The connectivities of leaf graphs of sets of points in the plane

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Let $U$ be a set of $n$ points in the plane. If no three points are collinear, then we say that $U$ is in general position. We can draw a graph of which vertices are points in $U$ and edges are straight line segments between two points in $U$. If any two edges in the graph do not intersect except at endpoints, then the graph is called a non-selfintersecting graph on $U$.

Y. Ikebe et al. [3] showed that any rooted tree with $n$ vertices can be embedded as a non-selfintersecting tree on a given set $U$, the root being mapped to an arbitrary specified point of $U$. Furthermore A. Kaneko and M. Kano extended the theorem for a forest with two rooted trees in [4]. A non-selfintersecting tree with $n$ vertices on $U$ is called an affine spanning tree on $U$. E. Campo and V. Glicia [2] showed that the tree graph on $U$ is connected. A tree graph on $U$ is defined in such a way that the vertex set is the set of all the affine spanning trees on $U$ and two affine spanning trees $t_1$ and $t_2$ are said to be adjacent if there exist edges $e_i \in E(t_i)$ such that $t_1 - e_1 = t_2 - e_2$.

In this paper, we study a spanning subgraph of the tree graph on $U$, called a leaf graph. An adjacency relation of a leaf graph is defined so that two affine spanning trees $t_1$ and $t_2$ are said to be adjacent if there exists $u \in U$ such that $t_1 - u = t_2 - u$ and this graph is connected. The authors studied the connectivity of the leaf graph of a 2-connected graph with minimum degree $k$ in [5]. We shall prove the following theorem in this paper.

**Theorem 1** Let $U$ be the set of points in the plane in general position. Then the leaf graph on $U$ is 2-connected.


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