

# RETAS: a restricted ETAS model inspired by Bath's law

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**Abstract:** *A restricted trigger model is proposed to analyse the temporal behaviour of some aftershock sequences. The conditional intensity function of the model is similar to that of the Epidemic Type Aftershock-Sequence (ETAS) model with the restriction that only the aftershocks of magnitude bigger than or equal to some threshold  $M_{tr}$  can trigger secondary events. For this reason we have named the model restricted epidemic type aftershock-sequence (RETAS) model. Varying the triggering threshold we examine the variants of the RETAS model which range from the Modified Omori Formula (MOF) to the ETAS model, including such models as limit cases.*

## 1 Introduction

Bath's law states that the difference between the main shock magnitude and the magnitude of the strongest aftershock is constant, on average 1.2 for Richter and 1.4 for Utsu (Ogata, 2001). By extending this principle to the subsequences generated by primary events in a trigger model, we get that the difference between the weakest primary event and the weakest event in the aftershock sequence must be at least 1.2. This suggests choosing the primary events among those of magnitude larger than a suitable threshold  $M_{tr}$ .

The aim of our work is to examine the class of trigger models obtained by varying  $M_{tr}$  between the cut-off  $M_0$  and the maximum magnitude  $M_{max}$ . At first we have simulated two data sets according to the ETAS and RETAS models respectively and we have checked that the true model was identified by the Akaike criterion. Then we have analysed two aftershock sequences showing a different behaviour: the first, generated by the strong earthquake of  $M = 7.8$  on April 4, 1904 in Kresna region, SW Bulgaria, is a typical mainshock-aftershock sequence, while the second is the seismic swarm started on September 26, 1997 in Umbria-Marche region, central Italy. Our idea is to look for the model that fits best the data according to the Akaike criterion and hence to identify which physical interpretation can be given to each different behaviour, among the following: the aftershock sequence was triggered by the only main shock, by every event or by just some selected events. Then we compare the results obtained through the stochastic modelling with the description of the seismic crisis provided by the seismological studies.

## 2 Trigger models

Let us consider the conditional intensity function of a generic trigger model:

$$\lambda(t|\mathcal{H}_t) = \mu + \sum_{\{m \in \mathcal{P}; t_m < t\}} \frac{K_m}{(t - t_m + c)^p}$$

being  $\mathcal{P} = \{m = i_k, k = 1, \dots, L\}$  the subset of primary events triggering offspring. This subset is the discriminant element among the different models of the class; in particular, when  $\mathcal{P}$  is reduced to the main shock we have the MOF, whereas if  $\mathcal{P}$  coincides with the entire sequence we have the ETAS model, of conditional intensity function  $\lambda_O$  and  $\lambda_E$  respectively:

$$\lambda_O = \frac{K}{(t+c)^p} \quad \lambda_E(t|\mathcal{H}_t) = \mu + \sum_{t_i < t} \frac{K_i}{(t-t_i+c)^p} \quad .$$

By borrowing from Bath's law the idea of a gap between the magnitude  $M_{tr}$  of the triggering event and that of the largest event generated, we have examined the model in which, like in Ogata (2001), the primary events are those having magnitude larger than or equal to a threshold  $M_{tr}$ , but, contrary to Ogata (2001), the parameters  $K_m$  obey the restriction  $K_i = K_0 e^{\alpha(M_i - M_0)}$  (Gospodinov and Rotondi (2005)). The conditional intensity function of our model turns out to be

$$\lambda(t|\mathcal{H}_t) = \mu + \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 e^{\alpha(M_i - M_0)}}{(t-t_i+c)^p} \quad .$$

Moreover we note that in this model, contrary to the original trigger model (Vere-Jones and Davies, 1966; Vere-Jones, 1970), each primary event belongs to the offspring of the preceding primary events. We have not fixed the size of the gap between  $M_{tr}$  and  $M_0$ , even better by varying  $M_{tr}$  we have examined all the models between the MOF and the ETAS model on the basis of the Akaike criterion. We recall that in the MOF  $\mu$  is generally set equal to 0 and  $K_0 e^{\alpha(M_1 - M_0)} = K$  for the identifiability of the model. After having chosen the best model among those proposed, we have evaluated its goodness of fit through the residual analysis.

### 3 Data analysis

We sketch just the analysis of the 1997 aftershock sequence in Umbria-Marche region, because it highlights better the properties of the RETAS model.

A seismic sequence struck Umbria-Marche region during September-October 1997. It was characterized by the occurrence of six earthquakes of magnitude larger than 5 in a period of 20 days. More than 2000 events were recorded from September 26, 1997 up to November 3, 1997 (*Waveforms, arrival times and locations of the 1997 Umbria-Marche aftershocks sequence*). This data set can be considered complete above magnitude 2.9 and above this cut-off it contains 508 events. Studies on different features of the sequence reveal that it is an untypical compound aftershock sequence for a magnitude 6 main shock. Deschamps *et al.* (2000) suggest that the strongest events delineate three subzones and are spatially close to the rupture zones of the main shocks.

By fitting the class of RETAS models to the aftershock sequence we have obtained that the best model is the ETAS model with  $M_{tr} = M_0 = 2.9$ . The  $\alpha$  parameter generally varies in the range 0.4 – 3.0 and, according to Ogata (1999, p. 487), its value is smaller in case of swarm-type activity than in ordinary mainshock and aftershock activities. In our case  $\alpha = 0.89$  classifies the sequence as swarm. The residual analysis reveals that the number of the observed events exceeds that of the expected ones nearly in the whole time interval under study.

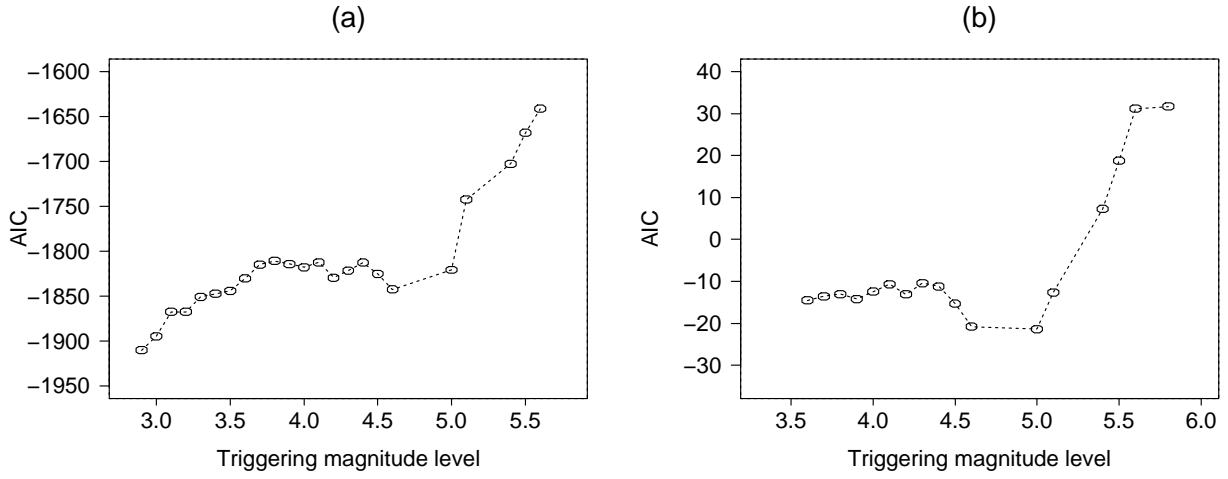


Figure 1: Umbria-Marche region:  $AIC$  value of the RETAS model for different  $M_{tr}$  triggering magnitudes with cut-off magnitude (a)  $M_0 = 2.9$  and (b)  $M_0 = 3.6$ . The best model is (a) the ETAS and (b) the RETAS model with  $M_{tr} = 5.0$ .

Figure 1a shows the trend of the AIC values with respect to different triggering magnitudes. Like in the analysis of the data set simulated from a RETAS model, we observe the presence of a local minimum in  $M_{tr} = 4.6$ . This observation led us to the idea of applying RETAS models with  $M_{tr} \geq 4.6$  to the subsequence with cut-off magnitude  $M_0 = 3.6$  so that the difference between  $M_{tr}$  and the new  $M_0$  is at least 1. By raising the threshold magnitude, secondary clusters appear generated by the six strongest events. The  $AIC$  values versus the triggering magnitudes are plotted in Fig. 1b. The residual analysis shows that, apart from a slight decrease of the activity between the 13th and the 16th day, the observed cumulative number of events is always within the error bounds.

Summarizing the algorithm we have applied is composed of two stages. The first consists in the following steps:

- i. order the set  $\mathcal{D} = \{M_i\}_{i=1}^N$  in increasing way and build the set of different magnitude levels  $\{Mag_k\}_{k=1}^K$  obtained by removing from  $\mathcal{D}$  the repeated values, so that  $Mag_k < Mag_{k+1}$ ,  $k = 1, \dots, K - 1$ ,  $\forall k Mag_k \in \mathcal{D}$ , and  $\forall i \exists k: M_i = Mag_k$ ; set  $k = 0$ ;
- ii.  $k \leftarrow k + 1$ ,  $M_{tr} = Mag_k$ ;
- iii. given the conditional intensity function

$$\lambda(t|\mathcal{H}_t) = \mu + \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p},$$

maximize  $\log \lambda(\cdot)$  and evaluate  $AIC_k$ ;

- iv. if  $k = 1$ , then set  $k_{best} = 1$  and  $AIC_{best} = AIC_1$ ; otherwise, if  $AIC_k < AIC_{best}$ , then set  $k_{best} = k$  and  $AIC_{best} = AIC_k$ ;
- v. if  $k < K$  then go to (ii), otherwise stop.

The final output, expressed by the value  $k_{best}$ , identifies the best model: ETAS if  $k_{best} = 1$ , MOF if  $k_{best} = K$  and RETAS with triggering magnitude  $Mag_{k_{best}}$  if  $1 < k_{best} < K$ .

If the ETAS model comes out to be the best model, it is interesting to examine whether particular correlations between the strongest aftershocks and the remaining seismicity emerge by raising the threshold  $M_0$ . In this second stage the analysis can be carried out through the following steps:

- i. plot the set of pairs  $\{(Mag_k, AIC_k)\}_{k=1}^K$ ;
- ii. if this set shows a monotonic trend, then stop; otherwise find the index of the local minimum, let's say  $j$ , set  $M_{tr} = Mag_j$  and  $M_0 \approx M_{tr} - 1$ , and go back to the first stage considering the subset formed by the events of size exceeding  $M_0$ .

## 4 Conclusions

We have found agreement between the physical interpretations of the results obtained through the stochastic modelling and the description of the seismic crisis provided by detailed seismological studies. It appears that the RETAS model can be applied for modelling compound seismic sequences in regions of complex tectonic structure with highly fractured earth crust.

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