

Title : Exact Statistics of the Gap and Time Interval Between the First Two Maxima of Random Walks and Lévy Flights

Abstract : I will present recent exact results for the statistics of the gap, G_n , between the two rightmost positions of a Markovian one-dimensional random walker (RW) after n time steps and of the duration, L_n , which separates the occurrence of these two extremal positions. The distribution of the jumps η_i 's of the RW, $f(\eta)$, is symmetric and its Fourier transform has the small k behavior $1 - \hat{f}(k) \sim |k|^\mu$ with $0 < \mu \leq 2$. For $\mu = 2$, the RW converges, for large n , to Brownian motion while for $0 < \mu < 2$, it corresponds to a Lévy flight of index μ . We compute the joint probability density function (pdf) $P_n(g, l)$ of G_n and L_n and show that, when $n \rightarrow \infty$, it approaches a limiting pdf $p(g, l)$. The corresponding marginal pdf's of the gap, $p_{\text{gap}}(g)$, and of L_n , $p_{\text{time}}(l)$, are found to behave like $p_{\text{gap}}(g) \sim g^{-1-\mu}$ for $g \gg 1$ and $0 < \mu < 2$, and $p_{\text{time}}(l) \sim l^{-\gamma(\mu)}$ for $l \gg 1$ with $\gamma(1 < \mu \leq 2) = 1 + 1/\mu$ and $\gamma(0 < \mu < 1) = 2$. For $l, g \gg 1$ with fixed $lg^{-\mu}$, $p(g, l)$ takes the scaling form $p(g, l) \sim g^{-1-2\mu} \tilde{p}_\mu(lg^{-\mu})$ where $\tilde{p}_\mu(y)$ is a (μ -dependent) scaling function.