Earthquakes clustering based on maximum likelihood estimation of point process conditional intensity function

Giada Adelfio and Marcello Chiodi

Dipartimento di Scienze Statistiche e Matematiche "Silvio Vianelli" (DSSM), University of Palermo, Italy

Abstract

In this paper we propose a clustering technique to separate and find out the two main component of seismicity: the background seismicity and the triggered one. The method here proposed assigns each earthquakes to the cluster of earthquakes or to the set of isolated event according to the intensity function. This function is estimated by maximum likelihood methods following the theory of point process and iteratively changing the assignment of the events. This technique develops non-parametric estimation methods and computational procedure for the maximization of the point process likelihood function.

1 Introduction

Because of the different seismogenic features controlling the kind of seismic release of clustered and background seismicity (Adelfio *et al.* (2005)), to describe the seismicity of an area in space, time and magnitude domains, it could be useful to study the features of independent events and strongly correlated ones, separately.

The two different kinds of events give different information on the seismicity of an area. For the short-term (or real-term) prediction of seismicity and to estimate parameters of phenomenological laws we need a good definition of the earthquake clusters. Furthermore the prediction of the occurrence of large earthquakes (related to the assessment of seismic risk in space and time) is complicated by the presence of clusters of aftershocks, that are superimposed to the background seismicity and shade its principal characteristics.

For this purposes the preliminary subdivision of a seismic catalog in background seismicity and clustered events is required. At this regard, a seismic sequences detection technique is presented; it is based on MLE of parameters that identify the conditional intensity function of a model that describe seismic activity as a clustering-process, which represents a slight modification of the ETAS model (Epidemic Type Aftershocks-Sequences model; Ogata (1988), Ogata (1998)).

2 Previous seismic clustering methods

Several methods are proposed to decluster a catalog. We could identify two main classes: methods that require the definition of a rectangular time-space window, with size depending on the mainshocks magnitude, around each mainshock (Gardner and Knopoff (1974)) and methods closed to the single linkage: they identify aftershocks by modelling an interaction zone about each earthquake assuming that any earthquake that occurs within the interaction zone of a prior earthquake is an aftershock and should be considered statistically dependent on it (Resenberg (1985)). The second class of methods has the advantage of not imposing a window for the final size or shape of the cluster, but the choice of coefficients defining the link distances in space and time could be influenced by researchers experience.

Ogata *et al.* (2002), to avoid such difficulties, proposes a stochastic method associating to each event a probability to be either a background event or an offspring generated by other events, based on the ETAS model for clustering patterns; a random assignment of events generates a thinned catalog, where events with a bigger probability of being mainshock are more likely included and a nonhomogeneous Poisson process is used to model their spatial intensity. This procedure identifies the two complementary subprocess of seismic process: the background subprocess and the cluster subprocess or the offspring process.

3 Conditional intensity function in point processes

To provide a quantitative evaluation of future seismic activity the conditional intensity function is crucial. It is proportional to the probability that an event with magnitude M will take place at time t, in a point in space of coordinates (x, y). The conditional intensity function of a space-time point process can be defined as:

$$\lambda(t, x, y|H_t) = \lim_{\Delta t, \Delta x, \Delta y \to 0} \frac{Pr_{\Delta t \Delta x \Delta y}(t, x, y|H_t)}{\Delta t \Delta x \Delta y} \tag{1}$$

where H_t is the space-time occurrence history of the process up to time t; $\Delta t, \Delta x, \Delta y$ are time and space infinitesimal increments; $Pr_{\Delta t \Delta x \Delta y}(t, x, y|H_t)$ represents the history-dependent probability that an events occurs in the volume $\{[t, t + \Delta t) \times [x, x + \Delta x) \times [y, y + \Delta y)\}$. The conditional intensity function completely identifies the features of the associated point process (i.e. if it is independent of the history but dependent only on the current time and the spatial locations (1) supplies a *nonhomogeneous Poisson process*; a constant conditional intensity provides a *stationary Poisson process*).

Other interesting point processes are defined by different conditional intensity functions. For example, we could consider probability models describing earthquakes catalogs as a realization of a branching or epidemic-type point process and models belonging to the wider class of Markov point processes, that assume previous events have an inhibiting effect to the following ones. The first type models could be identified with self-exciting point processes, while the second are represented by self-correcting processes as the strain-release model (Schoenberg and Bolt (2000)).

ETAS model is a self-exciting point process and represents the activity of earthquakes in a region during a period of time, following a branching structure. In particular it could be considered as an extension of the Hawkes model (Hawkes (1971)), which is a generalized Poisson cluster process associating to cluster centers a branching process of descendants.

4 The proposed clustering method

The technique of clustering that we propose leads to an intensive computational procedure, implemented by software R (R Development Core Team (2005)).

It identifies a partition of events $\mathcal{P}_{k+1}$ formed by $k+1$ sets: the one of background seismicity and those of clustered events (k disjoint sets). It iteratively changes the partition assigning events either to the background seismicity or to the $j-th$, ($j = 1, \dots, k$) cluster, given the current partition, on the basis of the likelihood function variation due to their moving from a set to another one. At each step ML estimation of seismicity parameters is performed.

4.1 Main steps

Firstly, a starting classification, found by a like single-linkage method procedure, needs.

Moving on, the parameters of the intensity function of ETAS model, describing clustering phenomena in seismic activity, are estimated, by ML, iteratively. Differently by ETAS model, in our approach space densities are estimated inside each cluster by a bivariate kernel estimator. The smoothing constant is evaluated with Silverman's formula (Silverman (1986)). Let $[T_0, T_{max}]$ and Ω_{xy} time and space domains of observation respectively; according to the theory of point process the likelihood function to be maximized is:

$$\log L = \sum_{i=1}^{n} \log \lambda(x_i, y_i, t_i) - \int_{T_0}^{T_{max}} \int_{\Omega_{xy}} \lambda(x, y, t) dx dy dt \tag{2}$$

where:

$$\lambda(x, y, t) = \lambda_t \mu(x, y) + \sum_{\substack{j=1 \\ (t_j < t)}}^{k} g_j(x, y) \frac{K_0 \exp[\alpha(m_j - m_0)]}{(t - t_j + c_j)^{p_j}} \tag{3}$$

In (3) t_j and m_j are time and magnitude of the mainshock of the cluster j respectively, $g_j(x, y)$ is the space intensity function of the cluster j (computed using the n_j points belonging to the $j-th$ cluster including the $j-th$ mainshock) and $\mu(x, y)$ is the background space intensity function (computed using the n_0 isolated points and the k mainshocks).

In the evaluation of (3) we can have two different parametrization: one with a common parameter c and a common parameter p for the whole partition and one with k parameters c_j and k parameters p_j, varying over each cluster, for each found partition. The choices can be compared at the end of the procedure.

To move events from their current position, the changes in the likelihood function (2) for each event of coordinates (x_i, y_i, t_i, M_i) needs: indeed, our iterative procedure moves seismic events assigning each earthquake either to the set of background seismicity or to the $j-th$ cluster, according to the term of (3) which is maximized.

The iterative procedure stops when the current classification does not change after a whole loop of reassignment.

5 Conclusion

Starting from a seismic catalog, the procedure here proposed returns a plausible separation of the components of seismicity and clusters that have a good interpretability.

This method could be the basis to carry out an analysis of the complexity of the seismogenetic processes relative to each sequence and to the background seismicity, separately. Indeed parameters that control the way in which strain energy is released could be strongly different through clusters and isolated events; this could be also observed in significant differences in the parameter estimates of a phenomenological model applied to sets of earthquakes relative to the two different processes.

References

Adelfio, G., Chiodi, M., De Luca, L., Luzio, D., and Vitale, M. (2005). Southern-tyrrhenian seismicity in space-time-magnitude domain. *Submitted for publication.*

Gardner, J. and Knopoff, L. (1974). Is the sequence of earthquakes in outhern california, with aftershock removed, poissonian? *Bulletin of the seimological Society of America, 64*, pages 1363–1367.

Hawkes, A. (1971). Spectra of some self-exciting and mutually exciting point processes. *Biometrika, 58, 1)*, pages 83–90.

Ogata, Y. (1988). Statistical models for earthquake occurrences and residual analysis for point processes. *Journal of the American Statistical Association, 83, 401*, pages 9–27.

Ogata, Y. (1998). Space-time point-process models for earthquake occurrences. *Ann. Inst. Statist. Math, vol. 50, no. 2*, pages 379–402.

Ogata, Y., Zhuang, J., and Vere-Jones, D. (2002). Stochastic declustering of space-time earthquake occurrences. *Journal of the American Statistical Association, 97, 458*, pages 369–379.

R Development Core Team (2005). *R: A language and environment for statistical computing.* R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.

Resenberg, P. (1985). Second-order moment of central california seismicity, 1969-1982. *Journal of Geophysical Research, vol. 90, no. B7*, pages 5479–5495.

Schoenberg, F. and Bolt, B. (2000). Short-term exciting, long-term correcting models for earthquake catalogs.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis.* Chapman and Hall, London.