

**Title : Exact Statistics of the Gap and Time Interval Between the First Two Maxima of Random Walks and Lévy Flights**

**Abstract** : I will present recent exact results for the statistics of the gap,  $G_n$ , between the two rightmost positions of a Markovian one-dimensional random walker (RW) after  $n$  time steps and of the duration,  $L_n$ , which separates the occurrence of these two extremal positions. The distribution of the jumps  $\eta_i$ 's of the RW,  $f(\eta)$ , is symmetric and its Fourier transform has the small  $k$  behavior  $1 - \hat{f}(k) \sim |k|^\mu$  with  $0 < \mu \leq 2$ . For  $\mu = 2$ , the RW converges, for large  $n$ , to Brownian motion while for  $0 < \mu < 2$ , it corresponds to a Lévy flight of index  $\mu$ . We compute the joint probability density function (pdf)  $P_n(g, l)$  of  $G_n$  and  $L_n$  and show that, when  $n \rightarrow \infty$ , it approaches a limiting pdf  $p(g, l)$ . The corresponding marginal pdf's of the gap,  $p_{\text{gap}}(g)$ , and of  $L_n$ ,  $p_{\text{time}}(l)$ , are found to behave like  $p_{\text{gap}}(g) \sim g^{-1-\mu}$  for  $g \gg 1$  and  $0 < \mu < 2$ , and  $p_{\text{time}}(l) \sim l^{-\gamma(\mu)}$  for  $l \gg 1$  with  $\gamma(1 < \mu \leq 2) = 1 + 1/\mu$  and  $\gamma(0 < \mu < 1) = 2$ . For  $l, g \gg 1$  with fixed  $lg^{-\mu}$ ,  $p(g, l)$  takes the scaling form  $p(g, l) \sim g^{-1-2\mu} \tilde{p}_\mu(lg^{-\mu})$  where  $\tilde{p}_\mu(y)$  is a ( $\mu$ -dependent) scaling function.