Ann. Inst. Statist. Math.
Vol. 54, No. 4, 840–847 (2002)

ON NEW MOMENT ESTIMATION OF PARAMETERS OF THE GAMMA DISTRIBUTION USING ITS CHARACTERIZATION*

TEA-YUAN HWANG[1] AND PING-HUANG HUANG[2]

[1] *Institute of Statistics, National Tsing Hua University, Hsinchu, 30043, Taiwan, R.O.C.*
[2] *Department of Statistics and Insurance, Aletheia University, Tamsui, Taipei, 25103, Taiwan, R.O.C.*

(Received April 5, 2001; revised July 16, 2001)

Abstract. In this paper, the more convenient estimators of both parameters of the gamma distribution are proposed by using its characterization, and shown to be more efficient than the maximum likelihood estimator and the moment estimator for small samples. Furthermore, the distribution of the square of the sample coefficient of variation is obtained by computer simulation for some various values of the parameters and sample size, and thus the simulated confidence interval of its shape parameter is established.

Key words and phrases: Sample coefficient of variation, shape parameter, moment estimator, gamma distribution.

1. Introduction

The gamma distribution is widely used and plays an important role in the reliability field and the survival analysis, therefore a successful estimation of its parameters will be very important. Unfortunately, there exist some difficulties in present estimation schemes. Maximum likelihood estimation method for its parameters are described in the literature by Johnson and Kotz (1970), Cohen and Norgaard (1977), Cohen and Whitten (1982), Harter and Moore (1965), Bowman *et al.* (1987) and Bowman and Shenton (1988). Also some difficulties and modified MLEs are mentioned in these papers. On the other hand, Bai *et al.* (1991) and Bowman and Shenton (1988) pointed out a high degree of deviation of the estimators from the parent distribution if one uses the methods involving the moments.

Hwang and Hu (1999, 2000) proved the independence of sample coefficient of variation V_n with sample mean $\bar{X}_n$ when random samples are drawn from gamma distribution. In the next section, we use this characterization to derive the expectation and the variance of V_n^2, and then propose the new moment estimators of the shape and the scale parameters of gamma distribution. Furthermore, by simulation, we compare in Section 3 the new estimators with the maximum likelihood estimator and usual moment estimator in term of mean square error.

For finding a simulated confidence interval of the shape parameter, the simulated distribution of V_n^2 will be derived in Section 4. In Hu (1990), a set of non-linear transformations of order statistics was devised to derive the sample distribution of V_n; its explicit probability density function has been obtained only for sample size $n = 3, 4$ and 5. In

*This paper was supported by a grant from the National Sciences Council of R.O.C.

Hwang and Lin (2000), the detailed c.d.f.s of V_n and V_n^2 under exponential population are presented for $n = 3, 4$ and 5 only. Until now it is still difficult to derive explicitly the sample distribution of V_n^2, thus simulation is used to find the sample distribution of normalized V_n^2 for shape parameters $= 0.5, 1.0, 1.5, 2.0$ and scale parameters $= 1, 0.5$ and 0.25 when $n = 5, 10, 15, 20$ and 25 respectively; it looks almost like gamma distribution for the cases mentioned above. Finally, the simulated confidence intervals for shape parameter are established.

2. New moment estimator of parameters of the gamma distribution

For deriving new moment estimator of parameters of the gamma distribution, we need the following theorem taken from Hu (1990) and Hwang and Hu (1999).

THEOREM 2.1. *Let $n \geq 3$ and let X_1, Λ, X_n be n positive i.i.d. random variables having a probability density function $f(x)$. Then the independence of the sample mean $\bar{X}_n$ and the sample coefficient of variation $V_n = S_n/\bar{X}_n$ is equivalent to that f is a gamma density where S_n is the sample standard deviation.*

The next result and Theorem 2.1 are useful in deriving the expectation and the variance of $V_n^2 = (S_n/\bar{X}_n)^2$, where $\bar{X}_n$ and S_n are respectively the sample mean and the sample standard deviation.

THEOREM 2.2. *Let $n \geq 3$ and let X_1, Λ, X_n be drawn from a population having a gamma density*

$$g(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \qquad x > 0, \alpha > 0, \beta > 0.$$

Then

$$E(\bar{X}_n^2) = \frac{(n\alpha + 1)n\alpha}{n^2\beta^2}$$

and

$$E(S_n^2) = \frac{\alpha}{\beta^2}$$

where $\bar{X}_n$ and S_n^2 are respectively their sample mean and sample variance.

PROOF. It is easy to prove that

$$E(X) = \frac{\alpha}{\beta}, \quad \mathrm{Var}(X) = \frac{\alpha}{\beta^2}, \quad V^2 = \frac{\mathrm{Var}(X)}{E^2(X)} = \frac{1}{\alpha},$$

$$E(X^k) = \frac{(\alpha + k - 1) \cdots (\alpha + 1)\alpha}{\beta^k} \quad \text{for} \quad k \geq 1, \tag{2.1}$$

and that $\bar{X}_n$ has the following p.d.f.

$$g(\bar{x}_n; \alpha, \beta) = \frac{(n\beta)^{n\alpha}}{\Gamma(n\alpha)} \bar{x}_n^{n\alpha-1} e^{-n\beta\bar{x}_n}$$

and moments

$$E(\bar{X}_n^k) = \frac{(n\alpha + k - 1) \cdots (n\alpha + 1)(n\alpha)}{n^k\beta^k} \quad \text{for} \quad k \geq 1. \tag{2.2}$$

Thus (2.1) and (2.2) together give the following relation:

$$\begin{aligned} E(S_n^2) &= \frac{1}{(n-1)} E\left[\sum_{i=1}^{n}(X_i - \bar{X}_n)^2\right] \\ &= \frac{1}{n-1}[E(X^2) - E(\bar{X}_n^2)] \\ &= \frac{\alpha}{\beta^2} \end{aligned}$$

and Theorem 2.2 is established.

Theorem 2.2 implies that the sample mean $\bar{X}_n$ and the sample variance S_n^2 are respectively the unbiased estimator of population mean α/β and population variance α/β^2, a property also possessed by the normal population. Thus we have the moment estimators $\hat{\alpha}_m$ and $\hat{\beta}_m$ of α and β as follows:

$$\hat{\alpha}_m = \frac{\bar{X}_n^2}{S_n^2}, \qquad \hat{\beta}_m = \frac{\bar{X}_n}{S_n^2}.$$

THEOREM 2.3. *Let $n \geq 3$ and let X_1, Λ, X_n be drawn from a population having a gamma density*

$$g(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \qquad x > 0, \alpha > 0, \beta > 0.$$

Then

$$E\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{n}{1+n\alpha}$$

where $\bar{X}_n$ and S_n^2 are respectively their sample mean and sample variance.

PROOF. By Theorem 2.1, we have

$$E(S_n^2) = E\left(\frac{S_n^2}{\bar{X}_n^2} \cdot \bar{X}_n^2\right) E\left(\frac{S_n^2}{\bar{X}_n^2}\right) \cdot E(\bar{X}_n^2)$$

and hence

$$E\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{E(S_n^2)}{E(\bar{X}_n^2)}.$$

Applying Theorem 2.2, to the above identity yields that

$$E\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{n}{1+n\alpha}$$

and Theorem 2.3 is established.

Note that $E(S_n^2/\bar{X}_n^2) \to \frac{1}{\alpha}$ as $n \to \infty$ and that $\frac{1}{\alpha}$ is the square of the coefficient of variation. Thus $S_n^2/\bar{X}_n^2$ is an asymptotically unbiased estimator of the square of the coefficient of variation.

By Theorem 2.3 , V_n^2 is the unbiased estimator of $\frac{n}{1+n\alpha}$, thus it seems reasonable to propose $\frac{1}{V_n^2} - \frac{1}{n}$ as the estimator of α, namely

$$\hat{\alpha}_c = \frac{1}{V_n^2} - \frac{1}{n}.$$

It is easy to show that $\hat{\alpha}_c > 0$. Therefore, by the identity $E(\bar{X}_n) = \frac{\alpha}{\beta}$ and moment estimation method approach, it seems also reasonable to propose

$$\hat{\beta}_c = \frac{\hat{\alpha}_c}{\bar{X}_n} = \frac{1}{\bar{X}_n}\left(\frac{1}{V_n^2} - \frac{1}{n}\right).$$

Note that $\hat{\alpha}_c \to \hat{\alpha}_m$ and $\hat{\beta}_c \to \hat{\beta}_m$ as $n \to \infty$, and their differences get bigger when the sample size n gets smaller.

The fact that $\hat{\alpha}_c$ and $\hat{\beta}_c$ are more convenient to be computed than the maximum likelihood estimators $\hat{\alpha}_L$ and $\hat{\beta}_L$ of α and β is quite trivial. For comparing the efficiency of $\hat{\alpha}_c$ and $\hat{\beta}_c$ with $\hat{\alpha}_L$ and $\hat{\beta}_L$, respectively, we apply the next theorem to derive the normalized behevior of V_n.

THEOREM 2.4. *Let $n \geq 3$ and let X_1, Λ, X_n be drawn from a population having a gamma density*

$$g(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \qquad x > 0, \alpha > 0, \beta > 0.$$

Then

$$\text{Var}(S_n^2) = \frac{\alpha}{\beta^4}\left[\frac{2n\alpha}{(n-1)^2} + \frac{6}{n}\right] \tag{2.3}$$

and

$$\text{Var}\left(\frac{S_n^2}{\bar{X}_n^2}\right) \frac{2\alpha(\alpha+1)}{(n-1)\left(\alpha+\frac{1}{n}\right)^2\left(\alpha+\frac{2}{n}\right)\left(\alpha+\frac{3}{n}\right)}. \tag{2.4}$$

PROOF. Since $M_X(t) = (1 - t/\beta)^{-\alpha}$, we have

$$E(X) = \alpha/\beta$$
$$E(X^2) = \alpha(\alpha+1)/\beta^2$$
$$E(X^3) = \alpha(\alpha+1)(\alpha+2)/\beta^3$$
$$E(X^4) = \alpha(\alpha+1)(\alpha+2)(\alpha+3)/\beta^4$$

and $M_{\bar{X}_n}(t) = (1 - \frac{1}{n\beta}t)^{-n\alpha}$ gives

$$E(\bar{X}_n) = \alpha/\beta$$
$$E(\bar{X}_n^2) = \alpha\left(\alpha+\frac{1}{n}\right)\Big/\beta^2$$
$$E(\bar{X}_n^3)\alpha\left(\alpha+\frac{1}{n}\right)\left(\alpha+\frac{2}{n}\right)\Big/\beta^2$$
$$E(\bar{X}_n^4) = \alpha\left(\alpha+\frac{1}{n}\right)\left(\alpha+\frac{2}{n}\right)\left(\alpha+\frac{3}{n}\right)\Big/\beta^4.$$

By using the above identities, we obtain

$$E(S_n^4) = \frac{\alpha}{\beta^4(n-1)^2}\left[(n^2-1)\alpha + 6\left(\sqrt{n} - \frac{1}{\sqrt{n}}\right)^2\right]$$

and combing the above identity and Theorem 2.2, we have

$$\mathrm{Var}(S_n^2) = \frac{\alpha}{\beta^4}\left[\frac{2n\alpha}{(n-1)^2} + \frac{6}{n}\right].$$

Next, the independence of $(S_n^2/\bar{X}_n^2)^2$ and $(\bar{X}_n)^2$ gives

$$E\left(\frac{S_n^4}{\bar{X}_n^4}\right) = \frac{E(S_n^4)}{E(\bar{X}_n^4)} \quad \text{and}$$

$$E\left(\frac{S_n^2}{\bar{X}_n^2}\right)^2 = \frac{E(S_n^4)}{E(\bar{X}_n^4)} = \frac{(n^2-1)\alpha + 6\left(\sqrt{n} - \frac{1}{\sqrt{n}}\right)^2}{(n-1)^2\left(\alpha + \frac{1}{n}\right)\left(\alpha + \frac{2}{n}\right)\left(\alpha + \frac{3}{n}\right)}.$$

Thus we have

$$\mathrm{Var}\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{2\alpha(\alpha+1)}{(n-1)\left(\alpha + \frac{1}{n}\right)^2\left(\alpha + \frac{2}{n}\right)\left(\alpha + \frac{3}{n}\right)}$$

and Theorem 2.4 is established.

Theorem 2.4 implies that both $\mathrm{Var}(S_n^2)$ and $\mathrm{Var}(V_n^2)$ tend to zero as $n \to \infty$. Thus S_n^2 and V_n^2 are respectively consistent estimators of $\frac{\alpha}{\beta^2}$ and $\frac{n}{n\alpha+1}$ for large samples. After some computations, we find the following inequality:

$$\frac{\mathrm{Var}(V_n^2)}{\mathrm{Var}(S_n^2)} < \left(\frac{n\beta}{n\alpha+1}\right)^4 = \left(\frac{\beta}{\alpha + \frac{1}{n}}\right)^4$$

$$\mathrm{Var}\,(V_n^2) < \mathrm{Var}(S_n^2), \qquad \beta \le \alpha + \frac{1}{n}.$$

Furthermore, the fact that $\mathrm{Var}(V_n^2) \to 0$ as $n \to \infty$ also confirms the reason: why V_n can always considered approximately as constant for large samples, and it can be used in checking experiment results and in estimating the standard deviation.

3. The comparison with previous estimators

In this section, the comparison of our estimators $(\hat{\alpha}_c, \hat{\beta}_c)$ with maximum likehihood estimators $(\hat{\alpha}_L, \hat{\beta}_L)$ and moment estimators $(\hat{\alpha}_m, \hat{\beta}_m)$ would be done in terms of mean square error by using the simulation procedures proposed by Greenwood and Durand (1960) which improved Thom (1958). Note that $(\hat{\alpha}_L, \hat{\beta}_L)$ are more difficult to compute than $(\hat{\alpha}_c, \hat{\beta}_c)$ and $(\hat{\alpha}_m, \hat{\beta}_m)$.

We have done more than 100,000 times simulation for $\alpha = 0.5, 1, 1.5, 2$ and $\beta = 1, 2, 4$ when $n = 5, 10, 15, 20$ and 25, and obtain the following conclusions:

(1) $(\hat{\alpha}_c, \hat{\beta}_c)$ is the best estimators of (α, β), $(\hat{\alpha}_L, \hat{\beta}_L)$ the next and $(\hat{\alpha}_m, \hat{\beta}_m)$ the worse for $n \le 25$, and the smaller n the better $(\hat{\alpha}_c, \hat{\beta}_c)$;

(2) $(\hat{\alpha}_L, \hat{\beta}_L)$ is the best estimators of (α, β), $(\hat{\alpha}_c, \hat{\beta}_c)$ the next and $(\hat{\alpha}_m, \hat{\beta}_m)$ the worse for $n > 25$, and the larger n the better $(\hat{\alpha}_L, \hat{\beta}_L)$.

4. The confidence interval for shape parameter

For deriving the confidence interval of the shape paramter, we need to study the behavior of V_n^2.

By Theorem 2.3 and Theorem 2.4, we construct the normalized distribution of V_n^2 under gamma distribution with various parameters values: $\alpha = 0.5, 1, 1.5, 2$ and $\beta = 1, 2, 4$ when $n = 5, 10, 15$ and 20 by 100,000 simulations . Its simulated c.d.f. are presented in Hwang (2000). Comparing our simulated results with the results presented in Hwang and Lin (2000) for $\alpha = 1$ and $\beta = 1$ when $n = 5$, they are quite same; for example $P(V_5^2 \leq 1.10) = 0.7599$ in Hwang and Lin (2000) while it is equal to 0.7630 in this paper. From the simulated results we conclude that V_n^2 looks almost like a gamma distribution for any α, β and any n. This conclusion is justified by both of the Kolmogorov-Smirnov test and χ^2 test.

Furthermore, we obtain also by simulations the frequencies of V_n^2 falling in one standard deviation; two standard deviation and three standard deviation interval (with its mean as their center) respectively from Hwang (2000) for $\alpha = 0.5, 1.0, 1.5, 2.0$ and $\beta = 1, 2, 4$ when $n = 5, 10, 15, 20, 25$ and 30. The results are presented in Table 1. The

Table 1.

n	α	1σ	2σ	3σ
5	0.5	72.620	94.710	98.870
	1.0	75.680	95.153	98.297
	1.5	76.010	95.313	98.220
	2.0	75.943	95.310	98.300
10	0.5	76.920	95.340	98.287
	1.0	76.323	95.530	98.440
	1.5	75.717	95.467	98.433
	2.0	74.897	95.833	98.707
15	0.5	77.160	95.550	98.443
	1.0	75.890	95.593	98.487
	1.5	74.853	95.773	98.553
	2.0	73.843	95.760	98.627
20	0.5	77.010	95.577	98.353
	1.0	75.047	95.967	98.700
	1.5	74.027	95.983	98.723
	2.0	72.993	95.827	98.657
25	0.5	76.813	95.657	98.410
	1.0	75.037	95.977	98.647
	1.5	73.227	95.680	98.730
	2.0	72.450	95.953	98.957
30	0.5	76.233	95.623	98.460
	1.0	73.780	95.820	98.683
	1.5	72.173	95.647	98.763
	2.0	71.990	95.697	98.803

behavior of sample mean of V_n^2 is also investigated, and the conclusion is the same as central limit theorem; this fact can be justified by any of the Kolmogorov-Smirnov test and χ^2 test.

By Theorem 2.1 and Theorem 2.3, we have the mean and the variance of V_n^2 as follows:

$$E\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{n}{1+n\alpha}$$

and

$$\sigma^2 = \mathrm{Var}\left(\frac{S_n^2}{\bar{X}_n^2}\right) = \frac{2\alpha(\alpha+1)}{(n-1)\left(\alpha+\frac{1}{n}\right)\left(\alpha+\frac{2}{n}\right)\left(\alpha+\frac{3}{n}\right)}.$$

For finding the confidence interval of α, we need to manipulate the following probabilities for various values of α and n,

$$\Pr\left(\frac{n}{n\alpha+1} - k\sigma \le \frac{S_n^2}{\bar{X}_n^2} \le \frac{n}{n\alpha+1} + k\sigma\right).$$

Since it is quite difficult to derive, we present its approximate probabilities in Table 1 and the conclusions would be drawn for some values of $\alpha = 0.5, 1.0, 1.5, 2.0$, and $n = 5, 10, 15, 20, 25$ and 30 as follows:

$$\Pr\left(\frac{n}{n\alpha+1} - \sigma \le \frac{S_n^2}{\bar{X}_n^2} \le \frac{n}{n\alpha+1} + \sigma\right) \cong 0.75,$$

$$\Pr\left(\frac{n}{n\alpha+1} - 2\sigma \le \frac{S_n^2}{\bar{X}_n^2} \le \frac{n}{n\alpha+1} + 2\sigma\right) \cong 0.95$$

and

$$\Pr\left(\frac{n}{n\alpha+1} - 3\sigma \le \frac{S_n^2}{\bar{X}_n^2} \le \frac{n}{n\alpha+1} + 3\sigma\right) \cong 0.98.$$

Here 0.75, 0.95 and 0.98 will be assumed to be the mean probabilities respectively for various α,β and n. Thus the approximated 75.5%, 95% and 98% confidence intervals for α could be concluded respectively as follows:

$$\left(\frac{1}{\frac{S_n^2}{\bar{X}_n^2}+\hat{\sigma}} - \frac{1}{n}, \frac{1}{\frac{S_n^2}{\bar{X}_n^2}-\sigma} - \frac{1}{n}\right), \quad \left(\frac{1}{\frac{S_n^2}{\bar{X}_n^2}+2\hat{\sigma}} - \frac{1}{n}, \frac{1}{\frac{S_n^2}{\bar{X}_n^2}-2\sigma} - \frac{1}{n}\right) \quad \text{and}$$

$$\left(\frac{1}{\frac{S_n^2}{\bar{X}_n^2}+3\hat{\sigma}} - \frac{1}{n}, \frac{1}{\frac{S_n^2}{\bar{X}_n^2}-3\hat{\sigma}} - \frac{1}{n}\right)$$

where $\hat{\sigma}^2 = \frac{2\hat{\alpha}_c(\hat{\alpha}_c+1)}{(n-1)\left(\hat{\alpha}_c+\frac{1}{n}\right)^2\left(\hat{\alpha}_c+\frac{2}{n}\right)\left(\hat{\alpha}_c+\frac{3}{n}\right)}$, and $\hat{\alpha}_c$ is the new moment estimator of α proposed by using Theorem 2.3.

After simplification of the following probability, we write

$$\Pr\left(\frac{n}{n\alpha+1} - k\sigma \le \frac{S_n^2}{\bar{X}_n^2} \le \frac{n}{n\alpha+1} + k\sigma\right)$$

as

$$\Pr\left(\frac{\bar{X}_n^2}{S_n^2}\left(1-\frac{k}{\sqrt{n-1}}\right)-\frac{1}{n}\leq\alpha\leq\frac{\bar{X}_n^2}{S_n^2}\left(1-\frac{k}{\sqrt{n-1}}\right)-\frac{1}{n}\right)$$

and the approximate 75%, 95% and 98% confidence intervals for α are

$$\left(\frac{\bar{X}_n^2}{S_n^2}\left(1-\frac{k}{\sqrt{n-1}}\right)-\frac{1}{n},\frac{\bar{X}_n^2}{S_n^2}\left(1-\frac{k}{\sqrt{n-1}}\right)-\frac{1}{n}\right),\qquad k=1,2,3,$$

respectively for large sample.

Acknowledgements

We thank the editor, associate editor, and the referees for their thoughtful comments. We also thank L. H. Chen, S. T. Kao and K. E. Chang, master students of Institute of Statistics of Tsing Hua University, for their carefully performing all simulations.

References

Bai, J., Jakeman, A. J. and McAleer, M. (1991). A new approach to maximum likelihood estimation of the three-parameter gamma and Weibull distributions, *Austral. J. Statist.*, **33**, 397–410.

Bowman , K. O. and Shenton, L. R. (1988). Properties of estimators for the gamma distribution, Marcel Dekker, New York.

Bowman, K. O., Shenton, L. R. and Lam, H. K. (1987). Simulation and estimation problems associated with the three-parameter gamma distribution, *Comm. Statist. Simulation Comput.*, **16**, 1147–1188.

Cohen, A. C. and Norgaard, N. J. (1977). Progressively censored sampling in the three-parameter gamma distribution, *Technometrics*, **19**, 333–340.

Cohen, A. C. and Whitten, B. J. (1982). Modified moment and maximum likelihood estimators for parameters of the three-parameter gamma distribution, *Comm. Statist. Simulation. Comput.*, **11**, 197–216.

Greenwood, J. A. and Durand, D. (1960). Aids for fitting the gamma distribution by maximum likelihood, *Technometrics*, **2**, 55–65.

Harter, H. L. and Moore, A. H. (1965). Maximum-likelihood estimation of the parameters of the gamma and Weibull populations from complete and from censored samples, *Technometrics*, **7**, 639–643.

Hu, C. Y. (1990). On signal to noise ratio statistics, Ph. D. Thesis, Institute of Statistics, National Tsing Hua University, Taiwan.

Hwang, T. Y. (2000). On the inference of parameters of gamma distribution, Tech. Report, 89-2118-M-007-014, National Science Council, Taiwan.

Hwang, T. Y. and Hu, C. Y. (1999). On a characterization of the gamma distribution: The independence of the sample mean and the sample coefficient of variation, *Ann. Inst. Statist. Math.*, **51**, 749–753.

Hwang, T. Y. and Hu, C. Y. (2000). On some characterizations of population distribution, *Taiwanese J. Math.*, **4**(3), 427–437.

Hwang, T. Y. and Lin, Y. K. (2000). On the distribution of the sample heterogeneity of molecular polymer, *Tamsui Oxford Journal of Mathematical Sciences*, **16**(2), 133–149.

Johnson, N. J. and Kotz, S. (1970). *Continuous Univariate Distributions-1*, Wiley, New York.

Thom, H. C. S. (1958). A note on the gamma distribution, Washington, *Monthly Weather Review*, **86**(4), 117–121, Office of Climatology, U. S. Weather, Washington D. C.