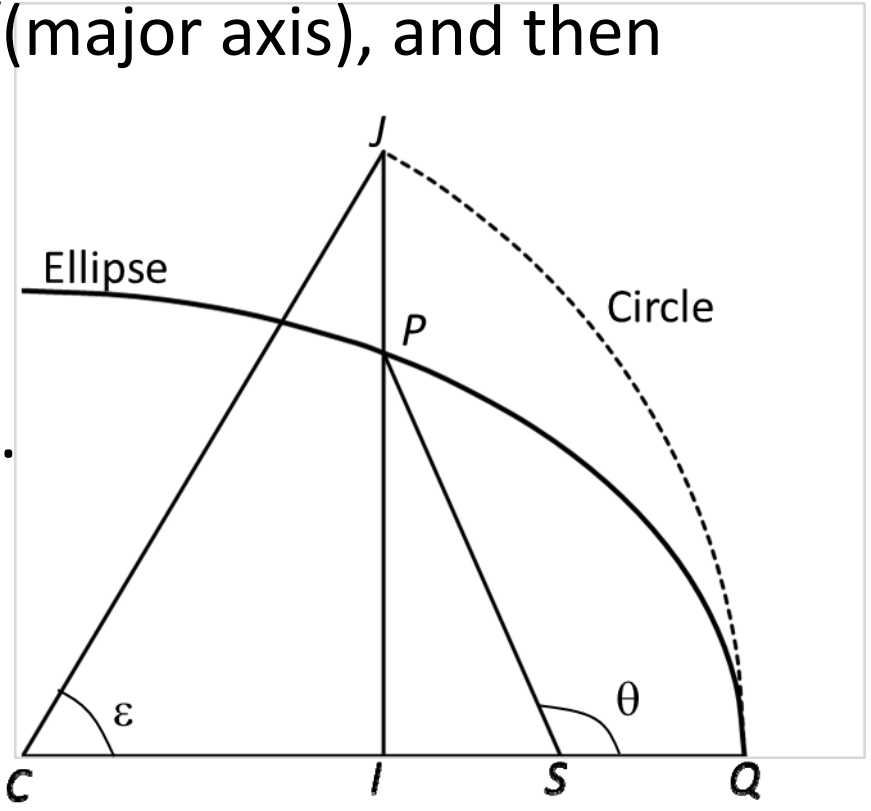
$M = \varepsilon - e \sin\varepsilon$,

where *e* denotes

the eccentricity of the ellipse.

- Relation between angles:

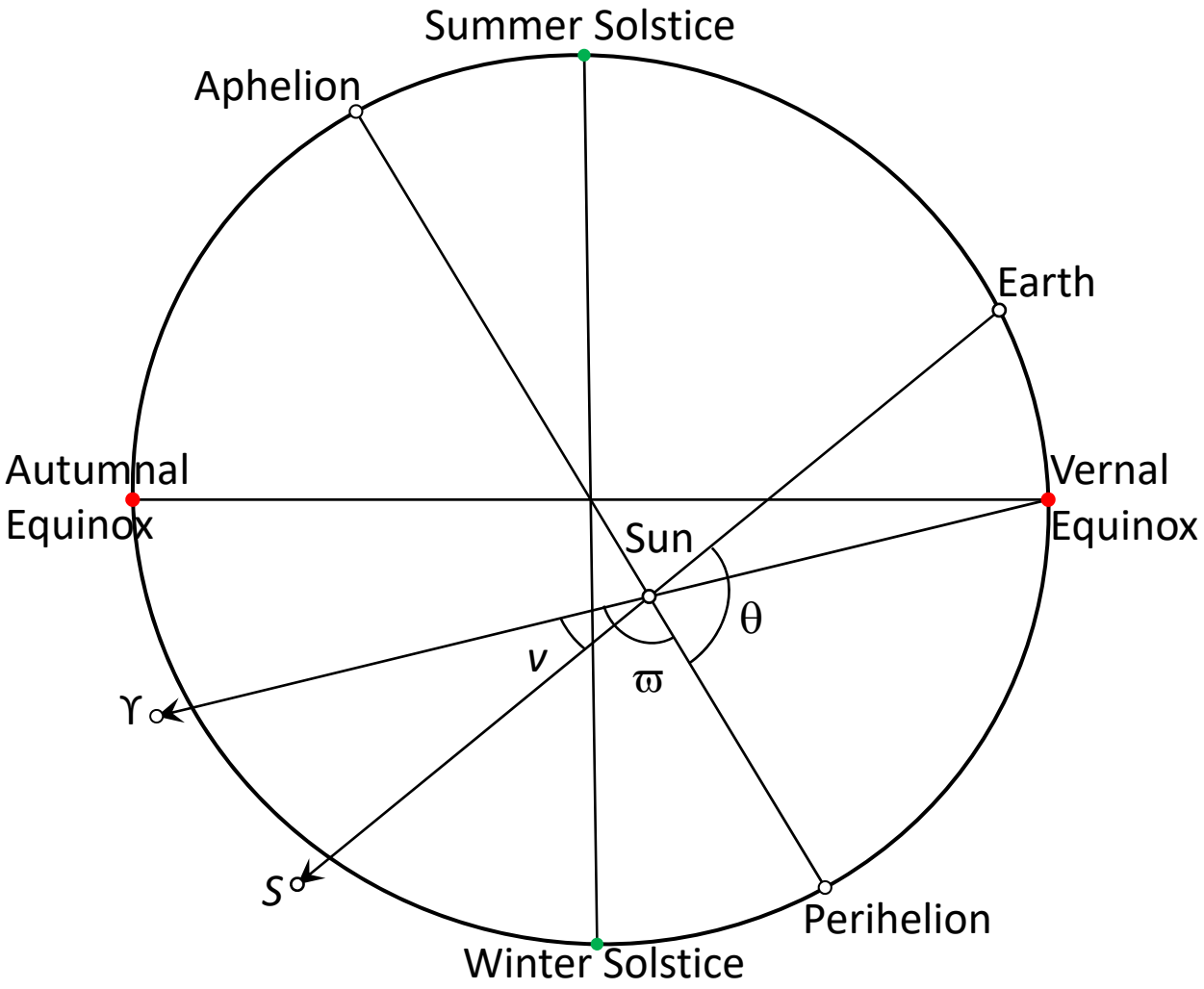
$\tan\theta/2 = \sqrt{((1+e)/(1-e))}\ \tan\varepsilon/2$



Orbit of Earth from Zenith

- Result:

$\varpi + \theta - \nu = \pi$.



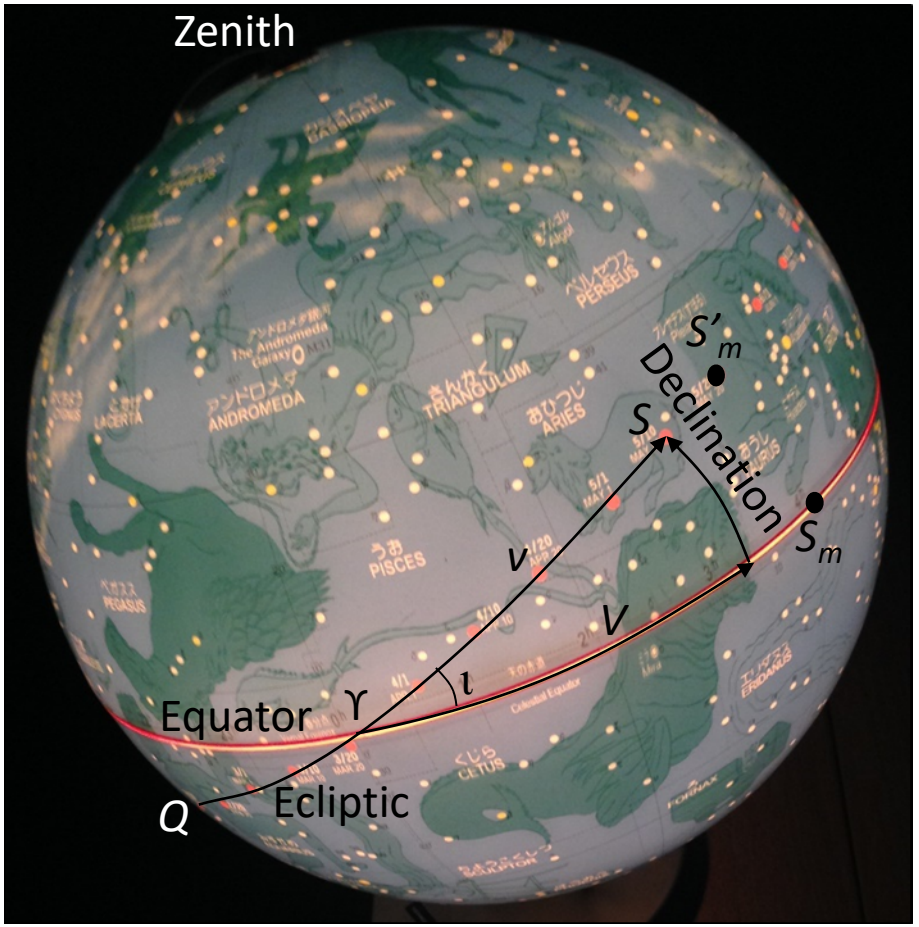
Relation between Space and Time

- Using the obliquity ι, relation between the ecliptic longitude *v*, i.e., ϒ*S*, and the corresponding right ascension *V* :

  $\tan V = \tan v \cos \iota$.

- Mean motions are the same:

  $\Upsilon S_m = \Upsilon S'_m$.



Equation of Time: definition

- Definition:

  $\Upsilon S_m - V$.

- Rewriting the above as follows:

$$\Upsilon S_m - V = \Upsilon S'_m - \Upsilon S + v - V$$
$$= QS'_m - QS + v - V$$
$$= M - \theta + v - V$$
$$= M - \pi + \varpi - V.$$

Equation of Time: algorithm

[Algorithm]

(1) Give the ecliptic longitude v.
Note $v = 0$ at ♈.

(2) Get the right ascension by
$V = \mathrm{atan}(\tan v \cos\iota)$.

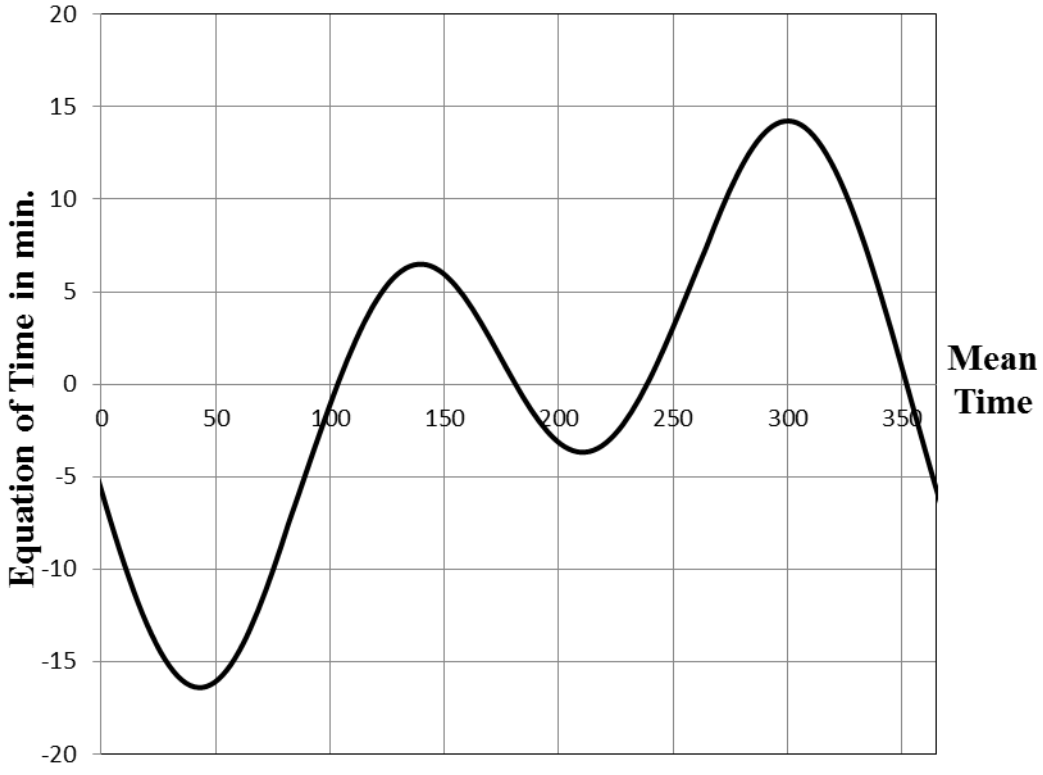
(3) Get the true anomaly by
$\theta = v + \pi - \varpi$.

(4) Get the eccentric anomaly by
$\varepsilon = 2\mathrm{atan}(\sqrt{((1-e)/(1+e))}\tan\theta/2)$.

(5) Get the mean anomaly by
$M = \varepsilon - e \sin\varepsilon$.

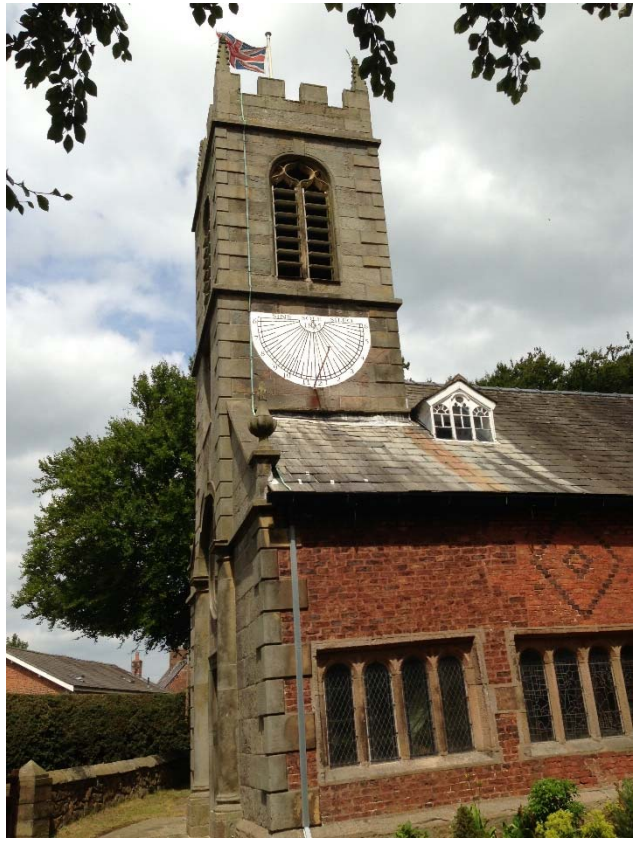
(6) Evaluate the equation of time by
$M - \pi + \varpi - V$.

(7) Repeat (1)-(6) for all year around.



Establishment of local time

- Now you can establish the local time with your observation by a sundial and correction due to the equation of time.



Exercise applying to St Helena (1)

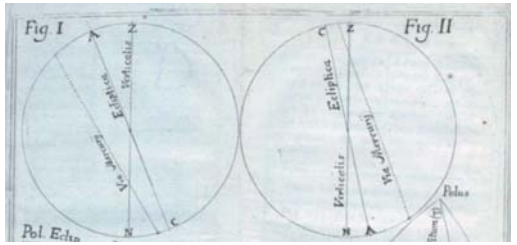
Transit of Mercury on 7 Nov 1677

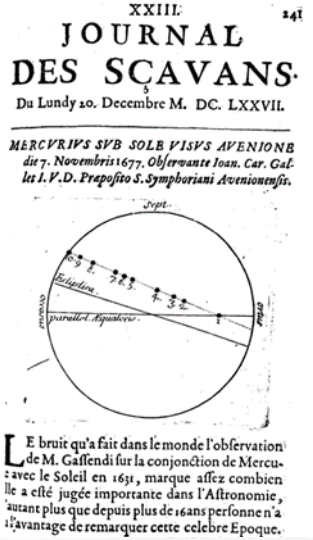
- Edmond Halley @ St Helena
- Jean Charles Gallet @ Avignon

St Symphorien Church

- Richard Towneley @ Towneley

Towneley Hall





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Du Lundy 20. Decembre M. DC. LXXVII.

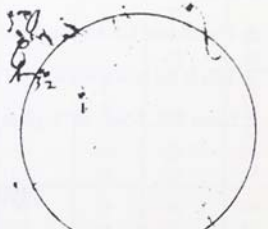
MERCVRIVS SVB SOLE VISVS AVENIONE die 7. Novembris 1677. Observante Ioan. Car. Gallet I.V.D. Præposito S. Symphoriani Avenionensis.

LE bruit qu'a fait dans le monde l'observation de M. Gassendi sur la conjonction de Mercure avec le Soleil en 1631, marque assez combien elle a esté jugée importante dans l'Astronomie, d'autant plus que depuis plus de 16 ans personne n'a eu l'avantage de remarquer cette celebre Epoque.

what wee observed by a minute watch set by my pendulum

2h. 00′ at 1
45 at 2
52 at 3

58 - 30″: ☿ Just went of. wee saw him just upon the limbe very exactly insomuch that wee could distinguish him to decay and as cleare when wee saw halfe the body on as when wee saw the whole hee appeared but of about this bignesse (o) when the suns picture was about 6 inches diameter, of a duskish red colour, wee made use of a 12 foot glasse but the charge was but small to take in all the picture and our



Result (1)

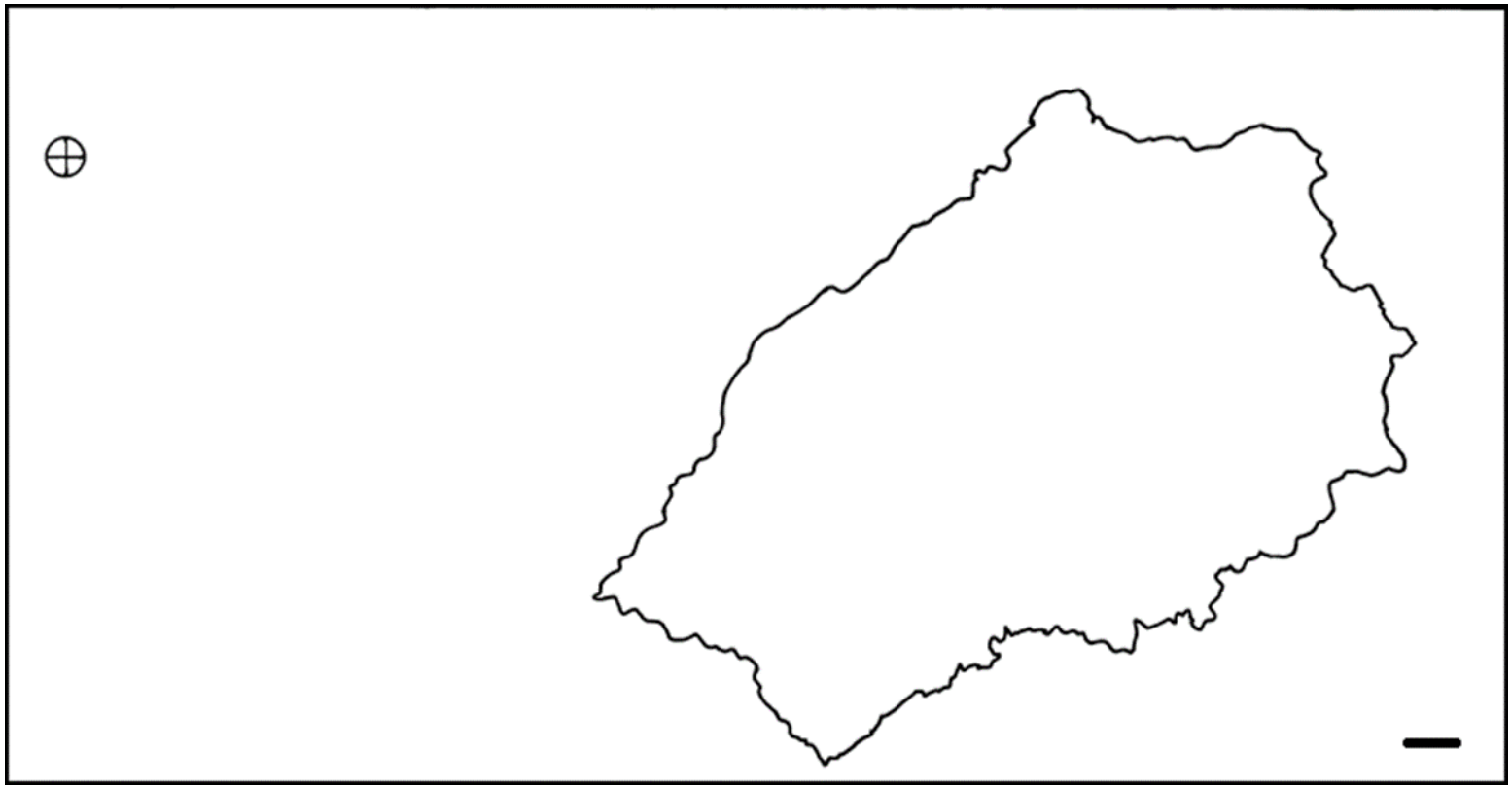
//time of emersion//	//longitude//
*Avignon $15^h26^m56^s$	$4^o48'46''$ E
*Towneley $14^h56^m36^s$	$2^o13'21''$ W
*St Helena $14^h41^m54^s$	'to be determined'

Translating the time difference into the longitude difference, we determine the longitude of St Helena:

$6^o26'44''$ W by use of Avignon data;

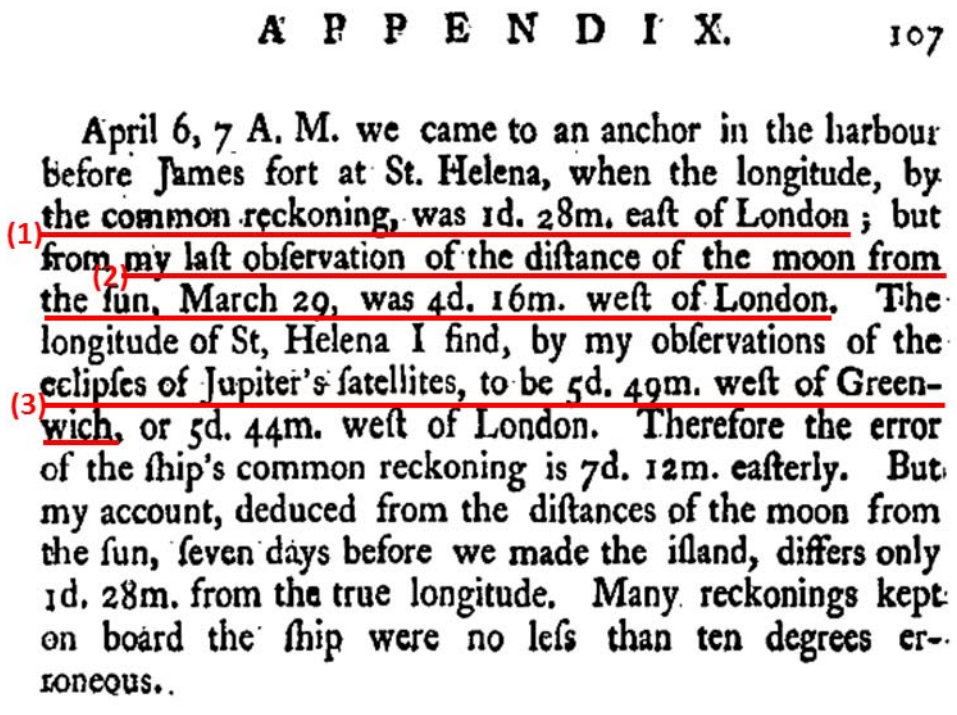
$5^o53'21''$ W by use of Towneley data.

•The latter is better, but this estimate points the place one island away from St Helena. The clock of Avignon gains two minutes or so.



Exercise applying to St Helena (2)

- Nevil Maskelyne's own account, 'The British Mariner's Guide,' (London, 1763):



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April 6, 7 A. M. we came to an anchor in the harbour before James fort at St. Helena, when the longitude, by the common reckoning, was 1d. 28m. east of London; but from my last observation of the distance of the moon from the sun, March 29, was 4d. 16m. west of London. The longitude of St, Helena I find, by my observations of the eclipses of Jupiter's satellites, to be 5d. 49m. west of Greenwich, or 5d. 44m. west of London. Therefore the error of the ship's common reckoning is 7d. 12m. easterly. But my account, deduced from the distances of the moon from the sun, seven days before we made the island, differs only 1d. 28m. from the true longitude. Many reckonings kept on board the ship were no less than ten degrees erroneous.

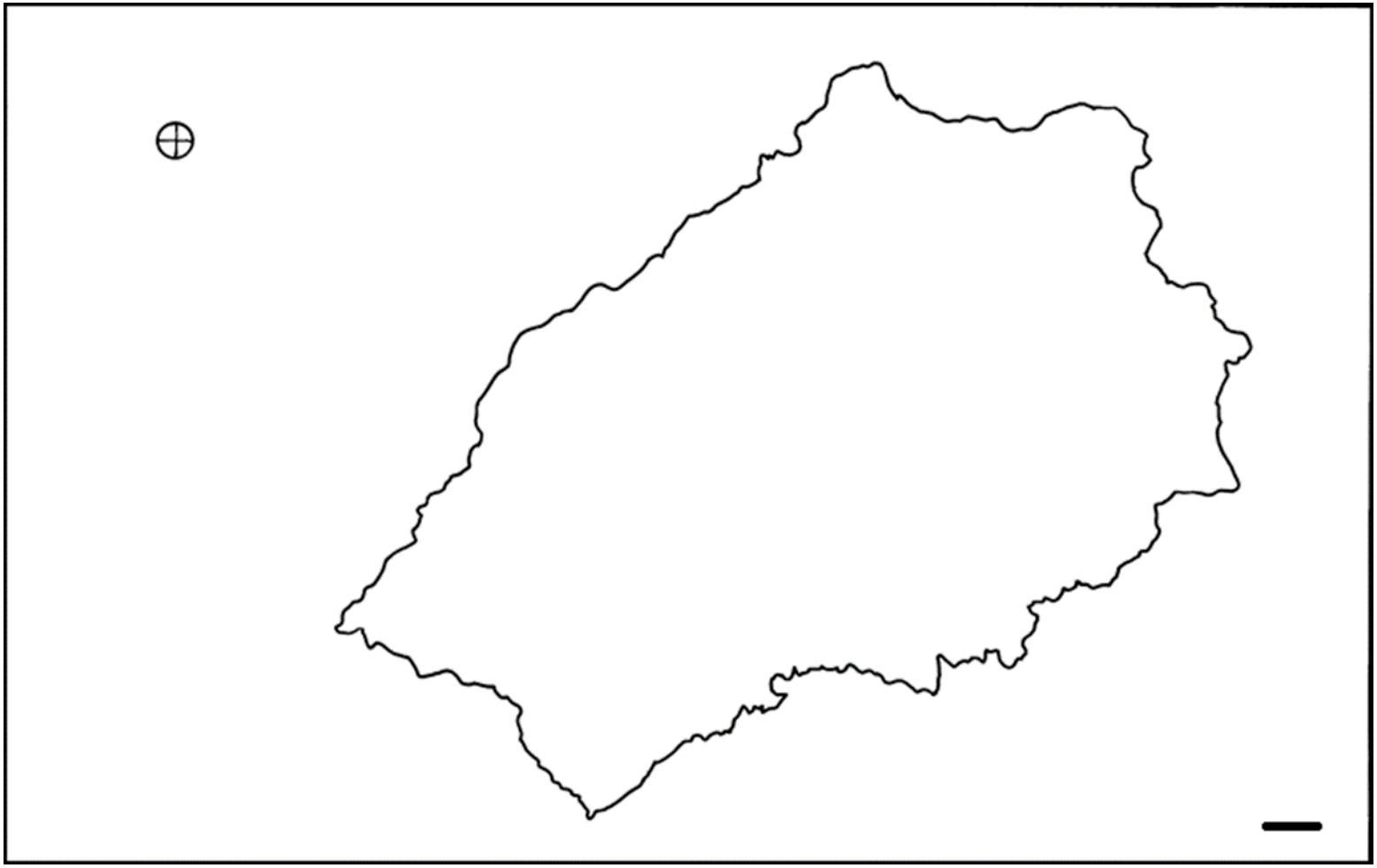
Result (2)

- Total Lunar Eclipse on 18 May 1761

//location//	//emersion//	//end of eclipse//	//longitude//
*St Helena	$10^h39^m23^s$	$11^h46^m52^s$	to be determined
*Cape	$12^h15^m37^s$	$13^h23^m42^s$	$1^h13^m35^s$ E
*Stockholm	$12^h15^m00^s$	$13^h21^m08^s$	$1^h12^m01^s$ E

- Taking the mean of two chances, we determine the longitude of St Helena by mean of two places:

  $5^o49' 05''$…much the same as Maskelyne's result.



Exercise applying to St Helena (3)

- Manuel J. Johnson's result by use of the Lunar Distance Method (1830-33):

The latitude: 15°55' 26"S; the longitude: 5°43' 39"W.

- This is the location of Ladder Hill Observatory in St Helena.

Result (3)

- Solar Eclipse on 27 July 1832
- Johnson's estimate of the conjunction at St Helena:

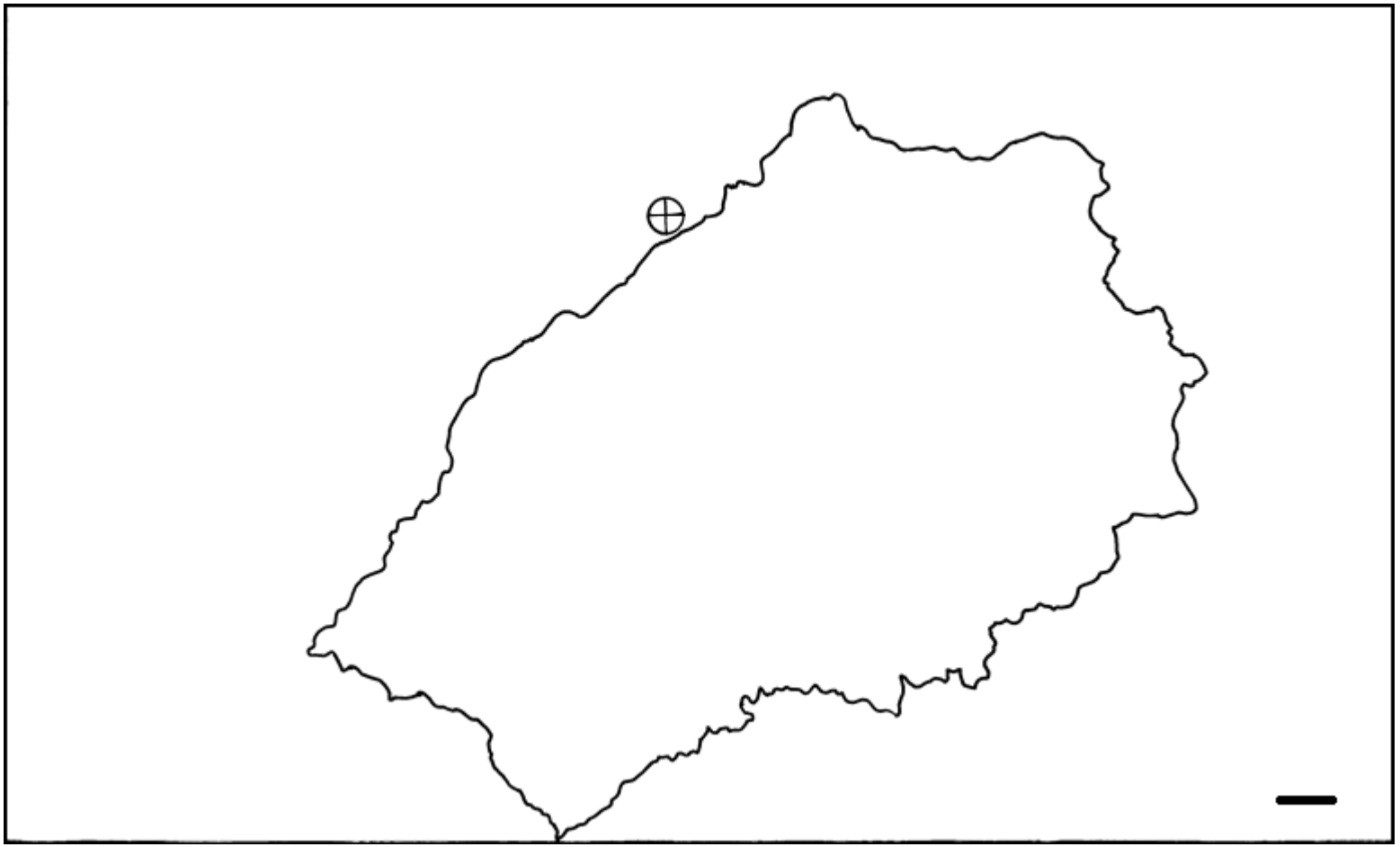
  $13^h39^m8.6^s$

  $14^h01^m00^s$ (UT) at Greenwich by NASA database

  $14^h01^m00^s - 13^h39^m8.6^s = 21^m51.4^s$,

That is the longitude is $5^o27'51"$W. This is very short.

- Method of estimating the conjunction is the source of the errors.



Conclusion

- It is important to establish local time with the connection to ‘Equation of Time,’ the phenomena geometrical as well as gravitational.
- By use of eclipses of Jupiter’s satellites, we get the most accurate method, but this is not used at sea.
- The second best is the Lunar Distance Method.

To be contined.

Conclusion continued

- Exercises:

(1) Edmond Halley’s observation of ‘the transit of Mercury:’ the result points the place one island away from St Helena.

(2) Nevil Maskelyne’s observation of the total lunar eclipse: the result points the place half an island away from St Helena.

(3) Manuel John Johnson’s observation of the total solar eclipse: the result is the worst (much too short); the linear method of estimating the conjunction is the source of errors.