

Toward urgent forecasting of aftershock hazard:

Simultaneous estimation of b -value of the Gutenberg-Richter's law of the magnitude frequency and changing detection rates of aftershocks immediately after the mainshock

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It is known that detection rate of aftershocks is extremely low during a period immediately after a large earthquake due to the contamination of arriving seismic waves. This has been made a considerable difficulty to attain the estimate of the empirical laws of aftershock decay and magnitude frequency immediately after the main shock. This paper presents an estimation method for predicting underlying occurrence rate of aftershocks of any magnitude range, based on the magnitude frequency model that combines Gutenberg-Richter's law with the detection rate function. This procedure enables real-time probability forecasting of aftershocks immediately after the mainshock, when a majority of large aftershocks are likely to occur.

$$f(M|b, \mu, \sigma) = \frac{10^{-bM} q(M|\mu, \sigma)}{\int_{-\infty}^{\infty} 10^{-bM} q(M|\mu, \sigma) dM} = e^{-\beta(M-\mu)-\beta^2\sigma^2/2} q(M|\mu, \sigma), \quad (\beta = b \ln 10)$$

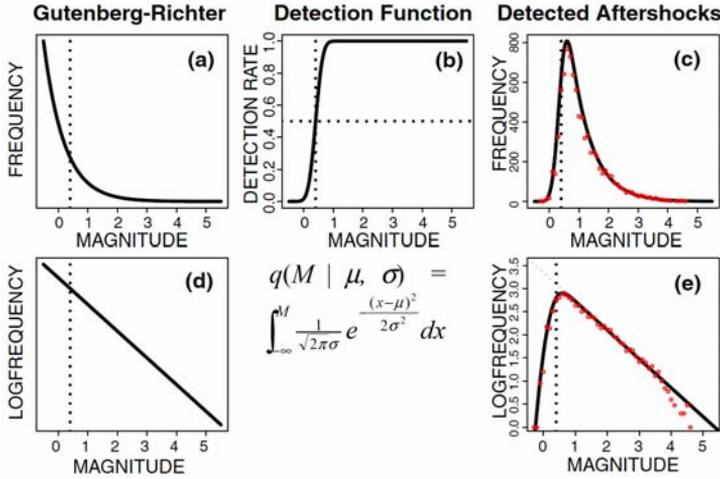


Fig. 1. Schematic diagrams showing the model of theoretical magnitude frequency of detected earthquakes. The exponential curve (a) and straight line (d) indicate the Gutenberg-Richter's relation of underlying magnitude frequency of earthquakes in linear and logarithmic scale, respectively. The solid curve in (b) shows the detection-rate (probability) of events of each magnitude in linear scale ranging between 0 and 1. The frequency distribution of the detected earthquakes in (c) is derived from the product of the functions in (a) and (b) (cf., Eq. (6)), the logarithmic scale of which is shown in (e). Vertical dotted lines in the panels indicate magnitude of 50% detection rate. Red circles in (c) and (e) indicate the actual frequencies of detected aftershocks of the 2003 Miyagi-Ken-Oki earthquake of M7.0 during the period of 350 days from 20th day since the main shock.

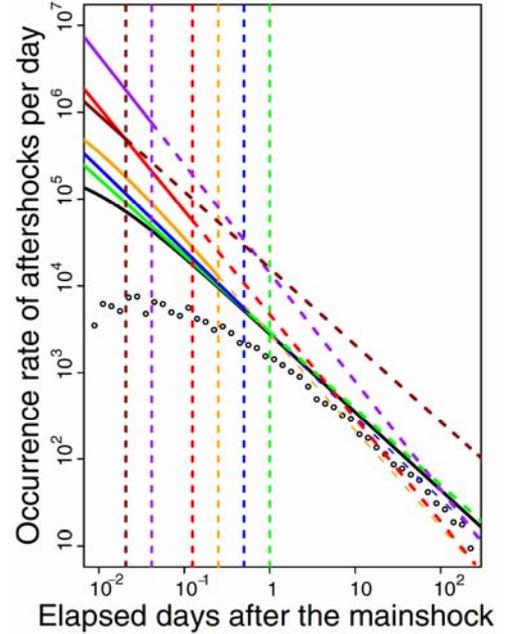


Fig. 3. Plots of frequency rate of all detected aftershocks per day (circles) versus elapsed time in doubly logarithmic scale, and estimated theoretical rate of all $M > 0$ events including undetected events (colored curves) for the observed period from the mainshock up until the updated time 1 hour, 3 hours, 6 hours, 12 hours, and 24 hours. The color corresponds to the case in Figure 2, and solid and dotted curve corresponds to the one during the observed period and extended period for the prediction, respectively.

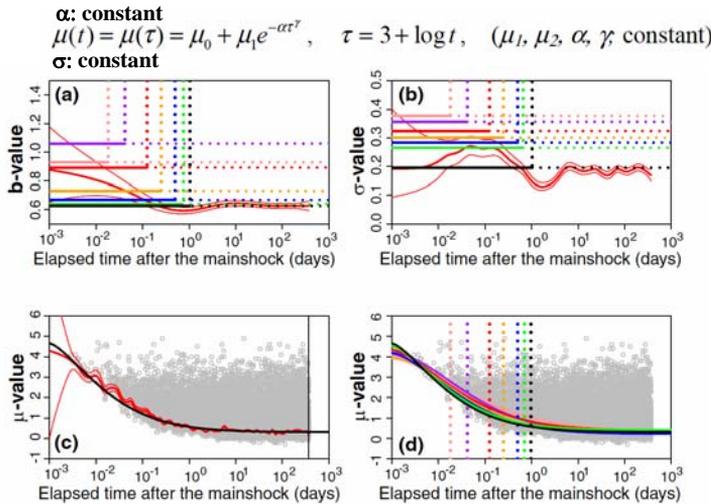


Fig. 2. Estimations of detected aftershocks of the 2003 Miyagi-Ken-Oki earthquake of M7.0. Panel (a) and (b) represent the estimated functions of b -values and σ -values for the standard deviation of partially detected magnitudes, respectively, and (c) and (d) indicate the estimated μ -values for the magnitude of 50% detection-rate. Horizontal axis of all panels is the elapsed time since the mainshock indicated in days with logarithmic scale. Grey color circles in (c) and (d) represent the magnitude data of the aftershocks against the elapsed time. Thick smooth red curve in (a), (b) and (c) shows the maximum posterior estimate of b -, σ - and μ -value, respectively, which is bounded by smooth thin red curves below and above, showing 2-fold standard errors due to the posterior distribution. Thick colored horizontal segments in (a) and (b), and thick curves in (d) respectively show the maximum likelihood estimates of b -values (constant), σ -values (constant), and the function $\mu(t)$ in equation (9); where color of dark red, purple, red, orange, blue, green and black represents the estimates obtained by fitting the observed magnitudes of all detected aftershocks during the period from the mainshock up until 30min, 1hour, 3hours, 6hours, 12hours, 24hours and 370days, respectively; time location of which are indicated by the corresponding colored vertical dotted lines.

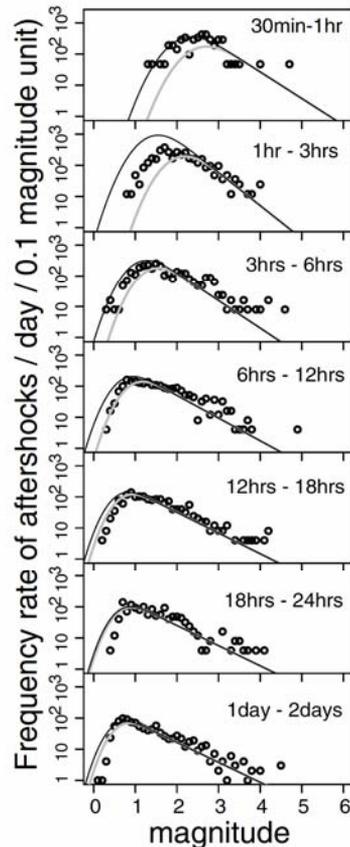


Fig. 4. Predicted frequencies and detected aftershocks of the 2003 Miyagi-Ken-Oki earthquake of M7.0. The solid curves represent the predicted magnitude frequency distribution in equation (10) using the estimated parameters shown in Table 1 and Figure 2. The dots plots are the empirical frequencies (rate per day) of the detected aftershocks that took place during the predicted intervals starting from the last time points of the periods of the estimation (cf., caption of Figure 2), namely, during the time intervals of [0.5, 1]hour, [1, 3]hours, [3, 6]hours, [6, 12]hours, [12, 18]hours, [18, 24]hours, and [1, 2]days (rate per day for each magnitude of 0.1 unit), respectively.