Are foreshocks really mainshocks with larger aftershocks?

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Summary

Earthquakes cluster in space and time. In retrospect, the largest earthquake in a cluster is called the mainshock. Earthquakes preceding the mainshock are called foreshocks and earthquakes following the mainshock are called aftershocks. No physical differences between these earthquakes are known. However, some researchers argue that foreshocks are triggered by the nucleation phase of the up-coming mainshock (e.g. Dodge et al. 1995). Therefore they could have different characteristics than aftershocks. In contrast most short term earthquake models treat all earthquakes consistently as if they obeyed a single triggering mechanism. (e.g. Felzer et al., 2003, Console et al., 2004, Zhuang, et al., 2004).

We review some empirical laws for earthquake clustering. They can be used to derive foreshock probabilities. Reasenberg and Jones (1989, 1994) developed such a model for California which gives a probability of 10% for an earthquake to be a foreshock. Actual observed foreshock rates are usually around 5%.

To investigate the discrepancy between the model prediction and the observations we review an earlier model for aftershock rates by Utsu (1969). We also present a new model based on the average number of aftershocks in a cluster using an example from global catalogue analysis. We derive model parameters for the new model from the Californian catalogue. The two later models predict foreshock rates that are consistent with the data.

We conclude that no separate triggering mechanism is required to derive foreshock rates from aftershock models and thus foreshocks can be regarded as mainshocks with larger aftershocks.

Empirical laws for aftershock occurrence

Empirical laws have been used successfully to model the short-term clustering of earthquakes. Omori's law describes the decay of earthquake activity with time

$$d N / dt = K / (t + t_C)^p \qquad (1)$$

where dN is the number of earthquakes in the time interval dt; K is a parameter that is proportional to the aftershock productivity; p describes the decay and takes values around 1.0; and c stands for a small time interval just after the mainshock (e.g. Utsu et al., 1995).

The Gutenberg-Richter relation describes the magnitude-frequency distribution

$$\log_{10} N(M) = a - b M. \qquad (2)$$

where N (M) is the number of earthquakes of magnitude M and a and b are parameters (e.g. Gutenberg and Richter, 1949).

Another empirical relation describes the increase of the average number N of aftershocks with mainshock magnitude M (e.g. Singh, and Suárez, 1988)

$$\log_{10} N(M) = c M - d. \qquad (3)$$

If the number of aftershocks scaled with the area of the aftershock occurrence, then the parameter c would be expected to be 1.0.

The Californian generic model

Reasenberg and Jones (1989, 1994) combined equations (1) and (2) to determine the rate of aftershocks of magnitude M and above, at time t following a main-shock of magnitude M_m:

$$R(t, M_m) = 10^{a' + b (Mm - M)} / (t + c)^p \qquad (4)$$

Reasenberg and Jones analysed 62 Californian earthquake sequences and derived the parameters a' = -1.67, p = 1.08, b = 0.91 and c = 0.05. These parameters have become known as the generic Californian aftershock model parameters (e.g. Gerstenberger et al., 2004).

The rate can be used to calculate the probability that at least one earthquake of magnitude M or above occurs in the time interval $[t_1, t_2]$ as follows:

$$P = 1 - \exp \int R(t,M)\, dt \qquad (5)$$

Equation (5) yields a probability of 10% that an earthquake is followed by one of the same magnitude or larger within one week. However, the model over-estimates foreshock occurrence in California by about a factor of 2.

Summary of observed foreshock rates

Table 1 compares some previously reported foreshock probabilities with recalculated values from our global catalogue. We found good agreement with previous observations when matching our space, time and magnitude window. All observations are significantly lower than predicted by the aftershock models. We note that we found some significant variation in foreshock probabilities between different tectonic settings which we do not show here though. However, all these probabilities were still smaller than expected from the model.

Table 1: Comparison of previously reported and recalculated foreshock probabilities in different magnitude, space and time windows.

Previously reported foreshock probabilities			Recalculated foreshock probabilities	
Location, catalogue threshold, space and time window	Magnitude restriction	Foreshock probability	Foreshock probability	Comments
California, (Jones, 1985)		6%	4.69±0.01 %	For magnitude 3.5 and larger
Global catalogue, completeness M ~ 5.6, 75 km and 7 days (Reasenberg, 1999)	MS ≥ 5.0	7.5%	4.2% 0.2%	The discrepancy between for MS ≥ 5.0 earthquakes can be explained by incomplete data in the Reasenberg catalogue.
	MS ≥ 6.0	2.3%	1.9±0.1%	
	MS ≥ 7.0	0.4%	0.5±0.1%	
New Zealand, 30 km and 5 days (Savage and Rupp, 2000)	MS ≥ 5.0	4.5 ± 0.7%	4.3±0.3%	The recalculated values of old subduction zones are compared. Old subduction zones have the highest foreshock and aftershock rates.
	MS – FS ≥ 1.0	0.8±0.3%	0.8±0.1%	

A Japanese model

According to Yamanaki and Shimazaki (1990), Utsu (1969) defined the rate of aftershock occurrence to be:

$$R(t, M_m) = 10^{b (Mm - Mo) - a} / (t + c)^p \qquad (6)$$

with the parameters a = 1.83, p = 1.3, b = 0.85 and c = 0.3. Using equation (5), the probability that an earthquake is followed by one of the same magnitude or larger within on week can be calculated to be 4.2%. This is significantly smaller than the result from the Californian generic model and agrees reasonably well with observation from other studies (see table 1).

A new model based on aftershock numbers

We fit equation (3) to the average number of aftershocks N_m above a chosen magnitude M_0 and in a time interval $[t_1, t_2]$ for which we expect the data to be complete. The modified Omori's law (equation 1) can be used to relate the expected number of aftershocks in one time interval to the expected number of aftershocks in a different time interval $[t_3, t_4]$. For this purpose we define $N_t = \gamma N_m$. Then γ can be expressed as

$$\gamma = \ln [(t_4 + c) / (t_3 + c)] / \ln [(t_2 + c) / (t_1 + c)] \qquad \text{for } p = 1 \qquad (7)$$

$$\gamma = [(t_4 + c)^{1-p} - (t_3 + c)^{1-p}] / [(t_2 + c)^{1-p} - (t_1 + c)^{1-p}] \qquad \text{for } p \neq 1 \qquad (8)$$

The expected rate in the time interval $[t_3, t_4]$ can be written as

$$R = 10^{c(M-M1)}\, 10^{-b (M - Mo)}\, \gamma \qquad (9)$$

The magnitude M_1 (= d/c according to equation 3) corresponds to the mainshock that on average produces one aftershock above the completeness magnitude M_0 within the chosen time interval $[t_1, t_2]$.

The Californian catalogue

For our analysis we used the catalogue of the ANSS (Advanced National Seismic Systems) for 1984-2004 between latitude 31-43 North and longitude 127-112 East. Data were restricted to events with depths shallower than 40 km. The completeness magnitude outside an on-going earthquake sequence can be as low as 1.2 (Woessner and Wiemer, 2005). However, as we are mainly interested in earthquake clusters we initially restricted the data to events above magnitude above 2.49. The catalogue included about 45,000 earthquakes.

Defining earthquake clusters

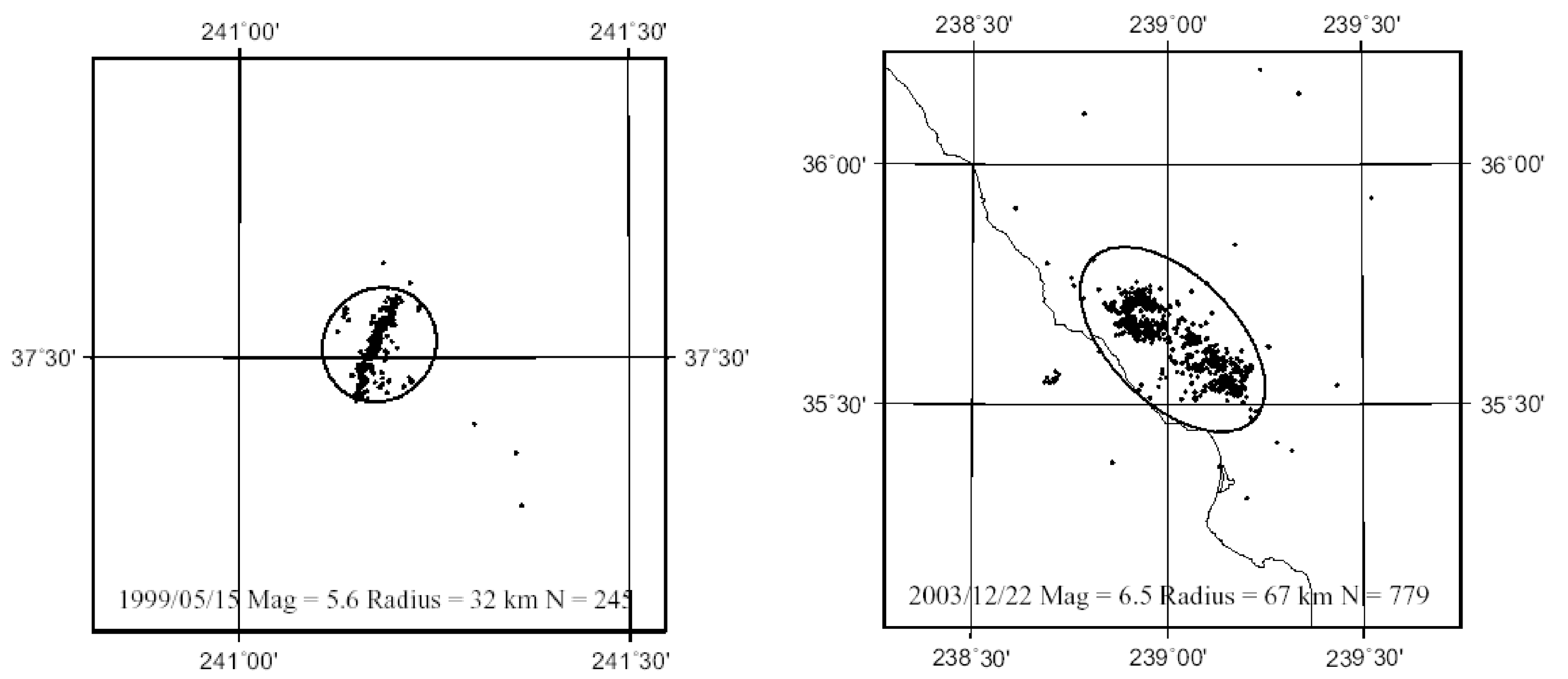
We defined earthquake clusters via a two step process. First the catalogue was searched for sequences containing at least one earthquake above a minimum magnitude. We trialled different search radii (as summarised in table 2) and different minimum magnitudes. The initial search radius is important for finding the optimal clusters. Each sequence with at least 3 earthquakes was fitted by an ellipse using the scatter of the epicentres (for more details see Christophersen, 2000). We visually inspected the ellipses by plotting each cluster centred on the middle of the ellipse and with map width of the search radius. Figure 1 shows two examples for the adjusted Uhrhammer window sizes. If the initial search radius is too small, earthquakes tend to occur close to the map's edge. If the search radius is too large, significant background is included and the ellipse does not fit the main cluster well.

In time a window of at most 10 days from the previous most recent event associated with the sequence was used to determine the duration of the sequence.

Table 2: Overview of different spatial search radii.

	Search radius in km for different methods						
Method\ magnitude	**3**	**4**	**5**	**5.5**	**6**	**6.5**	**7**
Christophersen, 2000			8	14	26	47	84
Gardner&Knophoff, 1985	22.5	30	40	47	54	61	70
Uhrhammer, 2005			20	30	45	67	100
Uhrhammer adjusted	4	9	20	30	45	67	100

Figure 1: Two examples of the ellipse fitting method. Each map is centred on the ellipse and had a half width of the search radius associated with the mainshock



Fitting the mean abundance and model results

Our data set of earthquake clusters covers mainshocks in the magnitude range 3.0 – 7.2. Figure 2 shows a scatter plot of aftershock magnitudes and their time of occurrence in days after the mainshock. As thoroughly discussed by Kagan (2004), the completeness magnitude sharply increases following a large earthquake for a period that depends on the size of the earthquake. For Landers there are hardly any earthquakes of magnitude 4.5 or below in the first 0.1 days and there are significant gaps between magnitude 2.5 and 3 for all of the first day. Smaller mainshocks on the contrary have shorter duration and therefore it is a challenge to find a time interval in which aftershocks are expected to be completely recorded. We compromised by fitting data in the time interval [0.1,10] days for mainshocks with magnitudes between 3.0 and 6.5. We counted earthquake of magnitude 2.6 and above. Figure 3 shows the mean aftershock number and a least squares fit with slope 0.94 and intercept of 3.95. M1, the magnitude that on average has one aftershock of magnitude 2.6 or larger in the time interval [0.1,10], is thus 3.72.

The b-value for all aftershocks within the ellipse over all earthquake clusters is 0.92 ±0.02. Assuming a p-value of 1.0 in the modified Omori's law the probability of an earthquake to be a foreshock in a time interval of one week is 4.9%. For the generic Californian p-value of 1.02, this probability is 4.8%. This is in excellent agreement with the observation from the data that an earthquake of magnitude 3.5 or larger has a probability of 4.69 ±0.01% of being a foreshock.

Figure 2: Scatter plots of aftershock magnitudes in time (a) for the Landers 1992 earthquake and (b) all mainshocks with magnitude between 5 and 5.5.

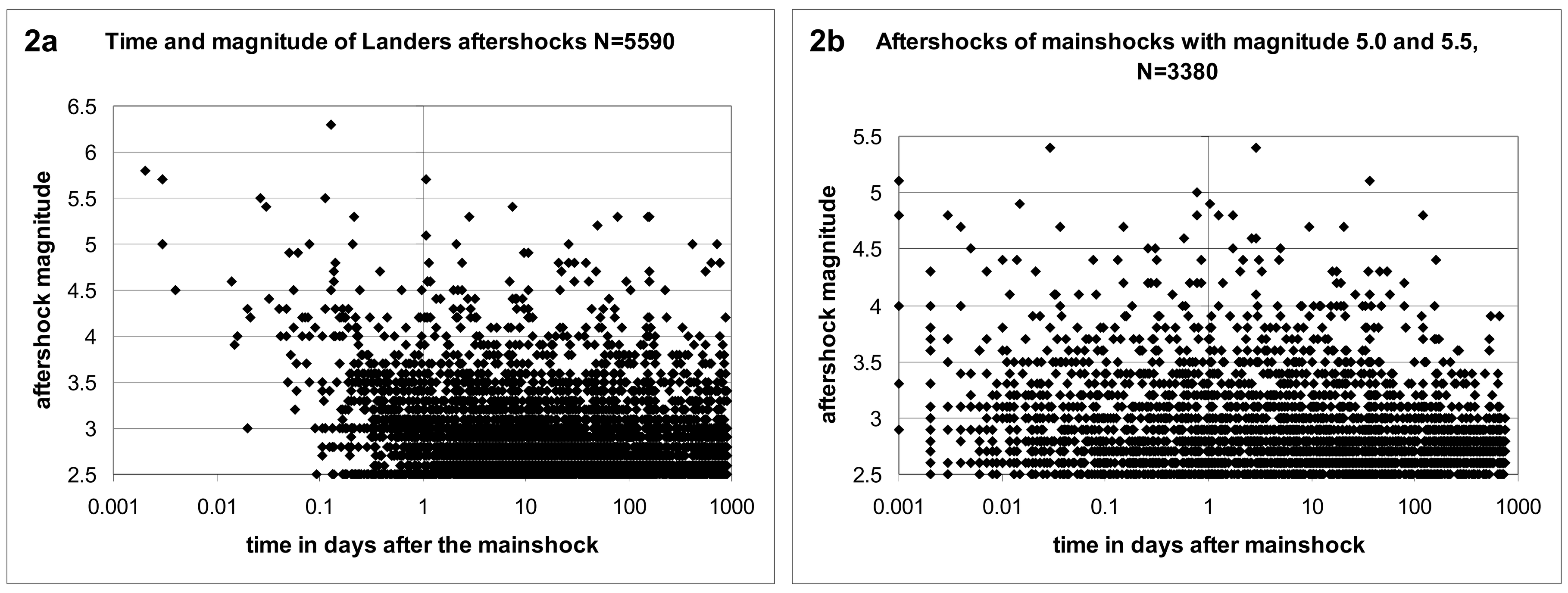
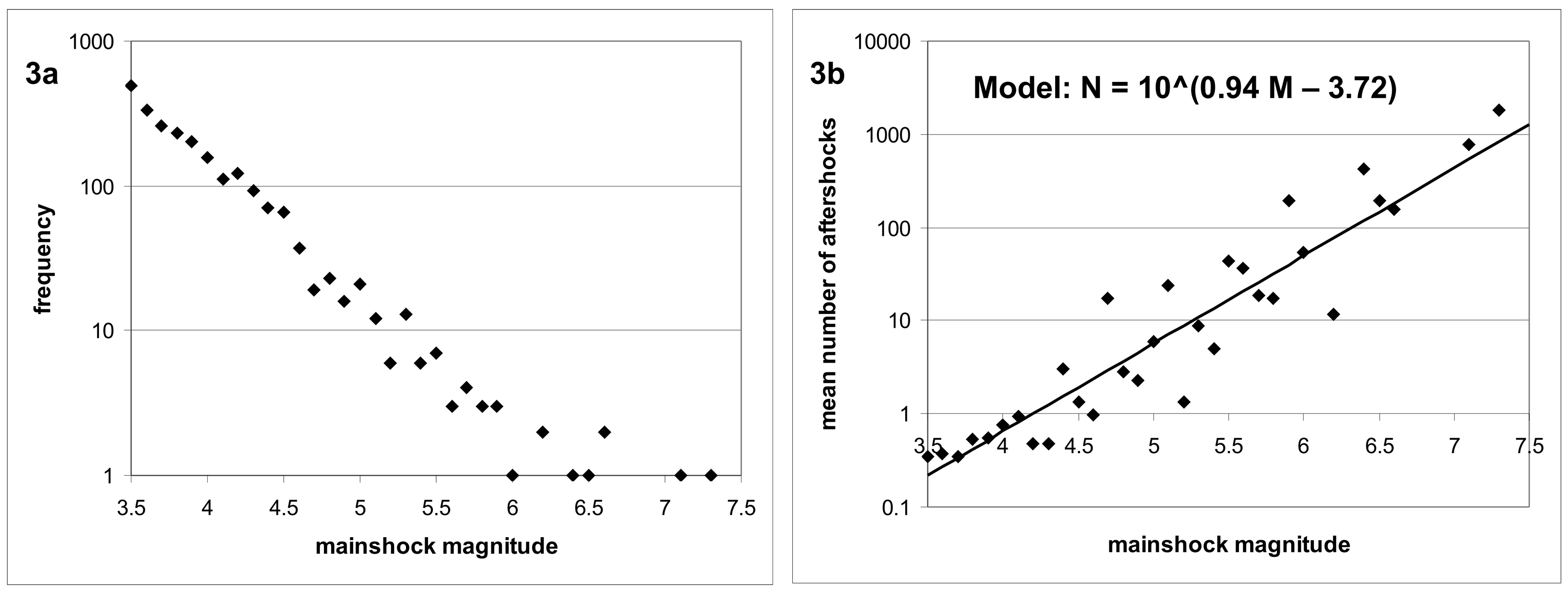


Figure 3: (a) The magnitude-frequency distribution of mainshocks and (b) The mean number of aftershocks per mainshock with the model fitted in the time interval [0.1,10].



Discussions and Conclusions

We explored the discrepancy between the foreshock prediction derived from the generic Californian aftershock model and observed foreshock rates. We showed that with a different model formulation and with different model parameters good agreement between foreshock prediction from aftershock models and foreshock observations can be found. The aftershock model include empirical laws of earthquake productivity, the decay of activity with time and the magnitude-frequency relation. While we have not investigated the characteristics of foreshocks for each of these relationships, the aggregate provides results that lead to the conclusion that foreshocks behave like mainshocks with larger aftershocks.

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References

Christophersen, A. (2000), *The probability of a damaging earthquake following a damaging earthquake*, PhD Thesis, Victoria University of Wellington.

Console, R., M. Murru, and A. M. Lombardi (2003) *Refining earthquake clustering models*, J. Geophys. Res. 108, 10.1029/2002JB002130

Dodge, D.A., G.C. Beroza, and W.L. Ellsworth (1995), *Foreshock sequence of the 1992 Landers, California earthquake and its implications for the earthquake nucleation process.* J. Geophys. Res. 101, 22371-22392.

Felzer, K. R., R.E Abercrombie, and G Ekstroem (2004), *A common origin for aftershocks, foreshocks, and multiplets*, Bull. Seism. Soc. Am. 94, 88-98.

Gardner, J.K., and L. Knophoff (1974), *Is the sequence of earthquakes in Southern California, with aftershocks removed Poisson*, Bull. Seism. Soc. Am. 64, 1363-1367.

Gerstenberger, M.C., Wiemer, S., L.M. Jones, and P.A. Reasenberg (2005), *Real-time forecasts of tomorrow's earthquakes in California*, Nature 435, doi:10.1038/nature03622.

Gutenberg, B., and C. F. Richter (1949), *Seismicity of the earth and associated phenomena*, Princeton University Press.

Jones, L.M. (1985), *Foreshocks and time-dependent earthquake hazard assessment in Southern California*, Bull. Seism. Soc. Am. 75, 1669-1679.

Kagan, Y.Y. (2004), *Short-term properties of earthquake catalogs and models of earthquake source*, Bull. Seism. Soc. Am. 94, 1207-1228.

Reasenberg, P.A., and L.M. Jones (1989), *Earthquake hazard after a mainshock in California*, Science 243, 1173-1176.

Reasenberg, P.A., and L.M. Jones (1994), *Earthquake Aftershocks: Update*, Science 265, 1251-1251.

Savage, M.K., and S.H. Rupp (2000), *Foreshock probabilities in New Zealand*, New Zealand Journal of Geology & Geophysics 43, 461-469.

Singh, S.K., and G. Suárez (1988), *Regional variation in the number of aftershocks (mb ≥ 5) of large subduction zone earthquakes (Mw ≥ 7.0)*, Bull. Seism. Soc. Am. 78, 230-242.

Uhrhammer (2005), *personal communication*.

Utsu, T. (1969), *Aftershocks and earthquake statistics (I) - some parameters which characterize an aftershock sequence and their interrelations*, J. Fac. Sci. Hokkaido Univ., Ser. VII, 3, 129-195.

Utsu, T., Y. Ogata, and R. S. Matsu'ura (1995), *The Centenary of The Omori Formula for a decay law of aftershock activity*, J. Phys. Earth 43, 1–33.

Woessner, J., and S. Wiemer (2005), *Assessing the quality of earthquake catalogues: Estimating the magnitude of completeness and its uncertainty,* Bull. Seism. Soc. Am. 95, 684-698, doi: 10.1785/0120040007.

Zhuang, J., Y. Ogata, and D. Vere-Jones (2004), *Analyzing earthquake clustering features by using stochastic reconstruction*, J. Geophys. Res. 109 B05301, 4755-4768.