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LIKELIHOOD ANALYSIS OF POINT PROCESSES AND ITS APPLICATIONS TO SEISMOLOGICAL DATA

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1. INTRODUCTION

The monograph by Cox and Lewis (1966) and the volume editated by Lewis (1972) showed the development of statistical techniques in point processes and whole range of applications. Moreover Vere-Jones (1970) emphasized the applications of point process models in seismological statistics [see Vere-Jones and Smith (1981) for a survey in statistical seismology]. However the maximum likelihood estimation procedure is not fully developed, despite the general agreement of providing a sensible method for parameter estimation and a sensitive testing of models. A key to the likelihood theory of point processes is the conditional intensity function (C.I.F.) defined by

$$\lambda(t|F_t) = \lim_{\Delta\to 0} \text{Prob}\{\text{Event is } [t,t+\Delta)|F_t\}/\Delta \tag{1.1}$$

where F_t is a family of informations (σ-fields) over the time interval $(0,t)$ of observations including the history of the point process itself at time t. A C.I.F. characterizes a point process completely. For example, if a C.I.F. is the function of time t only, the point process is Poisson. Once the C.I.F. is given, simulation of the corresponding point process is easily performed by the so called "thinning technique" (Ogata, 1981) which is an extension of the method of Lewis and Shedler (1979). Also once the C.I.F. is given, the likelihood for the realization in $(0,T)$ can be written down in the form

$$f_T(t_1,t_2,\cdots,t_n;n) = \{\prod_{i=1}^{n} \lambda(t_i|F_{t_i})\}\exp\{-\int_0^T \lambda(t|F_t)dt\} \tag{1.2}$$

wnere n is the number of events observed in the interval.

Thus it now becomes important to obtain good parametric models of C.I.F. My principal aims are to describe a class of flexible parametric models, like AR model in time series, for the statistical analysis of earthquake catalogues and the assesment of earthquake risks in some areas (see Vere-Jones, 1978). In this paper I will show some examples of systematic modelling of C.I.F. and their applications to the earthquake data.

2. MAXIMUM LIKELIHOOD AND MODEL SELECTION

Consider a C.I.F. $\lambda_\theta(t|F_t)$ which is parameterized by a n-dimensional vector $\theta=(\theta_1,\cdots,\theta_k)$, and a series of events $\{t_1,\cdots,t_n\}$ which is observed in the time interval $[0,T)$. Then the log likelihood of the statistical model

$$\log L(\theta;t_1,\cdots,t_n;T) = \sum_{i=1}^{n} \log \lambda_\theta(t_i|F_{t_i}) - \int_0^T \lambda_\theta(t|F_t)dt \tag{2.1}$$

is a function of the parameter θ only. The maximum likelihood estimator of θ is the value of the parameter which maximizes the log likelihood. In general it is not easy to derive the maximum likelihood estimates explicitly. If the second term in (2.1) can be expressed analytically in θ, then the gradient of the log likelihood function can be easily obtained. In that case the maximization of the likelihood can be carried out by using a standard non-linear optimization technique such as in Fletcher and Powell (1963).

Suppose that we have to choose the best model among amny competing models. Then it is natural to have a measure to see which model will most frequently reproduce, through simulations, similar features to the give data $\{t_1,\cdots,t_n\}$. The Akaike Information Criterion (Akaike, 1974)

$$\text{AIC} = (-2)\text{maximum log likelihood} + 2(\text{number of parameters})$$

is the most suitable for such purpose. Here log denotes natural logarithm, and a model with a smaller AIC is considered to be a better fit. The AIC is an estimate of the expected negentropy (Akaike, 1977) which is a natural extension of Boltzmann's probabilistic interpretation of the thermodynamic entropy as the logarithm of the probability of getting a sample distribution. Conventionally the model selection is realized by successively applying the likelihood ratio test to the nested sequence of models. The relationship between the AIC and the likelihood ratio statistic is discussed in Sakamoto and Akaike (1978).

3. ANALYSIS OF AFTERSHOCK OCCURRENCE

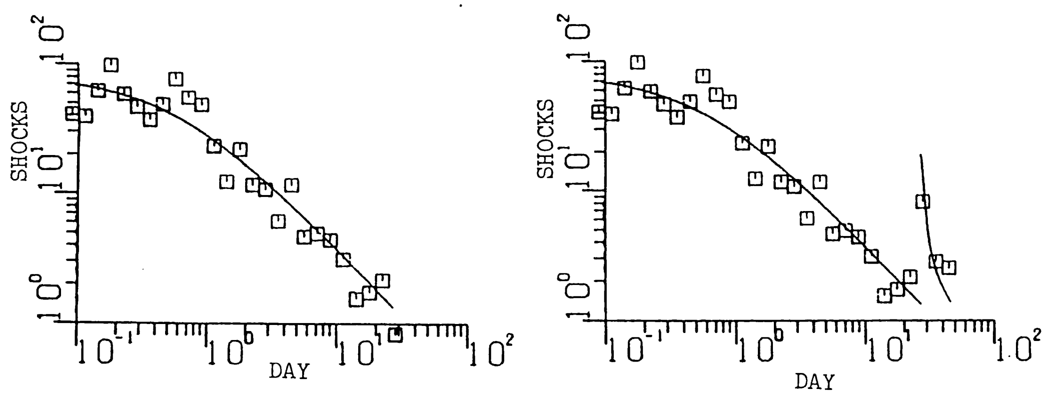
3.1. Traditional analysis

The frequency of aftershocks per unit time interval (one day, one month, etc.) is well represented by the modified Omori formula (Utsu, 1961)

$$n(t) = K(t+c)^{-p} \qquad (k,c,p\text{: parameters}) \tag{3.1}$$

where K depends on the magnitude of the main shock and the lower bound of the magnitude of aftershocks counted, while p is known to be independent of these. The value p is thought to reflect mechanical conditions of earth's crust. For example Mogi (1962) demonstrate a certain systematic regional distribution of p values in Japan. Estimates of the parameter p have been obtained in the following way: Plot n(t) versus time t on a log-log scaled plane and then fit an asymptotic straight line; the slope of the line is an estimate for p. The values of c can be determined by another graphical technique. For example the small squares in the first graph of Figure 1 are obtained by plotting n(t) for the time period up to 27 days immediately after the mainshock of the Tokachi earthquake in 1968. The occurrence time data of all aftershocks with the magnitude $M \geq 4.5$ for 45 days are listed in Appendix A1, based on the Seismological Bulletin of Japan Meteorological Agency (JMA).

FIG.1 FREQUENCY OF AFTERSHOCKS, Off Tokachi (1968)



3.2. Likelihood analysis

Consider now the occurrence times of aftershock sequence $\{t_1, t_2, \cdots, t_N\}$ in the time interval [S,T], where the origin of the time scale, t=0, corresponds to the occurrence time of the mainshock. Let us assume that the aftershock sequence is distributed according to a non-stationary Poisson process with the C.I.F.

$$\lambda(t;\theta) = K(t+c)^{-p}, \qquad \theta = (K,c,p), \tag{3.2}$$

which represent the modified Omori formula. Then from (2.1) the log likelihood function of the aftershock sequence is written by

$$\log L(K,c,p) = N \log K - p \sum_{i=1}^{N} \log (t_i+c) - K\, S(c,p), \tag{3.3}$$

where

$$S(c,p) = [(T+c)^{1-p} - (S+c)^{1-p}]/(1-p) \qquad \text{for } p \neq 1$$
$$\log (T+c) - \log (S+c) \qquad \text{for } p=1.$$

The maximum likelihood procedure has also the advantage of producing estimates for the asymptotic standard errors of parameters. In our case the Fisher information matrix is given by

$$J(\theta;S,T) = \int_S^T \frac{1}{\lambda(t;\theta)} \frac{\partial\lambda(t;\theta)}{\partial\theta'} \frac{\partial\lambda(t;\theta)}{\partial\theta} dt$$

$$= \int_S^T \begin{pmatrix} K^{-1}(t+c)^{-p} & -p(t+c)^{-p-1} & -(t+c)^{-p}\ln(t+c) \\ * & Kp^2(t+c)^{-p-2} & Kp(t+c)^{-p-1}\ln(t+c) \\ * & * & K(t+c)^{-p}\ln^2(t+c) \end{pmatrix} dt, \qquad (3.4)$$

and the inverse of the matrix (3.4) provides the variance-covariance matrix of the asymptotic standard error. Thus by the data given in A1 over the interval up to 27 days from the time origin, we obtained the following results.

TABLE 1

parameter	estimate	variance-covariance matrix		
K	63.66	$.1177\times10^4$	$.1800\times10^2$	$.7026\times10$
c	0.8799		.2942	.1054
p	1.227			$.4438\times10^{-1}$

The smoothed line in the first graph in Figure 1 is obtained by using (3.2) with the maximum likelihood estimates.

Smaller error variances are expected when the number of events is larger with very large mainshocks. However there might be the possibility of an estimation bias. One reason is that the rate of missing events in the beginning of a sequence can be rather large compared to the remainder. The other reason is that the early stages of the aftershock sequence can be more or less complicated. Thus in case of large samples the selection of the time interval [S,T] has to be taken into account. This defines a typical problem of model selection. For detail the reader is referred to Ogata (1982).

Now, from the data in A1 we might suspect that the Tokachi earthquake has secondary aftershocks, because of the strong clustering after about 27 days from the mainshocks. For such the case we consider a version of model of (3.2)

$$\lambda(t;\theta) = K_1(t+c_1)^{-p_1} + K_2(t-t_2+c_2)^{-p_2} H(t;t_2),$$
$$\theta = (K_1, c_1, p_1, K_2, c_2, p_2; t_2) \qquad (3.5)$$

where $H(t;t_2)$ is an indicator function defined by $H(t;t_2)=1$ for $t_2<t$, 0 otherwise. For the triggering shock of the secondary sequence, $t_2=27.5367$ day (which have the magnitude 7.2) was chosen. In this model we might be interested in knowing that whether $p_1=p_2$ holds or not. Using the AIC we can now compare the following these models;

AIC(no secondary aftershocks) = -2x255.4 + 2x3 = -504.8

AIC(a secondary aftershock $p_1=p_2$) = -2x337.8 + 2x6 = -663.6

AIC(a secondary aftershocks $p_1 \neq p_2$) = -2x338.2 + 2x7 = -662.4

which clearly suggest the existence of secondary aftershock rather with $p_1=p_2=1.06$. Parametric estimates are given in the Table 2 together with the estimated standard deviations which are obtained by the inverse of the Hessian matrix $-\partial^2 \log L/\partial\theta'\partial\theta$ of log likelihood function.

TABLE 2

parameter	K_1	c_1	p	K_2	c_2
estimate	44.58	0.5731	1.060	13.54	0.1103
s.d.	0.24	0.4113	0.085	0.22	0.6159

It can be also examined statistically by the similar modelling and model selection by AIC that whether a pair of aftershock sequences from different regions or eras have the same p value or not. This is quite similar situation to the problem of comparing the sample averages from normal distributions (see Ogata 1983).

4. LINEARLY PARAMETERIZED INTENSITY

Hereafter we consider a class of linearly parameterized C.I.F.,

$$\lambda_\theta(t|F_t) = \sum_{k=1}^{K} \theta_k Q_k(t|F_t), \qquad \theta = (\theta_1, \cdots, \theta_k), \qquad (4.1)$$

where $Q_k(t|F_t)$ are some statistics depending on the information F_t but independent of the parameters θ_k. One of the principal advantages of such parameterization is that the log likelihood function (2.1) has at most one maximum, which is seen from the fact that the Hessian matrix is everywhere negative definite in θ (see Ogata,1978, p.255). Therefore we do not have to worry about the initial guess of parameters. Moreover the second term in (2.1) is given by a linear combination of parameters. This enables us efficient calculations of likelihoods for the nested series $K=1,2,\cdots,K_{max}$. However the major disadvantage is that the C.I.F. in (4.1) can be negative. This is rather easy to take place when the number of parameter in (4.1) is large. Moreover this occasionally causes the difficulty in getting a maximum likelihood estimates, because the negativity of some t contributes the increase of likelihood due to the second term of (2.1). To avoid negative C.I.F. we have to impose restrictions on the parameters. In this paper we use a certain smooth penalty function $R(\theta)$ which takes the value 0 in the restricted region but takes large values on the outside, and then we minimize the following function

$$G(\theta) = -\log L(\theta) + R(\theta). \tag{4.2}$$

4.1. Trend/cyclic/clustering decomposition of earthquake risk

A systematic trend analysis by the likelihood procedure was carried out by MacLean (1974) and Lewis & Shedler (1976) fitting non-stationary Poisson processes with exponential polynomial intensity, which implies the numerical integrations of the second term in (2.1). Similarly the exponential trigonometric intensity for cyclic effect and its mixture with trend were suggested in Lewis (1970) and Lewis & Shedler (1976). However these were not computationally efficient to see the shape of the periodical change at a known frequency. Ozaki (1981) noted that in many cases a linear trigonometric parameterization

$$\lambda(t) = \sum_{k=1}^{K} A_k \sin(\omega_k t+\psi_k) \tag{4.3}$$

is useful, where the frequencies ω_k are suitably chosen and fixed, while the other parameters is to be optimized. For the clustering effect such as aftershocks or earthquake swarms, a "contagious" process in which the C.I.F. takes the form

$$\lambda(t|F_t) = \mu + \sum_{t_i<t} g(t-t_i); \quad g(x) \geq 0, \quad x > 0, \tag{4.4}$$

where the parameter $\mu(>0)$ can be interpreted as the rate of an underlying stationary Poisson process initiating clusters, and the function $g(x)$ (which we would like to call the response function of a point) measures the increase in risk due to an earthquake occurring at a time x time units before the time of measurement t. This model was introduced by Hawkes (1971) and has been fitted to earthquake data by Hawkes & Adamopoulos (1973) through the "spectral-likelihood". The fitting of a Hawkes model through the likelihood (2.1) was carried out by Ozaki (1978). It is important from the prediction viewpoint to estimate the response function. Akaike suggested a parameterization by the Laguerre type polynomial

$$g_M(x) = \sum_{m=1}^{M} a_m x^{m-1} e^{-cx} \tag{4.5}$$

in order to compute the likelihood efficiently (Ogata & Akaike, 1982 and Vere-Jones & Ozaki, 1982).

The model which we will use here takes form

$$\lambda(t|F_t) = \mu + P_J(t) + c_K(t) + \sum_{t_i<t} g_M(t-t_i). \tag{4.6}$$

The first component represents the evolutionary trend and is given by

$$P_J(t) = \sum_{j=1}^{J} \alpha_j \phi_j(2t/T-1) \tag{4.7}$$

where T is the total length of the observed interval and $\phi_j(x)$ are orthogonal polynomial expansion on [-1,1] such as the Legendre polynomial. The second component stands for the cyclic effect with a known fixed cycle length T_0 and is expressed by the Fourier expansion

$$c_K(t) = \sum_{k=1}^{K} \{\beta_{2k-1}\cos(2k\pi t/T_0) + \beta_{2k}\sin(2k\pi t/T_0)\} \tag{4.8}$$

The clustering effect corresponds to the last term of (4.6) in which we take the parameterization (4.5). If the scaling parameter c in (4.5) is fixed then the model (4.6) is linearly parameterized. The purpose of the present section is both examining the existence and estimating the shape of each component. This is possible by comparing the AIC values among the triplets (J,K,M).

It is not easy to define an explicit constraint in terms of parameters which ensures the non-negativity of C.I.F. throughout the time interval. Since the last term in (4.6) stands for clustering effect we put the following sufficient constraints for each (J,K,M)

$$g_M(x) \geq 0 \qquad \text{for } x \geq 0, \text{ and}$$

$$\mu + P_J(y) + C_K(y) \geq 0 \qquad \text{for } 0 \leq y \leq T. \tag{4.9}$$

In order to construct a computationary feasible penalty function in (4.2), we here choose suitable partitions of $[0,\infty)$ and $[0,T]$ to reduce (4.9) to linear constraints of parameters. That is to say, we charge a penalty if any integrals of $g_M(x)$ over the subintervals in $[0,\infty)$ are negative valued. Similar penalty is made for the second inequality in (4.9). More detail is described in Ogata & Katsura (1983).

The occurrence time data in Appendix A2 is selected from the Seismological Bulletin of JMA with the following restrictions; time interval between 1965 and 1980, shallow depth ($H \leq 60$ kilometer), Richter magnitude $M \geq 3.5$, and rectangle region from 131°E to 137°E and from 34°N to 38°N. The most shocks took place in the Inner Zone of the Southwest Japan. Here this is reproduced after the transformation into i-day, i.e., the time scale unit is one-day with the origin 0 of the time being equated to the 1st of January 1965. Before fitting the model (4.6) to the data, we had to fit the simpler model (4.4) to find a guess of the optimal exponential coefficient c in the response function (4.5). Orders M up to 15 were examined. M=9 attained the minimum AIC and the maximum likelihood estimate is c=1.854 [see Ogata, Akaike and Katsura (1982) detail for the calculation procedure]. In order to examine the effect of seasonality, T_0=365.25 in addition to M=9, c=1.854, was fixed in (4.6), and all the pairs (K,J) was examined up to 5 and 15, respectively. We obtained the second part of Table 3 below of AIC values. Also the first part was similarly obtained with the restriction of M=0, i.e., no effect of clustering.

TABLE 3 SOUTHWEST JAPAN DATA

clustering restrictions	M=0				M=9 (c=1.854)			
other restrictions	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0
minimum AIC, attained (K,J)	3037.7 (0,0)	3017.3 (0,9)	3015.2 (4,0)	3001.6 (4,9)	2854.1 (0,0)	2853.5 (0,1)	2849.5* (4,0)	2851.0 (4,1)

Comparing the AIC values, existence of clustering is clear. Moreover it is suggested that the seasonal effect exists but that the evolutionary trend is constant, since the overall minimum AIC is attained by (M,K,J)=(8,4,0). This was confirmed by comparing the similar tables for some other possible values c. Figure 2 displays the estimated shape of the clustering and seasonality components of the minimum AIC model (the trend component is not included since it is just constant μ=0.0562).

It is suggested in Oike (1977) that the monthly distribution of frequency of shallow shocks in the Inner Zone of the Southwest Japan shows the similar pattern to the rates of change of mean monthly precipitation. In the first graph of Figure 3 we plotted the averaged annual distribution of precipitations (broken line) over 30 years from 1941 through 1970 at Takamatsu, Shikoku Island, and its smoothed line obtained through BAYSEA programed by Akaike and Ishiguro (1980). This is more or less the common seasonal pattern throughout the Southwest Japan, except the area along the coast of Japan Sea where it is snowy in winter. The second graph shows the increasing rate (derivative) of the smoothed precipitation, which has similar variations to the seasonal component of Figure 2, especially at the typhoon season around early September and next to the dry season in winter. As is seen here, it is understood that the drastic change of rainfall can be a triggar of earthquakes.

FIG.2 DECOMPOSED COMPONENTS

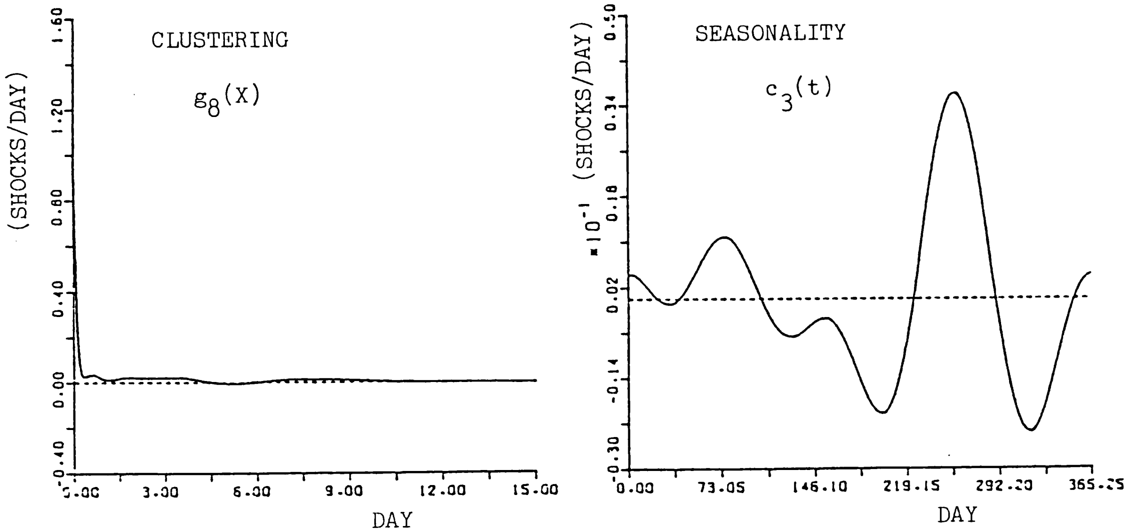
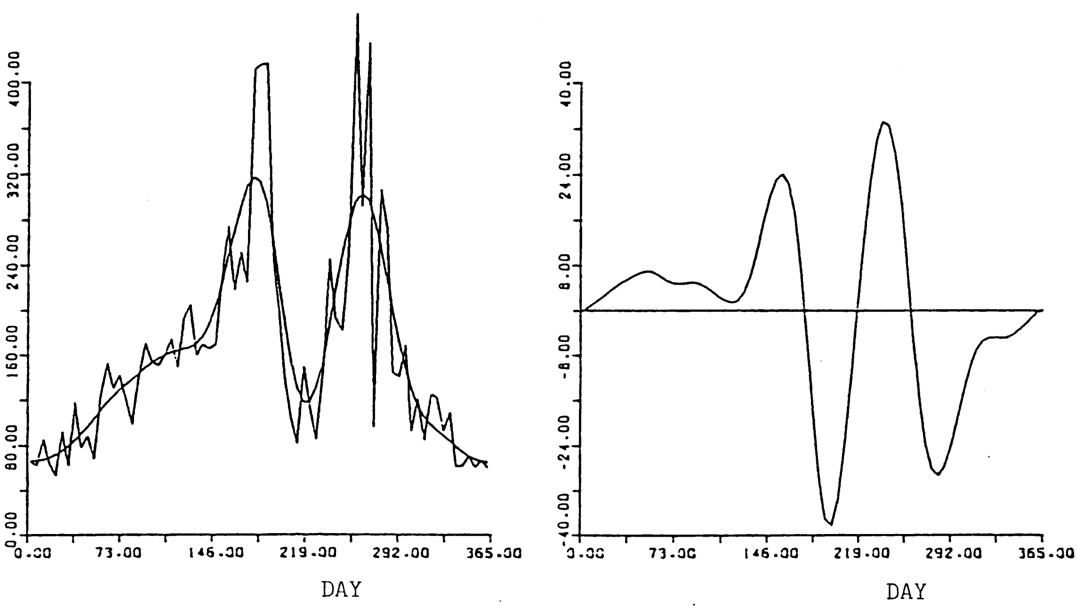


FIG.3 PRECIPITATION AVERAGE AND ITS CHANGE RATE



4.2. Relationship between shallow and deep seismicity

We extend the model (4.6) to the following

$$\lambda(t|F_t) = \mu + P_J(t) + c_K(t) + \sum_{t_i<t} g_M(t-t_i) + \sum_{u_j<t} h_N(t-u_j) \qquad (4.10)$$

where $\{u_j\}$ is another series of events considered as an input of the C.I.F. system, and the response function $h_N(x)$ is parameterized similarly to (4.5),

$$h(x) = \sum_{n=1}^{N} b_n x^{n-1} e^{-dt} \qquad (4.11)$$

If there is no causal relation between the input $\{u_j\}$ and the output $\{t_i\}$, the response function will be $h(x)=0$ for all $x\geq 0$. Otherwise we are interested in knowing an approximate shape of $h(x)$.

This model was applied to earthquake occurrence data supplied by Seismological observatory, Geophysics Division, DSIR of New Zealand for shallow and deep earthquakes in a region covering the North Island area, New Zealand, from 1946 through 1980. The area used was a quadrilateral with vertices at the points (40°S, 170°E), (35°S, 175°E), (39°S, 177°W) and (44°S, 178°E), and the shocks with magnitudes 5.5 or over were classified in the following two groups; shallow events with the depth $H\leq 40$km and deep events with $H\geq 150$km. Thus Appendix A3 is reproduced after transformation into i-day, i.e., the time scale unit is one-day with origin 0 of the time being equated to the 1st of January 1946. Since several aftershock events or swarms seems to be contained mainly in the earliest part of shallow earthquakes, we removed all such shocks that are clustering both in space (within a sphere with radius 30km) and time (within three months interval), except only one shock with the largest magnitude in each cluster. Similar process was made for the deep group of shocks. The numbers with an asterisk in Appendix A3 are such events to be omitted. In order to confirm that no clustering events are included in both the shallow and deep groups of events, we could have fitted (4.10) to each data. Instead, to save the computing time, we first fitted (4.4) with (4.5) of several orders to find a guess for the scaling parameter c and the order $M\geq 1$, and then fitted the model (4.6) for all pairs (K,J) up to 5 and 9, respectively, by the same way as in the previous section. The values of AIC to each models are listed in Tables 4 and 5.

TABLE 4 SHALLOW SHOCKS

clustering restrictions	M=0				M=1 ($c=0.312\times10^{-2}$)			
other restrictions	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0
minimum AICs, attained (K,J)	743.8 (0,0)	724.4* (0,8)	741.6 (1,0)	725.4 (1,7)	738.3 (0,0)	729.4 (0,7)	735.6 (1,0)	729.4 (1,7)

TABLE 5 DEEP SHOCKS

clustering restrictions	M=0				M=1 ($c=0.180\times10^{-2}$)			
other restrictions	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0	K=0 J=0	K=0 J≠0	K≠0 J=0	K≠0 J≠0
minimum AICs, attained (K,J)	1014.2 (0,0)	1011.8* (0,1)	1015.6 (1,0)	1013.6 (1,1)	1016.0 (0,0)	1015.2 (0,1)	1016.2 (2,0)	1016.5 (1,1)

Especially it is suggested that the seasonal effect does not exist in both the shallow and deep earthquakes with magnitude $M \geq 5.5$, although we cannot deny it very definitely for the shallow group because of the small difference between the minimum and second minimum AICs. (Incidentally it was found that the shallow shocks with smaller magnitude (say $M \geq 5.0$) clearly showed the seasonality.) Thus these preliminary results lead us to the use of the following simpler model than (4.10) to investigate the causal relations between the two series of shocks.

$$\lambda(t|F_t) = \mu + P_J(t) + \sum_{u_j<t} h_N(t-u_j) \qquad (4.12)$$

First, the series of shallow series is considered as the output $\{t_i\}$ and deep ones as the input $\{u_i\}$. A very practical and computationary efficient procedure is realized by restricting the exponential coefficient d in (4.11) to some finite number of candidate values [say, $d_j=[(\sqrt{5}-1)/2]^j$, $j=8,9,\cdots,20$ in the present case] and by comparing AIC values among the all doublets (N,J) up to N=5 and J=9, respectively. The Table 6 below presents the minimum AIC values with selected orders (N,J) and d_j among each restrictions.

TABLE 6 SHALLOW SHOCKS

restrictions	N=0,J=0	N=0,J≠0	N≠0,J=0	N≠0,J≠0
minimum AICs	743.9	725.0*	745.9	728.5
d_j attained	———	———	any	0.107×10^{-3}
attained (N,J)	(0,0)	(0,7)	(1,0)	(1,8)

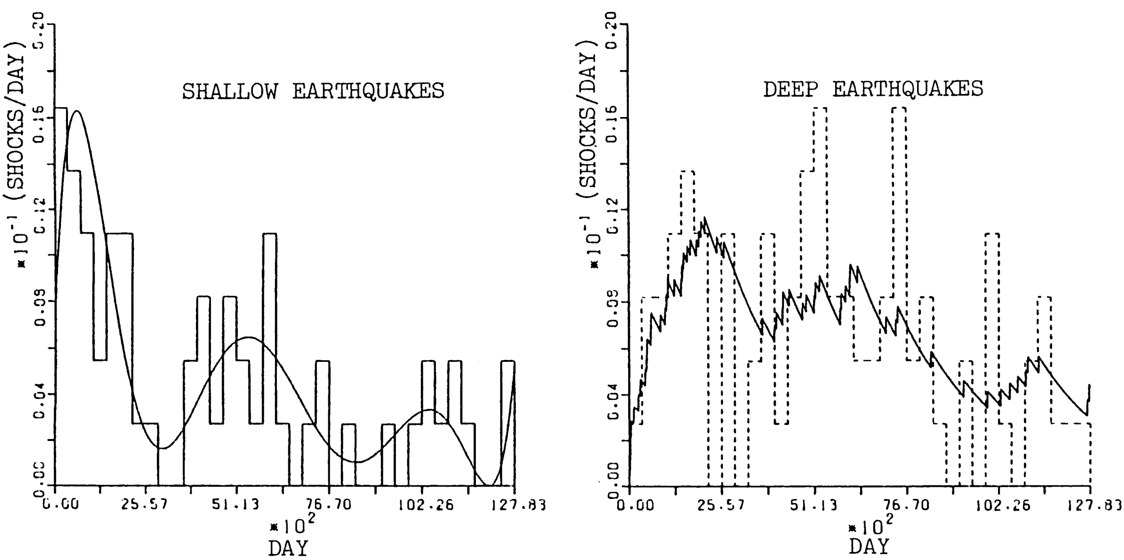
Comparing the Table 6 with Table 4, the overall minimum AIC was attained at the model of only trend components with J=7. This may suggest that the occurrence of the shallow earthquakes is not stimulated by the deep earthquakes. The first graph in Figure 4 below displays the shape of the estimated trend of shallow shocks. Histograms for number of shallow shocks in yearly intervals are also included.

To see what happens in the opposite direction, the values of AIC of the models (4.12) with the deep series as output $\{t_i\}$ and the shallow series as input were calculated for all different pairs of orders (N,J) up to N=5 and J=9, respectively, at each fixed $d_j=[(\sqrt{5}-1)/2]^j$, j=8,9,$\cdots$,19. Table 7 lists the AIC values of orders (N,J) with $d_{15}=0.453\text{x}10^{-3}$ which contains the overall minimum AIC among the different d_j. Thus the overall minimum AIC is attained at J=0 and N=1, and the estimated parameters of the model (4.12) are μ=0.000 (shocks/day), $c=0.453\text{x}10^{-3}$ (1/day) and $b_1=0.727\text{x}10^{-3}$ (shocks/day). The second graph in Figure 4 displays the estimated intensity (earthquake risk) of deep series with histograms for numbers of deep shocks in yearly intervals.

TABLE 7 DEEP SHOCKS

restrictions	N=0,J=0	N=0,J≠0	N≠0,J=0	N≠0,J≠0
minimum AICs	1014.2	1011.8	1007.8*	1009.9
d_j attained	———	———	$0.453\text{x}10^{-3}$	$0.453\text{x}10^{-3}$
attained (N,J)	(0,0)	(0,1)	(1,0)	(1,1)

FIG.4 ESTIMATED INTENSITIES AND YEARLY HISTOGRAMS



The results of the above two analyses clearly show that the earthquake occurrences in deep region, northern New Zealand, significantly receive one-way stimulation from earthquake occurrences in shallow region. Some readers may be remind of the results by Ogata, Akaike and Katsura (1982) in which the existence of the opposite one-way stimulation is concluded by the similar analysis of Utsu data (Utsu, 1975) in Japan. It was found after the present analysis that these two types of relationship between the shallow and deep seismicity in the sestern Pacific region was already discussed by Mogi (1973). Especially he found that the seismic activity in Mariana and Tonga areas gradually migrated from the shallow to the deep regions within the descending lithosphere, while the opposite migration is found in the Kurile-Kamchatka and northern Japan island-arc region. The migration rate or speed along the deep seismic zone of Tonga arc suggested by Mogi is about 45km per year, with which the above estimated response impulse function seems to be consistent (the mean distance between shallow and deep group of shocks of the data A3 is about 180km). These may indicate that the similar tendency is kept within the Tonga-Kermades-New Zealand techtonic zone.

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SUMMARY

The conditional intensity function is known to characterize a point process, and also provides the explicit form of the likelihood function. In this paper classes of flexible parametric models are developed to carry out the maximum likelihood calculation efficiently. Model selections are performed by using the Akaike Information Criterion. The modified Omori model is fitted to investigate the reducing rate of aftershock frequencies. Then the linearly parameterized intensity models are fitted to carry out the decomposition of the intensity of events into components of evolutionary trend, clustering and periodicity (seasonality), and also to investigate the causal relationship betwen two data sets of point processes. For the demonstration of the use of these models, Japanese and New Zealand earthquake data are considered.

RESUME

Il est connu que la fonction d'intensité conditionelle caractérise un processus ponctuel, et elle donne aussi une forme explicite de la fonction de vraisemblance. Dans cet article nous développons des modèles paramétriques flexibles pour executer les calculations du maximum de vraisemblance d'une manière efficace. Pour la sélection d'un modèle nous nous servons du critère d'information de Akaike. Le model de Omori modifié est ajusté pour examiner le degré de réduction dans la fréquence des répliques d'un séisme. Ensuite nous ajustons des modèles où l'intensité dépend linéairement des paramètres pour effectuer la décomposition de l'intensité des évènements en composants d'une tendance évolutionnaire, des agglomérations et des périodicités saisonnières. En plus, nous étudions à l'aide de ces modèles les rapports de cause entre deux processus ponctuels. Pour démontrer l'usage de ces modèles, nous considérons des données des tremblements de terre au Japon et en Nouvelle-Zélande.

APPENDIX A1 AFTERSHOCK, OFF TOKACHI (1968)

0.01100	0.04369	0.08969	0.11621	0.14363	0.15708
0.17309	0.18245	0.18492	0.19645	0.20540	0.23708
0.24163	0.27717	0.29173	0.31325	0.33174	0.33979
0.35182	0.40982	0.46422	0.47268	0.48997	0.49773
0.51948	0.52165	0.52568	0.52831	0.54211	0.55122
0.56068	0.58583	0.60519	0.62755	0.64223	0.64785
0.66695	0.68950	0.69397	0.71203	0.74614	0.76936
0.77273	0.80197	0.80810	0.81478	0.82781	0.84341
0.85905	0.92775	0.96027	0.96360	0.98300	1.06913
1.07900	1.13786	1.15800	1.18794	1.23307	1.34233
1.41237	1.47533	1.50953	1.58627	1.63436	1.69381
1.72789	1.74532	1.79169	1.83837	1.90785	1.93630
2.02421	2.16822	2.17161	2.26080	2.34640	2.50454
2.55484	2.61444	2.69351	2.76961	2.98542	3.05856
3.08916	3.38830	3.51007	3.51543	3.59393	3.89434
4.07141	4.10237	4.14293	4.15874	4.19909	4.22666
4.25323	4.27662	4.57145	4.65105	4.69091	4.96440
5.07029	5.14060	5.59815	5.61047	5.63574	6.17669
6.41875	6.44076	6.58421	6.62539	6.77814	6.93738
7.56702	7.84637	7.96353	8.54567	8.55380	8.79083
8.86494	8.95041	9.46113	9.53404	9.56256	10.55232
10.70331	10.73993	10.92384	11.22847	11.40914	12.48086
12.57044	12.77230	12.92481	13.33782	14.34405	15.79381
16.00492	16.40479	17.29936	18.20746	19.59813	19.61244
19.68538	20.26890	20.35707	21.15397	21.73091	21.85302
22.91530	23.08034	23.83741	24.54115	24.71548	25.07625
26.84277	27.53669	27.56146	27.56833	27.56978	27.57589
27.57998	27.58546	27.58830	27.59719	27.60761	27.61828
27.62505	27.64888	27.65299	27.69053	27.71048	27.74997
27.75473	27.78295	27.78458	27.79136	27.80853	27.81350
27.85259	27.88108	27.89537	27.91885	27.96951	27.99536
28.03750	28.05332	28.20552	28.33283	28.46351	28.58843
28.64424	28.77940	28.84838	28.95079	28.99805	29.00622
29.03314	29.10370	29.21953	29.32843	29.46091	30.06984
30.11277	30.46851	30.79464	31.24376	31.45073	31.82627
32.46113	32.66390	32.67176	32.75118	32.75589	32.89470
33.28421	33.33858	33.68177	34.03428	34.71836	34.76673
35.30988	35.72465	35.76244	36.64848	37.01642	37.34667
37.42843	38.17289	38.18864	38.56786	40.94748	41.21214
41.39927	41.81761	42.68266	42.87870	42.93399	43.05714
43.36220	43.37167	43.56603	43.72753	44.14713	

APPENDIX A2 SOUTHWEST JAPAN, 1965-1980

0.55185	56.65478	59.01948	59.16178	62.18566
64.05680	65.12051	92.32569	95.79133	98.72842
127.12600	147.70163	148.72135	165.43222	195.80255
196.36056	200.54450	285.52526	345.66464	350.28550
351.89263	361.15919	379.80892	388.74662	430.37357
433.53381	435.06099	441.51291	471.15394	479.59011
490.00923	510.32620	530.72962	539.14792	544.89024
579.44229	585.34047	611.20516	634.90734	640.23018
645.35955	672.23544	680.93440	682.96206	726.03466
731.23352	752.66424	772.16355	829.99117	833.48425
846.61218	876.71747	901.88184	903.21791	911.79545
921.73438	948.86906	972.23092	986.40002	994.93872
1001.19558	1001.34239	1005.59203	1064.89725	1081.61927
1114.48029	1118.70253	1130.71283	1139.48016	1156.44833
1156.45170	1164.91750	1166.76617	1173.52309	1179.44324
1184.16978	1184.17336	1184.21202	1187.11158	1207.27947
1217.44484	1218.93505	1224.97367	1225.00237	1245.82624
1254.34396	1288.85767	1296.30555	1298.72971	1325.67515
1325.67583	1325.68042	1325.87783	1328.32205	1329.55334
1330.68395	1334.91578	1334.95340	1335.68149	1338.46816
1341.26278	1342.00045	1345.06682	1349.61746	1356.65760
1364.28576	1371.20434	1374.19173	1383.16239	1419.75774
1448.28505	1462.90059	1497.37353	1507.85904	1509.28969
1509.40968	1509.61797	1534.02095	1534.43275	1602.03184
1611.82267	1625.04391	1628.07315	1631.64834	1631.67208
1646.76105	1649.32815	1650.48924	1650.80683	1697.33640
1713.24800	1713.49728	1714.75652	1715.15177	1721.09402
1736.42365	1745.20457	1751.07522	1757.97728	1766.45231
1782.40752	1789.95391	1798.90806	1814.88640	1824.05900
1828.61271	1884.50997	1896.90339	1897.93545	1900.04505
1905.52766	1907.28831	1918.41820	1922.07066	1935.75074
1937.98594	1952.71955	1962.57998	1966.71480	1967.29349
2020.51029	2034.52809	2036.30898	2039.06502	2042.95791
2058.15813	2058.24565	2058.58172	2097.79973	2152.04680
2159.55232	2179.64903	2216.38497	2316.21012	2325.55703
2333.35761	2337.19204	2362.76923	2441.32421	2449.92620
2455.42316	2483.22290	2488.15882	2518.67510	2558.78623
2576.62651	2576.64644	2588.77666	2590.92311	2619.01751
2644.28946	2660.18686	2660.19307	2675.74525	2676.43205
2692.85181	2694.55578	2704.85473	2764.63290	2780.25576

(to be continued)

(continued from the previous page)

2799.70438	2799.71344	2802.89975	2806.72866	2813.58704
2813.63042	2824.46666	2848.20496	2857.13280	2889.70304
2916.18634	2928.49221	2932.09026	2932.44707	2943.82783
2956.33346	2975.66524	2976.54425	2977.79843	2989.81326
3011.76138	3018.72086	3053.96415	3100.24275	3158.79924
3176.74081	3185.47297	3185.64119	3185.84667	3186.42340
3188.16275	3191.15034	3191.15064	3199.74833	3221.57226
3250.38737	3266.96397	3304.28662	3304.54439	3310.33258
3324.18685	3327.74423	3327.88124	3338.59219	3348.46860
3372.93329	3407.25617	3469.37662	3514.92551	3526.61785
3539.35063	3548.98228	3598.06678	3607.11951	3640.75782
3712.69058	3724.95575	3739.68287	3743.56293	3779.01948
3801.09786	3807.88796	3825.95069	3831.85249	3874.78978
3915.68364	3918.44677	3942.83565	3943.99778	3952.58585
3977.95725	4008.65548	4029.39121	4035.80705	4044.46137
4052.40273	4069.48951	4077.86940	4121.26526	4124.34496
4222.40844	4224.91823	4245.36516	4249.43524	4268.60349
4274.90441	4278.36109	4291.31875	4294.17949	4308.20432
4332.25341	4349.82554	4364.16630	4372.33647	4378.45107
4386.11664	4387.32612	4398.55300	4398.60136	4398.68947
4422.42947	4442.90365	4504.05767	4504.17173	4505.81063
4532.18817	4563.58544	4598.03987	4600.11427	4601.18561
4602.08987	4602.12206	4603.73318	4609.87719	4611.02270
4611.19981	4612.20087	4638.28655	4643.51537	4655.68277
4674.97260	4711.53715	4741.65435	4754.25089	4774.83453
4786.14808	4799.34710	4830.48213	4834.11215	4840.46120
4843.02679	4853.35591	4877.91041	4893.94684	4894.80278
4902.21103	4902.21313	4902.21466	4902.21662	4902.22050
4902.22509	4902.22925	4902.25219	4902.26457	4902.26589
4902.26915	4902.27705	4902.31592	4902.45701	4902.80177
4902.91548	4903.00243	4911.01123	4912.48898	4935.47644
4937.98475	4958.57329	4974.33823	4993.06744	4993.13424
4996.64507	5026.83155	5027.55475	5030.13227	5030.16836
5040.66680	5048.47794	5060.75177	5086.35009	5146.00717
5148.13041	5157.75353	5164.19986	5169.74059	5172.34240
5180.82862	5183.82193	5187.44198	5188.93280	5193.68007
5211.92563	5213.51724	5214.58261	5219.85166	5262.43953
5264.31457	5295.34517	5344.18178	5361.84670	5367.39926
5371.42530	5373.42047	5398.68750	5399.07227	5401.32360
5437.35839	5474.99587	5479.94386	5483.00279	5500.25971
5561.45085	5591.59966	5619.43774	5619.71275	5674.48433
5710.19205	5732.86537	5738.77498	5739.50621	5739.59513
5748.89628	5755.10489	5758.31471	5814.00892	5833.95446
5837.95174	5842.76251			

APPENDIX A3 NEW ZEALAND DATA, 1946-1980

SHALLOW EARTHQUAKES

34.908	42.262	56.230	128.174	266.278
311.268	448.856	489.087	501.296	603.568
603.681*	604.630*	611.587*	615.917*	622.385
676.945*	874.725	1005.915	1043.208	1070.347*
1073.845	1257.219	1447.400	1467.598*	1472.867
1472.868*	1494.671*	1519.791	1532.402*	1610.841
1708.086	1866.144	1938.285	2000.196	2101.735
2430.445	2632.467	3681.363	3681.418*	3724.655*
4013.600	4069.021	4240.217	4250.242	4413.273
4781.975	4889.290	5110.999	5146.098	5255.043
5839.992	5866.285	5973.188	6088.845	6108.631
6113.611*	6310.362	7105.389	7367.999	7417.284
8339.064	9248.605	9912.613	10285.583	10535.444
10752.425	10986.126	11035.749	11035.754*	11339.237
12695.648	12745.182			

DEEP EARTHQUAKES

412.409	556.215	717.670	736.894	873.507
957.848	1109.196	1134.730	1146.810	1244.759
1473.426	1511.448	1628.664	1662.199	1704.013
1837.492	1912.080	2026.420	2133.849	2688.808
2702.054	2715.859	2828.067	3485.090	3611.878
3662.236	3671.751	3690.440	4003.238	4089.383
4489.158	4662.318	4726.294	4771.451	4853.052
4900.721	4959.862	5063.231	5194.064	5194.067*
5198.978	5198.979*	5296.561	5324.660	5375.129
5435.262	5511.523	5685.388	5746.591	5920.673
6013.262	6116.473	6231.035	6378.694	6626.075
6717.932	7020.256	7280.352	7281.754	7373.293
7378.701	7482.428	7482.908	7509.451	7667.964
7792.451	8005.724	8159.379	8203.749	8339.574
8690.684	9175.337	9235.026	9865.579	9874.840
10034.950	10222.256	10413.649	11091.209	11312.851
11395.522	11436.076	11545.617	11952.446	12357.397
12773.841				