Temporal Power-laws on Preseismic Activation and Aftershock Decay
Affected by Transient Behavior of Rocks

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Abstract: A constitutive law for rock behaviors including the brittle, transient and
steady-state behaviors is derived to recognize the temporal seismicity patterns on surface
displacement or prior and subsequent to mainshocks, and formulated as the relaxation modulus (i.e., the ratio of stress to strain) following a temporal power-law relaxation. This
constitutive law is transformable into the empirical power-law creep equation, and the
exponent of the constitutive law is the valuables reflecting the fractal structures of rocks in
response to the different deformation mechanisms. By the time-scale invariance in our law,
the temporal seismicity pattern is recognized on the surface displacement owing to major
large earthquake with many small events, on the power-law increase in the cumulative
Benioff strain-release as the preseismic activation, and on the power-law aftershock decay
represented by the modified Omori’s law. The exponents of the temporal power-laws on the
cumulative Benioff strain-release and the aftershock decay are linked to that of the
constitutive law of rocks, and change coupled with the fractal structure of the crustal rocks.

1. Introduction

We study the constitutive law for the rock behavior in order to recognize temporal
seismicity patterns on surface displacement and prior or subsequent to mainshocks. To this
end, the law requires to include the brittle behavior as well as viscoelastic behavior of rocks,
and to express the transient response of rocks, i.e., the response to the sudden change in the
stress and strain-rate such as earthquakes. The brittle behavior has been formulated by the
damage mechanics and fibre-bundle model (e.g., Turcotte et al., 2003; Nanjo et al, 2005),
and the transient behavior is derived as the temporal power-law relaxation called the long
time tail or temporal fractal relaxation (e.g., Nagahama, 1994; Kawada and Nagahama,
2004). However, the constitutive law satisfying both of the behaviors has not been derived.

The preseismic activation given by the increase in cumulative Benioff strain-release and
the aftershock decay represented by modified Omori’s law follow the temporal power-laws.
Although these processes prior or subsequent to major earthquakes are kinds of the behavior
of the crustal rocks, the processes have not been related with the temporal fractal property of
the transient behavior of rocks. On the other hand, in the percolation model of rock fracture
processes (e.g., Chelidze, 1993) and the theory on the power-law activation of spinodal
instability (e.g., Rundle et al., 2000), the temporal power-law increase in cumulative Benioff
strain-release is formulated with the settled exponent independent on the size or location of
the events. However, the analytical results of the exponents do not become the fixed values
(e.g., Bowman et al., 1998), and it has never been discussed what affects the change in the
exponent well.

In this presentation, from the constitutive law for the transient behavior of rocks, we
recognize the temporal seismicity pattern of surface displacement, aftershock decay and
cumulative Benioff strain-release. In particular, we state what property of the rocks affects
the exponents of the temporal power-laws on the cumulative Benioff strain-release and aftershock decay based on the constitutive law.

2. Constitutive law for transient behavior of rocks

The transient behavior of viscoelastic materials is generally formulated based on the relaxation modulus \( E(t) \) ( \( t \) is the deformation time), viz., \( \sigma / \dot{\varepsilon} \). For analytical reason, the secant modulus \( E_s(t) = \sigma / \dot{\varepsilon} \) in stead of \( E(t) \) is utilized. The analyses on the experimental data of the transient behaviors of halite (Shimamoto, 1987; Nagahama, 1994), marble and lherzolite (Kawada and Nagahama, 2004) yields the formulations:

\[
\frac{\sigma}{\varepsilon} = E_s(\xi) = \frac{E'}{g(\varepsilon)} \xi^{-\beta}, \quad \xi = \frac{t}{C} \exp\left(-\frac{Q}{RT}\right),
\]

where \( \sigma \) is the applied stress, \( \varepsilon \) is the strain, \( g(\varepsilon) \) is the strain-dependent function (\( g(\varepsilon) = 1 + a \varepsilon \)), \( \beta \) is the positive exponent, \( Q \) is the activated energy, \( R \) is the universal gas constant, \( T \) is the absolute temperature, \( E' \) and \( a \) are the material constants, and \( C \) is a constant. Moreover, \( \xi \) stands for the temperature reduced time obtained by normalizing the various temperature behaviors. The power-law of \( \xi \) (or \( t \)) in Eq. (1) shows the time-scale invariance of the rock behavior, called the temporal fractal property or long time tail (e.g., Nagahama, 1994). Equation (1) leads to \( E(\xi) \propto \xi^{-\beta} \) (e.g., Kawada and Nagahama, 2004).

The temporal power-law decay function is simply considered as the superposition of exponential decay functions with different lifetimes \( \lambda \). Then, \( E(t) \) is derived also by the Laplace transformation of the distribution function of \( \lambda \) as follows;

\[
E(t) = \int_0^\infty D(\lambda) \exp\left(-\frac{t}{\lambda}\right) d\lambda,
\]

where \( D(\lambda) \) is the distribution function of \( \lambda \). When \( D(\lambda) \propto \lambda^{\beta-1} \), \( E(t) \) shows the temporal power-law relaxation. This means that the structural fractal property of the rocks given by \( D(\lambda) \propto \lambda^{\beta-1} \) yields the long time tail behavior of rocks and \( \beta \) is related to the fractal distribution of elements composing the rocks (e.g., Nagahama, 1994).

When \( \dot{\varepsilon} = \dot{\varepsilon} / t \), Eq. (1) generates a relation (e.g., Kawada and Nagahama, 2004):

\[
\dot{\varepsilon} = \left(\frac{g(\varepsilon)}{\varepsilon E'}\right)^{\frac{1}{\beta}} \left(\frac{\varepsilon}{C}\right)^{\frac{1}{\beta}} \sigma^{1/\beta} \exp\left(-\frac{Q}{RT}\right).
\]

Equation (3) shows the stress power-law relation with thermal activated process, and is reduced to the empirical equation for the steady-state flow law. Then, \( 1/\beta \) is the stress exponent reflecting the deformation mechanism, and the empirical steady-state creep equation is a special case of Eq. (3) from the mathematical perspective. In this sense, the power-law relation between \( E(\xi) \) and \( \xi \) (or \( E(t) \) and \( t \)) yields the relation \( \dot{\varepsilon} \propto \sigma^{1/\beta} \), and represents the transient behavior as well as the steady-state behavior (e.g., Kawada and Nagahama, 2004).

The brittle behavior of rocks has been formulated by the continuum damage mechanics and fibre-bundle model (e.g. Turcotte et al., 2003). In particular, Nanjo and Turcotte (2005) pointed out that the brittle behavior of rocks is expressed by the stress power-law \( \dot{\varepsilon} \propto \sigma^{1/\beta} \) based on 1D fibre-bundle model. Hence, the power-law \( \dot{\varepsilon} \propto \sigma^{1/\beta} \) or \( E(t) \propto t^{-\beta} \) represents the brittle as well as viscoelastic (transient and steady-state) behaviors of rocks. Based on this constitutive law, we investigate the temporal seismicity pattern in the next section.
3. Various power-laws on the temporal seismicity patterns

3.1 Surface displacement time series

The surface displacement time series symbolizes a temporal seismicity pattern as a summation of transient responses of surface in various time-scales to the earthquakes. The transient responses in a-few-year scale to only a major large earthquake have been often analyzed by experimental steady-state creep equation of rocks: \( \dot{\varepsilon} \propto \sigma^{1/\beta} \) with the exponent \( 1/\beta = 3.5 (\beta \approx 0.29) \) of which the time series observed by the GPS (e.g., Freed and Bürgmann, 2004) is analyzed. On the other hand, the time series due to major earthquakes with small events can be recognized by the temporal fractal property on our constitutive law of rocks in Eq. (1) with \( \beta = 0.95 \) of which the time series observed by the creep meter (e.g., Wesson, 1987) is analyzed.

3.2 Modified Omori’s law

The temporal fractal decay of aftershocks is represented by the modified Omori’s law:

\[
\frac{dN}{d\tau} = B(\tau + c)^{-p},
\]

(4)

where \( \tau \) is the time elapsed after the main shock, \( dN/d\tau \) is the rate of occurrence of aftershocks greater than a magnitude, \( p \) is the positive exponent, and \( B \) and \( c \) are constants. Based on the continuum damage mechanics (e.g., Nanjo et al, 2005), the modified Omori’s law is related to the stress power-law in Eq. (3) under \( \beta = 1 - 1/p \).

3.3 Cumulative Benioff strain-release

Seismic activation has been observed prior to a lot of major earthquakes, and been quantified by the time series of cumulative Benioff strain-release \( \Omega \), i.e., the summation of the square root of the energy release at earthquake sequence like

\[
\Omega = \Omega_c - \frac{K}{s} (t_c - t)^s,
\]

(5)

where \( t_c \) is the occurrence time of the earthquake, \( \Omega_c \) is the Benioff cumulative strain-release at \( t = t_c \), \( s \) is a positive exponent, and \( K \) is a constant (e.g., Bowman et al., 1998). \( s \) is approximately equal to 0.25 (e.g., Rundle et al., 2000). Based on the fibre-bundle model and continuum damage model (e.g., Turcotte et al., 2003), the power-law increase in the Benioff strain release is linked to the stress power-law in Eq. (3) by \( \beta = 2(s - 1)/(3s - 2) \).

In this section, we show that the temporal seismicity pattern on the surface displacement and prior or subsequent to mainshocks can be recognized by the long time tail of the transient behavior of rocks.

4. Discussion

In the percolation model of rock fracture processes (e.g., Chelidze, 1993) and the theory on the power-law activation of spinodal instability (e.g., Rundle et al., 2000), the exponent of the temporal power-law on the cumulative Benioff strain-release is settled regardless of the location or size of the objective events, though \( s \)-value is not fixed in the analytical results (e.g., Bowman et al., 1998). In Section 3.3, \( s \) in Eq. (5) is related to \( \beta \) in Eqs. (1) and (3). \( \beta \) is linked to the structural fractal property of rocks as noted in Section 2, and the stress sensitive exponent \( 1/\beta \) depends on the deformation mechanisms such as the brittle failure and dislocation creep, and ranges from 1 to 60 (Table 1). Therefore, the change in \( \beta \) affects that in \( s \) (or \( p \) in Eq. (4)). Namely, we point out that \( \beta \) is regulated by the fractal structures of rocks corresponding to the different deformation mechanisms, and that the change in the structures affects that in the exponents \( s \) and \( p \).
Table 1. Experimental data of stress exponents $1/\beta$ and the corresponding deformation mechanisms (modified from Nagahama (1994) and Nakamura and Nagahama (1999)).

<table>
<thead>
<tr>
<th>$1/\beta$</th>
<th>Mechanism</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>Diffusion creep</td>
<td>Anorthite</td>
</tr>
<tr>
<td>2.6</td>
<td>Dislocation</td>
<td>Quartzite</td>
</tr>
<tr>
<td>3.0</td>
<td>Dislocation</td>
<td>Dunite</td>
</tr>
<tr>
<td>6.7</td>
<td>Dislocation</td>
<td>Halite</td>
</tr>
<tr>
<td>15.0</td>
<td>Primary creep</td>
<td>Halite</td>
</tr>
<tr>
<td>27.0</td>
<td>Brittle failure</td>
<td>Plagioclase</td>
</tr>
<tr>
<td>32.0</td>
<td>Brittle failure</td>
<td>Granite</td>
</tr>
<tr>
<td>60.0</td>
<td>Brittle failure</td>
<td>Sandstone</td>
</tr>
</tbody>
</table>

5. Conclusion
The temporal seismicity pattern on the surface displacement and prior or subsequent to the mainshocks can be recognized by the time-scale invariance of the transient behavior of rocks. The exponents of the temporal power-laws on the cumulative Benioff strain-release and modified Omori’s law change linked with the fractal structure of crustal rocks in response to the different deformation mechanisms.

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7. References