Infinitesimal analysis of set-valued functions and applications to spacial statistics and image analysis

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Abstract

The problem under consideration started with a study of spacial change-point problem, or change-set problem as we called it: suppose there is a set K, such that our observations inside this set behave differently then anywhere outside it. Given n observations, how can we test that a given K is the correct one against its small perturbation $K(\varepsilon)$ as an alternative?

In huge amount of publications on the change-set problem, almost all devoted to the estimation of the change-set, we could not find such an object as an "alternative set" $K(\varepsilon)$. One reason for this porbably is that it is not easy to realize how to describe small changes in sets.

We developed an appropriate notion of the derivative of set-valued function in (Khmaladze, 2007) and used it to build a version of contiguity theory in (Einmahl, Khmaladze, 2011) for statistical problems where the parameter of interest is a set.

One single result here is that if Φ_n is a point process with increasing intenstiy n, and the symmetric difference $K(\varepsilon)\Delta K$ shrinks and "vanishes" as $\varepsilon \to 0$, then the sequence $\Phi_n(K(\varepsilon)\Delta K)$ lives in the limit on the derivative set $dK(\varepsilon)/d\varepsilon$:

$$\Phi_n(K(\varepsilon)\Delta K) \to \Psi(dK(\varepsilon)/d\varepsilon, \text{ as } n \to \infty, \varepsilon \to 0.$$

In the talk we present the main framework of this approach. We hope that some discussions would lead to further applications.