

# **Introduction to Extreme Value Theory**

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## Sample extreme:

Let  $X_1, X_2, X_3, \dots$  be independent random variables, all with the same distribution function  $F$ .

Consider  $Y_n := \max(X_1, X_2, \dots, X_n) = X_{n,n}$  for  $n = 1, 2, \dots$

Probability distribution function of  $Y_n$ :

$$\begin{aligned} P\{Y_n \leq x\} &= P\{X_1 \leq x, X_2 \leq x, \dots, X_n \leq x\} \\ &\stackrel{\text{indep.}}{=} P\{X_1 \leq x\} P\{X_2 \leq x\} \dots P\{X_n \leq x\} \\ &\stackrel{\text{same}}{=} F^n(x). \\ &\text{distr.} \end{aligned}$$

Limit theory: what can we say about

$$P\{Y_n \leq x\} \quad \text{as} \quad n \rightarrow \infty?$$

$$\text{If } F(x) < 1, \text{ then } P\{Y_n \leq x\} = F^n(x) \rightarrow 0$$

$$\text{If } F(x) = 1, \text{ then } P\{Y_n \leq x\} = 1 \rightarrow 1.$$

Hence we get a degenerate limit (adopts only two values) which is not very interesting. Hence we put  $Y_n$  on the right scale and location i.e. we consider

$$\frac{Y_n - b_n}{a_n}$$

with  $b_n$  some sequence of real numbers (location correction) and  $a_n$  some positive numbers (scale correction).

Then

$$P\left\{\frac{Y_n - b_n}{a_n} \leq x\right\} = P\{Y_n \leq a_n x + b_n\} = F^n(a_n x + b_n).$$

We try to find sequences  $\{b_n\}$  and  $\{a_n\}$  such that

$$\lim_{n \rightarrow \infty} F^n(a_n x + b_n) \text{ exists } =: G(x) \quad (1)$$

where  $G$  is a non-degenerate distribution function i.e.  $G$  adopts at least 3 values (**extreme value condition**).

We are going to find all possibilities for  $G$ !

In fact we look at 2 questions:

1. What probability distribution functions  $G$  can occur as a limit in (1)?
2. For each of the  $G$  found in (1): what are the conditions on the original distribution function  $F$  such that (1) holds with this given  $G$ ? ( $F$  is in the “domain of attraction of  $G$ ”,  $F \in \mathcal{D}(G)$ )

## Preliminary calculations:

$$F^n(a_n x + b_n) \rightarrow G(x) \text{ (for all } x \text{ with } : 0 < G(x) < 1) \quad (1)$$



$$-n \log F(a_n x + b_n) \rightarrow -\log G(x) \text{ (for } x : 0 < -\log G(x) < \infty)$$

This can hold only if  $\log F(a_n x + b_n) \rightarrow 0$ .

Now recall the limit  $\lim_{s \rightarrow 0} \frac{-\log(1-s)}{s} = 1$

and apply with  $s := 1 - F(a_n x + b_n)$ .

We get  $\frac{-\log F(a_n x + b_n)}{1 - F(a_n x + b_n)} \rightarrow 1$

hence



$$n(1 - F(a_n x + b_n)) \rightarrow -\log G(x), n \rightarrow \infty. \quad (2')$$

With some effort it can be proved that this also holds when we replace  $n$  by a continuous parameter  $t$ :



$$t(1 - F(a(t)x + b(t))) \rightarrow -\log G(x), t \rightarrow \infty, t \text{ real}. \quad (2)$$

Hence (1)  $\Leftrightarrow$  (2). I want to derive a third equivalent form for the convergence.

This goes via the **inverse function**

**Lemma** Suppose  $f_n(x)$  is non-decreasing in  $x$  for all  $n$ . Consider  $f_n^{\leftarrow}(x)$ , the inverse function of  $f_n$  ( $n = 1, 2, \dots$ ).

Suppose  $\lim_{n \rightarrow \infty} f_n(x) = g(x)$  for all  $x \in (a, b)$

Then  $\lim_{n \rightarrow \infty} f_n^{\leftarrow}(x) = g^{\leftarrow}(x)$  for all  $x \in (g(a), g(b))$

where  $g^{\leftarrow}$  is the inverse function of  $g$ . □

**(picture)**



We apply this to

$$f_n(x) := \frac{1}{n(1 - F(a_n x + b_n))}$$

and

$$g(x) := \frac{1}{-\log G(x)}.$$

According to (2') we have  $f_n(x) \rightarrow g(x)$  for all  $x$ .

Hence  $f_n^{\leftarrow}(x) \rightarrow g^{\leftarrow}(x)$  for all  $x$ .

What are  $f_n^{\leftarrow}$  and  $g^{\leftarrow}$  in this case? First  $f_n(x)$ :

$$\begin{aligned}
y = f_n(x) &\Leftrightarrow y = \frac{1}{n(1 - F(a_n x + b_n))} \\
\Leftrightarrow ny &= \frac{1}{1 - F(a_n x + b_n)} \Leftrightarrow F(a_n x + b_n) = 1 - \frac{1}{ny} \\
\Leftrightarrow a_n x + b_n &= F^{\leftarrow}\left(1 - \frac{1}{ny}\right) \Leftrightarrow x = \frac{F^{\leftarrow}\left(1 - \frac{1}{ny}\right) - b_n}{a_n}.
\end{aligned}$$

Hence

$$f_n^{\leftarrow}(x) = \frac{F^{\leftarrow}\left(1 - \frac{1}{nx}\right) - b_n}{a_n}$$

Simpler notation :  $U(x) := F^{\leftarrow} \left( 1 - \frac{1}{x} \right).$

equivalently  $U(x) := \left( \frac{1}{1-F} \right)^{\leftarrow} (x).$

This was the inverse of  $f_n(x)$ . Now about the inverse of  $g$  :

$$y = g(x) \Leftrightarrow y = \frac{1}{-\log G(x)} \Leftrightarrow G(x) = e^{-\frac{1}{y}}$$

$$\Leftrightarrow x = G^{\leftarrow} \left( e^{-\frac{1}{y}} \right).$$

**Conclusion:** (1)  $\Leftrightarrow$  (2)  $\Leftrightarrow$

$$\lim_{\substack{n \rightarrow \infty \\ \swarrow \\ \text{integer}}} \frac{U(nx) - b_n}{a_n} = G^{\leftarrow} \left( e^{-\frac{1}{x}} \right) \quad \text{for } x > 0 \quad (3')$$

$\Downarrow$

$$\lim_{\substack{t \rightarrow \infty \\ \swarrow \\ \text{continuous variable}}} \frac{U(tx) - b_t}{a_t} = G^{\leftarrow} \left( e^{-\frac{1}{x}} \right) \quad \text{for } x > 0. \quad (3')$$

$\Downarrow$  (subtract the same with  $x = 0$ )

$$\lim_{t \rightarrow \infty} \frac{U(tx) - U(t)}{a(t)} = G^{\leftarrow} \left( e^{-\frac{1}{x}} \right) - G^{\leftarrow} \left( e^{-1} \right) \quad \text{for } x > 0. \quad (3)$$

## *Theorem Equivalent are:*

$$1) \lim_{n \rightarrow \infty} F^n(a_n x + b_n) = G(x)$$

$$2) \lim_{t \rightarrow \infty} t(1 - F(b(t) + xa(t))) = -\log G(x)$$

$$3) \lim_{t \rightarrow \infty} \frac{U(tx) - U(t)}{a(t)} = G^{\leftarrow}(e^{-\frac{1}{x}}) - G^{\leftarrow}(e^{-1})$$

Soon we shall see the use of this theorem. We proceed to identify the limit  $G(x)$ .

The complete class of possible limit distributions  $G$  is given in the next theorem.

**Theorem** (*Fisher and Tippett 1928, Gnedenko 1943*)

Suppose that for some distribution function  $F$  we have  $F^n(a_n x + b_n) \rightarrow G(x)$ , non-degenerate, for all continuity points  $x$ .

Then  $G(x) = G_\gamma(ax + b)$  for some  $a > 0$  and  $b$  where

$$G_\gamma(x) := \exp\left\{-\left(1 + \gamma x\right)^{-\frac{1}{\gamma}}\right\}$$

for all  $x$  with  $1 + \gamma x > 0$  and where the parameter  $\gamma$  can have any real value (for  $\gamma = 0$  read the formula as  $\exp\{-e^{-x}\}$ ).

## Remark

*There are 3 parameters,  $\gamma$ ,  $a$ ,  $b$  but  $\gamma$  is the only important one, the other two just represent scale and location. They are arbitrary since by changing the sequences  $\{a_n\}$  and  $\{b_n\}$ , one can get any  $a > 0$  and  $b$ .*

**Proof:** We found

$$F^n(a_n x + b_n) \xrightarrow{n \rightarrow \infty} G(x) \Leftrightarrow \frac{U(tx) - U(t)}{a(t)} \rightarrow G^{\leftarrow}\left(e^{-\frac{1}{x}}\right) - G^{\leftarrow}\left(e^{-1}\right) =: D(x)$$

Note :  $D(1) = 0$  . Take  $x, y > 0$  and write the identity

$$\begin{array}{ccccccc} \frac{U(tyx) - U(t)}{a(t)} & = & \frac{U(tyx) - U(ty)}{a(ty)} \cdot \frac{a(ty)}{a(t)} & + & \frac{U(ty) - U(t)}{a(t)} \\ (t \rightarrow \infty) \downarrow & & \downarrow & & \downarrow & & \downarrow \\ D(xy) & & D(x) & \Rightarrow & A^*(y) > 0 & \Leftarrow & D(y) \\ & & & & \text{(say)} & & \end{array}$$



Hence  $D(xy) = D(x)A^*(y) + D(y)$  for all  $x, y > 0$ .

We have to solve this functional equation.

We write  $D(e^{s+t}) = D(e^s)A^*(e^t) + D(e^t)$  for all real  $s, t$ .

Introduce  $H(s) := D(e^s)$  &  $A(t) := A^*(e^t)$ .

Then

$$H(t+s) = H(s)A(t) + H(t) \quad \forall t, s \text{ real}$$

$$\& H(0) = D(1) = 0, \quad A(0) = 1$$

or

$$H(t+s) - H(t) = H(s) \cdot A(t)$$

Write this as

$$\frac{H(t+s) - H(t)}{s} = \frac{H(s) - H(0)}{s} \cdot A(t).$$

Now  $H$  is monotone hence  $\exists t$  where  $H'(t)$  exists.

The equality above shows that  $H'(0)$  exists hence  $H'(t)$  exists for all  $t$ .

**Conclusion**

$$H'(t) = H'(0) \cdot A(t).$$

Since  $H$  cannot be constant, this implies  $H'(0) > 0$  .

Write  $Q(t) := H(t)/H'(0)$  .

Note  $Q(0) = 0$  ,  $Q'(0) = 1$  ,  $Q'(t) = A(t)$  .

We know

$$H(t+s) - H(t) = H(s)A(t)$$

hence

$$Q(t+s) - Q(t) = Q(s)A(t) = Q(s)Q'(t)$$

Write again

$$Q(t+s) - Q(t) = Q(s)Q'(t)$$

and, equivalently,

$$Q(t+s) - Q(s) = Q(t)Q'(s) .$$

Subtract, then

$$Q(t) - Q(s) = Q(t)Q'(s) - Q(s)Q'(t)$$

i.e.

$$Q(t) \frac{Q'(s) - 1}{s} = \frac{Q(s)}{s} (Q'(t) - 1) = \frac{Q(s) - Q(0)}{s} (Q'(t) - 1) .$$

Hence ( $s \rightarrow 0$ )

$$Q(t)Q''(0) = Q'(0)(Q'(t) - 1) = Q'(t) - 1$$

We know that  $Q'$  exists hence we differentiate the equation and get

$$Q'(t) Q''(0) = Q''(t)$$

hence

$$(\log Q')'(t) = \frac{Q''(t)}{Q'(t)} = Q''(0) =: \gamma \in \mathbb{R} \quad \text{for all } t.$$

Now we just work backwards.

Since  $Q'(0) = 1$ , by integration we get

$$\log Q'(t) = \gamma t \quad \text{i.e.} \quad Q'(t) = e^{\gamma t}$$

and (since  $Q(0) = 0$ ) again by integration

$$Q(t) = \int_0^t e^{\gamma s} ds = \frac{e^{\gamma t} - 1}{\gamma} .$$

(but if  $\gamma = 0$  we get  $Q(t) = t$ ) .

We go through the transformations

$$Q \rightarrow H \rightarrow D \rightarrow G^{\leftarrow} \rightarrow G$$

In order to identify the function  $G$ .

$Q \rightarrow H$ : Note that  $H(0) = 0$ . Write  $a := H'(0)$ .

$$H(t) \stackrel{\text{def.}}{=} H'(0) Q(t) = a \cdot \frac{e^{\gamma t} - 1}{\gamma}$$

and ( $H \rightarrow D$ )

$$D(t) \stackrel{\text{def.}}{=} H(\log t) = a \cdot \frac{t^{\gamma} - 1}{\gamma} .$$

$D \rightarrow G^{\leftarrow}$  : going further back recall that

$$D(t) = G^{\leftarrow} \left( e^{-\frac{1}{t}} \right) - G^{\leftarrow} (e^{-1})$$

hence (write  $b := G^{\leftarrow} (e^{-1})$  )

$$G^{\leftarrow} \left( e^{-\frac{1}{t}} \right) = b + a \cdot \frac{t^{\gamma} - 1}{\gamma}.$$



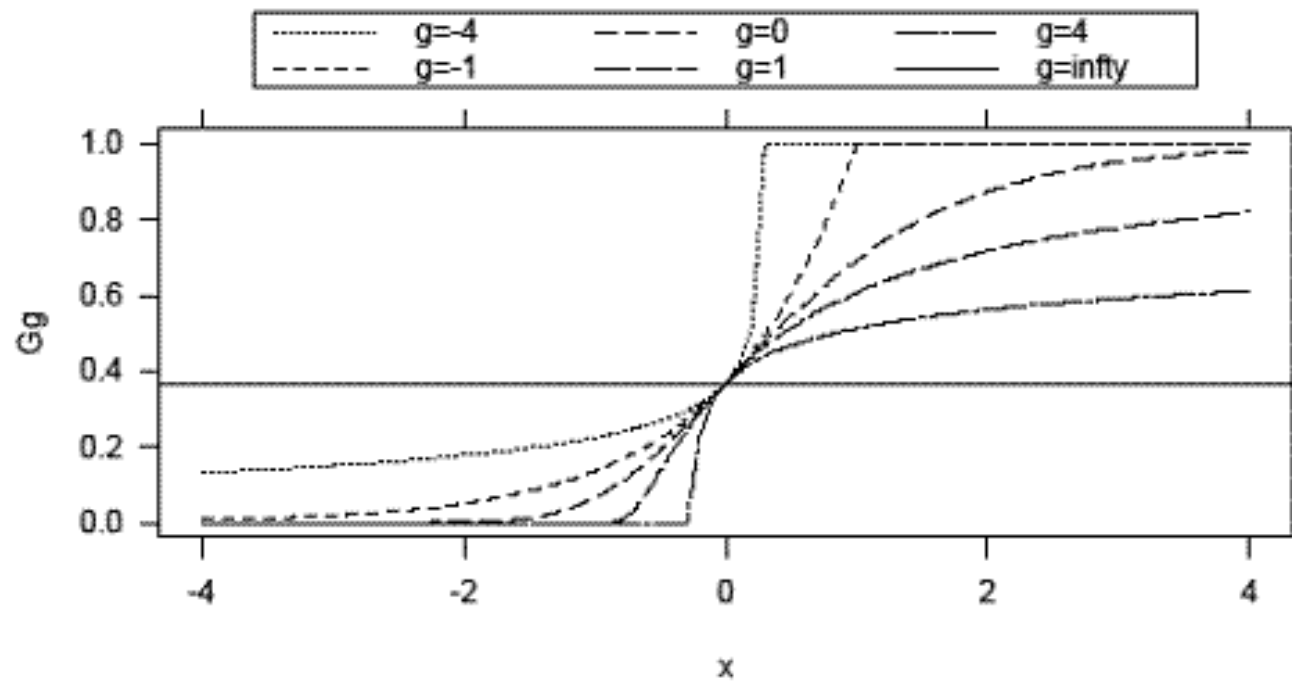
$G^{\leftarrow} \rightarrow G$ : apply  $G$  to both sides:

$$\exp\left\{-\frac{1}{t}\right\} = G\left(b + a\frac{t^{\gamma} - 1}{\gamma}\right).$$

Replace  $t$  by  $(1 + \gamma a^{-1}(x - b))^{\frac{1}{\gamma}}$ . We get

$$\exp\left\{-\left(1 + \gamma\frac{x - b}{a}\right)^{-\frac{1}{\gamma}}\right\} = G(x),$$

*Quod erat demonstrandum.*



Consider the graphs of  $G_\gamma$ .

Note that if  $\gamma < 0$

$$G_\gamma(x) = 1 \quad \text{for } x \geq -\frac{1}{\gamma}.$$

That means that no value beyond  $-\frac{1}{\gamma}$  is possible.

Define in general for a prob. dist. function  $F$

$$x^* = x^*(F) := \max \{x : F(x) < 1\} \leq \infty.$$

Note that for  $G_\gamma$  :

$$\begin{cases} \gamma > 0 \Rightarrow x^*(G_\gamma) = \infty \\ \gamma < 0 \Rightarrow x^*(G_\gamma) < \infty. \\ \gamma = 0 \Rightarrow x^*(G_\gamma) = \infty \end{cases}$$

If  $F^n(a_n x + b_n) \rightarrow G_\gamma(x)$ , for  $F$  we have similar behaviour

$$\begin{cases} \gamma > 0 \Rightarrow x^*(F) = \infty \\ \gamma < 0 \Rightarrow x^*(F) < \infty \\ \gamma = 0 : \text{can be both} \end{cases}$$

Hence :  $\gamma < 0 \Rightarrow x^*(F) < \infty$ .

*We consider the cases,  $\gamma > 0$ ,  $\gamma = 0$ ,  $\gamma < 0$  separately.*

$$1) \quad \gamma = 0: \quad G_0(x) = \exp(-e^{-x}).$$

Note that  $0 < G_0(x) < 1$  for all  $x$  hence the distribution has no lower or upper bound (all real values are possible). Also, since

$$\lim_{y \rightarrow 0} \frac{1 - e^{-y}}{y} = 1, \quad \text{we have with } y = e^{-x} :$$

$$\lim_{x \rightarrow \infty} \frac{1 - G_0(x)}{e^{-x}} = 1.$$

Hence the **tail** of the distribution ( $= 1 - G_0(x)$ ) goes down to zero very quickly. This means for example that all moments exists (are finite). We say that the distribution is **light tailed**.

2)  $\gamma > 0$ : Note that  $G_\gamma(x) < 1$  for all  $x$  hence there is no upper bound. Also, we see

$$\lim_{x \rightarrow \infty} \frac{1 - G_\gamma(x)}{x^{-\frac{1}{\gamma}}} = \gamma^{-\frac{1}{\gamma}} > 0$$

hence the tail is approximately a power function  $x^{-\frac{1}{\gamma}}$ .

This means that  $1 - G_\gamma(x)$  goes to zero much more slowly than in the case  $\gamma = 0$ .

In particular some moments are not finite. We say that in this case the distribution is **heavy tailed**.

**Note:** often in finance we have this case  $\gamma > 0$ .

3)  $\gamma < 0$ : Note that  $G_\gamma(x) = 1$  for all  $x \geq -1/\gamma$ .

Hence no values larger than  $-1/\gamma$  are possible.

We say that the distribution is **short tailed**.

**Note:** In environmental data we often find  $\gamma$  close to zero. In financial data we often find  $\gamma$  positive.



In some cases we can simplify the formula for  $G_\gamma$ :

1)  $\gamma > 0$ : In the formula

$$G_\gamma(x) = \exp\left\{-\left(1 + \gamma(ax + b)\right)^{-\frac{1}{\gamma}}\right\}$$

we can choose  $a = 1/\gamma$  and  $b = 1/\gamma$ . Then

$$G_\gamma(x) = \exp - x^{-\frac{1}{\gamma}}$$

In this case one simplifies by writing  $\alpha$  for  $1/\gamma$  and we get (traditionally)

$$\Phi_\alpha(x) = \exp - (x^{-\alpha}) \text{ for } x > 0 \text{ (and } = 0 \text{ for } x \leq 0).$$

In this form it is referred to as the **Fréchet class of extreme value distributions** ( $\alpha > 0$ ).

2)  $\gamma < 0$ : Take  $a = -\frac{1}{\gamma}$  and  $b = -\frac{1}{\gamma}$

in the formula

$$G_\gamma(x) = \exp\left\{-\left(1 + \gamma(ax + b)\right)^{-\frac{1}{\gamma}}\right\}$$

and write  $\alpha$  (again!) for  $-\frac{1}{\gamma}$ .

Then we get

$$\Psi_\alpha(x) = \exp\left[-\left(-x^{-\alpha}\right)\right] \text{ for } x < 0 \text{ (and } = 1 \text{ for } x \geq 1).$$

In this form it is referred to as the **reverse-Weibull class of distributions** ( $\alpha > 0$ ).

3)  $\gamma = 0$ :

$$G_\gamma(x) = \exp\{-e^{-x}\}.$$

This one is sometimes called the **Gumbel distribution**.

We are now able to reformulate the Theorem:

## Theorem

For  $\gamma \in \mathbb{R}$  the following statements are equivalent:

1) There exist real constants  $a_n > 0$  and  $b_n$  real, such that

$$\lim_{n \rightarrow \infty} F^n(a_n x + b_n) \rightarrow G_\gamma(x) = \exp\left(- (1 + \gamma x)^{-\frac{1}{\gamma}}\right), \quad (4)$$

for all  $x$  with  $1 + \gamma x > 0$ .

2) *There exists a positive function  $a$  such that for  $x > 0$*

$$\lim_{t \rightarrow \infty} \frac{U(tx) - U(t)}{a(t)} = \frac{x^\gamma - 1}{\gamma}, \quad (5)$$

*where for  $\gamma = 0$  the right-hand side is interpreted as  $\log x$ .*

3) *There exists a positive function  $a$  such that*

$$\lim_{t \rightarrow \infty} t(1 - F(a(t)x + U(t))) = (1 + \gamma x)^{-\frac{1}{\gamma}}, \quad (6)$$

*for all  $x$  with  $1 + \gamma x > 0$ .*

4) *There exist a positive function  $f$  such that*

$$\lim_{t \uparrow x^*} \frac{1 - F(t + xf(t))}{1 - F(t)} = (1 + \gamma x)^{-\frac{1}{\gamma}}, \quad (7)$$

*for all  $x$  which  $1 + \gamma x > 0$ , where  $x^* = \sup\{x : F(x) < 1\}$ .*

*Moreover (4) holds with  $b_n := U(n)$  and  $a_n := a(n)$ . Also (7) holds with  $f(t) = a(1/(1 - F(t)))$ .*

**Remark:**

We say that  $F \in D(G_\gamma)$  if the conditions of the Theorem hold for  $F$ . The parameter  $\gamma$  is called the extreme value index.

The class of distributions satisfying the condition is very wide.

The condition reflects a property of the far tail of  $F$ .

Let us look at three cases:  $\gamma > 0$ ,  $\gamma = 0$  and  $\gamma < 0$ .

$$\boxed{\gamma > 0}$$

It can be proved that in that case one can take  $f(t) = \gamma t$  in (7).

Hence  $F \in D(G_\gamma)$  with  $\gamma > 0$  if and only if

$$\lim_{t \rightarrow \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\frac{1}{\gamma}} \quad \text{for } x > 0$$

(“ $F$  has regularly varying tail”).



Such distribution function is called “heavy tailed” since

$$E\left(\max(X, 0)\right)^\alpha = \begin{cases} < \infty & \text{if } a < \frac{1}{\gamma} \\ = \infty & \text{if } a > \frac{1}{\gamma} . \end{cases}$$

Hence not all moments exist.

**Sufficient condition:**

$$\lim_{x \rightarrow \infty} \frac{x F'(x)}{1 - F(x)} = \frac{1}{\gamma} .$$

Examples: Cauchy's distribution  
Any Student distribution  
Pareto distribution  $F(x) = 1 - x^{-\frac{1}{\gamma}}$ ,  $x > 1$

$$\gamma = 0$$

## Sufficient condition:

$$\lim_{x \uparrow x^*} \frac{F''(x)(1-F(x))}{(F'(x))^2} = -1$$

where  $x^* := \sup \{x \mid F(x) < 1\} \leq \infty$ .

“Light tailed” since  $E(\max(X, 0))^a < \infty \quad \forall a > 0$

Examples: Normal distribution  
 Exponential distribution  
 Any Gamma distribution  
 Lognormal distribution  
 $F(x) = 1 + e^{1/x}$  for  $x < 0$

$$\boxed{\gamma < 0}$$

Then the probability distribution has an upper bound:

$$F(x) = \begin{cases} = 1 & \text{for } x \geq \text{some } x^* \\ < 1 & \text{for } x < x^* . \end{cases}$$

It can be proved that one can take  $f(t) = -\gamma(x^* - t)$ .

Leads to a simple criterion:

$$\lim_{t \downarrow 0} \frac{1 - F(x^* - tx)}{1 - F(x^* - t)} = x^{-\frac{1}{\gamma}} \quad \text{for } x > 0$$

(is again a kind of regular variation condition)

“Short tailed”

Examples: uniform distribution  
any Beta distribution

A sufficient condition valid for all domain of attraction:

If 
$$\lim_{t \uparrow x^*} \frac{F''(x)(1-F(x))}{F'(t)^2} = -\gamma - 1, \quad \text{then } F \in D(G_\gamma).$$

A necessary and sufficient condition (provided that  $x^* > 0$ ) is :

If 
$$\lim_{t \uparrow x^*} \frac{(1-F(t)) \int_t^{x^*} \int_y^{x^*} (1-F(x)) x^{-2} dx dy}{t^2 \left( \int_t^{x^*} (1-F(x)) x^{-2} dx \right)^2} = \begin{cases} 1 + \gamma & \text{if } \gamma > 0 \\ \frac{1-\gamma}{1-2\gamma} & \text{if } \gamma \leq 0, \end{cases}$$

then  $F \in D(G_\gamma)$ .

There are probability distributions that are not in any domain of attraction.

Examples:

geometric distribution  $F(x) = 1 - e^{-[x]}$  for  $x > 0$

Poisson distribution  $F(x) = \sum_{n=0}^x \lambda^n \frac{e^{-\lambda}}{n!}$  for  $x \geq 0$

von Mises' example  $F(x) = e^{-x - \sin x}$  for  $x \geq 0$

**Remark**

Let  $X$  be a r.v. with distribution function  $F$ .

Relation (7) can be reformulated as follows:

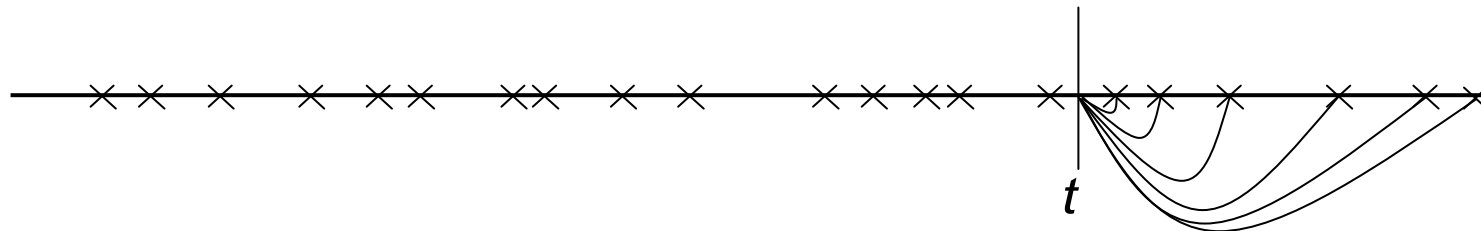
$$P\left\{\frac{X - t}{f(t)} > x \mid X > t\right\} \rightarrow (1 + \gamma x)^{-\frac{1}{\gamma}} \quad (t \rightarrow \infty) \text{ for } x > 0.$$

(Generalized Pareto distribution)

(model for residual life time)

# View towards applications

$n$  observations,  $t$  large





The overshoots of  $t$  are i.i.d. observations and they follow approximately a generalized Pareto distribution  $1 - (1 + \gamma x)^{-\frac{1}{\gamma}}$ ,  $\gamma \in \mathbb{R}$ .

They can be used to estimate the parameter of the Pareto distribution.

Then we can use the fitted Pareto distribution to estimate the distribution function beyond the observations.

In fact we take  $t$  to be one of the observations say, the  $k$ -th highest observation  $X_{n-k,n}$ .

We should choose  $k$  in such way, that  $k$  depends on  $n$ ,

$$k = k(n) \rightarrow \infty \quad (\text{allowing the use of CLT})$$

$$\frac{k(n)}{n} \rightarrow 0 \quad (\text{implies staying in the tail}).$$

Then we use only

$$X_{n-k,n}, X_{n-k+1,n}, \dots, X_{n,n}$$

for estimating the parameter of the Pareto distribution and also for estimating the probability of extreme events beyond the range of the sample.

The 8th Conference on Extreme Value Analysis  
July 8-12, 2013  
Fudan University, Shanghai, China

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We are pleased to announce that the 8th Conference on Extreme Value Analysis will take place from July 8 to 12, 2013 at Fudan University, Shanghai, China.

Organizers: Deyuan Li, Liang Peng, Zhengjun Zhang, Ming Zheng

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Topics:

- Univariate, multivariate, infinite dimensional extreme value theory
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- Heavy tails in actuarial sciences
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History: Previous EVA conferences have been held in Leuven, Belgium (2001), Lyon, France (2011), Vimeiro, Portugal (1983), Aveiro, Portugal (2004), twice in Gothenburg, Sweden (1998 and 2005), Bern, Switzerland (2007), Fort Collins, USA (2009).

