Ann. Inst. Statist. Math.
Vol. 54, No. 1, 100–113 (2002)

BAYESIAN SAMPLING PLANS FOR EXPONENTIAL DISTRIBUTION BASED ON TYPE I CENSORING DATA*

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(Received February 14, 2000; revised August 15, 2000)

Abstract. We study variable sampling plans for the exponential distribution based on type I censoring data. Using a suitable loss function, a Bayesian variable sampling plan (n_B, t_B, δ_B) is derived. For certain prior distributions and loss functions, the numerical values of the Bayesian sampling plans and the associated minimum Bayes risks are tabulated. In terms of Bayes risks, comparisons between the proposed Bayesian sampling plans (n_B, t_B, δ_B) and the "Bayesian" variable sampling plans $(n_0, t_0, \delta^L_{T_0})$ of Lam (1994, *Ann. Statist.*, **22**, 696–711) have been made. The numerical results indicate that under the same conditions, the proposed Bayesian sampling plan is superior to that of Lam in the sense that the Bayes risk of (n_B, t_B, δ_B) is less than that of $(n_0, t_0, \delta^L_{T_0})$.

Key words and phrases: Sampling plan, type I censoring, Bayes risk, Bayesian procedure.

1. Introduction

It is well-known that quality control of products is essential to the manufacturers since it directly affects the market sale and the profits of the manufacturers. In the research area of quality control, there are a lot of sampling plans (see Wetherill (1977)) that may be applied for various criteria. Among these, the decision-theoretic approach is more reasonable and realistic because the sampling plan is determined by making an optimal decision on the basis of some realistic criterion such as maximizing the return or minimizing the risk. The research work along this approach has generated vast literature. However, most statisticians working on this problem are confined to a linear loss function (see Fertig and Mann (1974) and Wetherill and Köllerström (1979)). Hald (1967, 1981), Lam (1988*a*, 1988*b*, 1994) and Chen and Lam (1999*a*, 1999*b*), among others, use polynomial loss functions. In most situations, it is more reasonable and acceptable to consider a polynomial loss since it approximates a loss more closely and it includes linear loss as a special loss. Interested readers are referred to Hald (1981) and Lam (1988*a*, 1988*b*, 1994) for the motivation of the usage of polynomial losses.

Suppose we are given a batch of lifetime components for acceptance sampling. The quality of an item in the batch is measured by its lifetime X. In order to estimate the quality of the components of the batch, a sample of size n items are put on life test at the outset and are not replaced on failure. Because of highly developed technology

*This research was partially supported by NSC 89-2118-M-032-026 and NSC 90-2218-E-032-012 from National Science Council, R.O.C.

in engineering, most products have high reliabilities. Thus, usually, it may take a long time to observe the complete lifetime data. Due to the time restriction, the experiment terminates at a pre-specified time t. The failure time of an item is observable if it fails before or by time t. If an item still functions at the close of the experiment its failure time is not observable. The item is said to be censored at time t. This type of time censoring is known as type I censoring. Type I censoring scheme has received much attention in the statistical literature, see Bartholomew (1963), Spurier and Wei (1980), Mann *et al.* (1982), Yang and Sirvanci (1977) and Huang and Chen (1992), among many others.

Let $X_1, \ldots, X_n$ denote the lifetimes of the n components put on a life test experiment. It is assumed that $X_1, \ldots, X_n$ are mutually independent, follow an exponential distribution having expected lifetime $\theta = \lambda^{-1}$. We denote such an exponential distribution by $E(\lambda)$. Let $X_{(1)} \leq \cdots \leq X_{(n)}$ be the order statistics of $X_1, \ldots, X_n$. Since the sample is subject to type I censoring at time t, the true observations are: $Y_i = \min(X_{(i)}, t)$, $i = 1, \ldots, n$. Then, $M \equiv M(n,t) = \max\{i \mid X_{(i)} \leq t, i = 1, \ldots, n\}$ is the number of failures by time t. Thus, for the given sample size n and the censoring time t, M and $\boldsymbol{Y}(M,t) = (Y_1, \ldots, Y_M)$ are the observable random variables and $Y(n,t,M) = \sum_{i=1}^{M} Y_i + (n-M)t$ is the total lifetime of the n items up to time t. It is known that: If $M > 0$, the maximum likelihood estimator (MLE) of the expected lifetime θ is given by $\hat{\theta}_{ML} = \frac{Y(n,t,M)}{M}$; and if M=0, then $\hat{\theta}_{ML} = nt$ (see Sinha (1986)).

For each (n,t), a decision function $\delta(\ \mid n,t)$ is a function defined on the observed value $(m, \boldsymbol{y}(m,t))$ of the random variables $(M, \boldsymbol{Y}(M,t))$ such that $\delta(m, \boldsymbol{y}(m,t) \mid n,t)$ is the probability of accepting the batch. The determination of n, t and $\delta(\ \mid n,t)$ is called a sampling plan which is denoted by (n,t,δ). Suppose that the parameter λ follows a prior distribution G over the parameter space $\Omega = (0,\infty)$. Then, the performance of a sampling plan (n,t,δ) should be evaluated based on its associated Bayes risk $r(n,t,\delta)$.

Lam (1994) studied a type of sampling plan (n,t,δ_T^L) based on $\hat{\theta}_{ML}$ given by

$$\delta_T^L(m, \boldsymbol{y}(m,t) \mid n,t) = \begin{cases} 1, & \text{if } \hat{\theta}_{ML} \geq \mathrm{T}, \\ 0, & \text{otherwise}\ ; \end{cases} \tag{1.1}$$

where $T = T(n,t)$ is some values between 0 and nt. Using the loss $L(a,\lambda,n,t)$ of (2.4) with $C_2 = 0$ where C_2 is the cost per unit time for life test, Lam (1994) attempted to find the (optimal) sampling plan, say $(n_0, t_0, \delta_{T_0}^L)$ such that $r(n_0,t_0,\delta_{T_0}^L) = \inf r(n,t,\delta_T^L)$ among all sampling plans of the type (n,t,δ_T^L).

We have observed that Lam's (optimal) sampling plan $(n_0,t_0,\delta_{T_0}^L)$ possesses certain defects:

(1) Though $\hat{\theta}_{ML}$ has certain optimal property, and δ_T^L of (1.1) is a natural decision function, it is not a Bayes decision function in general and therefore, it is not the optimal sampling plan.

(2) Since the cost of time t is not included in the loss of Lam (1994), for n being fixed, the best choice of censoring time should be $t = \infty$ from which we can observe the complete lifetime data. Based on these complete lifetime data, we can choose a suitable T value to reduce the cost of making a wrong decision. In other words, the t_0 values reported in Tables of Lam (1994) are not the optimal censoring time.

We may consider two competitors $(n_0,\infty,\delta_{T^*}^L)$ and (n_B^*,∞,δ_B) that may perform better than $(n_0,t_0,\delta_{T_0}^L)$. Here, n_0 is the "optimal" sample size obtained from Lam (1994), T^* is a positive value satisfying $r(n_0,\infty,\delta_{T^*}^L) = \inf_{T>0} r(n_0,\infty,\delta_T^L)$, and (n_B^*,∞,δ_B)

is the Bayesian sampling plan developed in Section 4 for $t = +\infty$ case. In the loss $L(a, \lambda, n, t)$ of (2.4), with $C_2 = 0$ and $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$, we have that $r(n_B^*, \infty, \delta_B) \leq r(n_0, \infty, \delta_{T^*}^L) \leq r(n_0, t_0, \delta_{T_0}^L)$ with strict inequality occurring in many cases (see Section 4). Thus, it is clear that the "optimal" sampling plan $(n_0, t_0, \delta_{T_0}^L)$ indeed is neither optimal nor Bayes.

As addressed previously, a reasonable sampling plan should take the cost of censoring time t into account. In fact, Chen and Lam (1999*a*, 1999*b*) have considered the cost of censoring time t. They assume the rate of time-consuming cost in the loss function. For finding a suitable censoring time t, the cost of the censoring time t should be included in the loss function. Therefore, in the paper, it is assumed that C_2, the cost per unit time, is positive.

In this paper, our goal is to seek an optimal sampling plan (n_B, t_B, δ_B) possessing the property that $r(n_B, t_B, \delta_B) = \inf r(n, t, \delta)$ among the class of all sampling plan (n, t, δ). We set up a decision-theoretic formulation of the problem of acceptance sampling in Section 2. A Bayesian sampling plan is derived. In Section 3, we consider a special case where $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ is considered. We provide an explicit presentation of the Bayes risk of a sampling plan $r(n, t, \delta_B(\ \mid n, t))$. Based on this expression, a numerical approximation for finding the optimal sample size n_B and the optimal censoring time t_B is proposed. In Section 4, we discuss a sampling plan for the case $C_2 = 0$. Some numerical results are computed to compare the performance of the three sampling plans $(n_B^*, \infty, \delta_B)$, $(n_0, \infty, \delta_{T^*}^L)$ and $(n_0, t_0, \delta_{T_0}^L)$. The numerical results indicate that $(n_B^*, \infty, \delta_B)$ is the best among the three sampling plans in terms of Bayes risks and $(n_0, \infty, \delta_{T^*}^L)$ is the second.

2. The model and a Bayesian sampling plan

For the fixed sample size n and the censoring time t, let M denote the number of failures by time t and let $\boldsymbol{Y}(M, t) = (Y_1, \ldots, Y_M)$ be the observable lifetimes of the M failed components. Then, given λ, $(M, \boldsymbol{Y}(M, t))$ has a joint probability density function

$$(2.1) \qquad f(m, y_1, \ldots, y_m \mid \lambda) = \frac{n!}{(n-m)!}\lambda^m \exp\left\{-\lambda\left(\sum_{j=1}^m y_j + (n-m)t\right)\right\}$$
$$= \frac{n!}{(n-m)!}\lambda^m \exp\{-\lambda y(n, t, m)\}$$

where $y(n, t, m) = \sum_{j=1}^m y_j + (n-m)t$ and $(n-m)t \leq y(n, t, m) \leq nt$. Note that $(M, Y(n, t, M))$ are sufficient for λ. It is assumed that the parameter λ is a realization of a positive random variable Λ, having a prior density $g(\lambda)$ over $(0, \infty)$. Then, the marginal joint probability density function of $(M, \boldsymbol{Y}(M, t))$ is

$$(2.2) \qquad f(m, y_1, \ldots, y_m) = \int_0^\infty f(m, y_1, \ldots, y_m \mid \lambda) g(\lambda) d\lambda$$
$$= \frac{n!}{(n-m)!}\int_0^\infty \lambda^m \exp\{-\lambda y(n, t, m)\} g(\lambda) d\lambda.$$

The posterior probability density of λ given $(M, \boldsymbol{Y}(M, t)) = (m, \boldsymbol{y}(m, t))$ is:

$$(2.3) \qquad g(\lambda \mid m, \boldsymbol{y}(m, t)) = \frac{f(m, \boldsymbol{y}(m, t) \mid \lambda) g(\lambda)}{f(m, \boldsymbol{y}(m, t))}$$

$$= \frac{\lambda^m \exp\{-\lambda y(n,t,m)\} g(\lambda)}{\int_0^\infty \lambda^m \exp\{-\lambda y(n,t,m)\} g(\lambda) d\lambda}.$$

From (2.3), it is clear that the posterior density $g(\lambda \mid m, \boldsymbol{y}(m,t))$ depends on $(m, \boldsymbol{y}(m,t))$ only through m and $y(n,t,m)$. Thus, we may also denote it by $g(\lambda \mid m, y(n,t,m))$.

Suppose that a batch of lifetime components is presented for acceptance sampling. Let a denote an action on this problem of acceptance sampling. When $a=1$, it means that the batch is accepted; and when $a=0$, it means to reject the batch. For the given sample size n, censoring time t and parameter λ, the loss of taking action a is defined as:

$$L(a,\lambda,n,t) = ah(\lambda) + (1-a)C_3 + nC_1 + tC_2, \tag{2.4}$$

where C_1, C_2 and C_3 are positive constants,

C_1: the cost per item inspected,

C_2: the cost per unit time used for life test,

C_3: the loss due to rejecting the batch,

and $h(\lambda)$: the loss of accepting the batch. Since $\theta = \lambda^{-1}$ is the expected lifetime, larger λ indicates smaller θ. So, usually, we require $h(\lambda)$ to be positive and increasing in λ for $\lambda > 0$. Also, to ensure the Bayes risk to be finite, it is assumed that $\int_0^\infty h(\lambda) g(\lambda) d\lambda < \infty$.

Using the loss $L(a,\lambda,n,t)$, the Bayes risk of a sampling plan (n,t,δ) is:

$$\begin{aligned} (2.5)\ r(n,t,\delta) &= E_\Lambda E_{M,\boldsymbol{Y}(M,t)|\Lambda}\{nC_1 + tC_2 + C_3 + \delta(M,\boldsymbol{Y}(M,t) \mid nt)[h(\Lambda) - C_3]\} \\ &= nC_1 + tC_2 + C_3 + r_1(\delta \mid n,t) \end{aligned}$$

where,

$$\begin{aligned} (2.6)\quad r_1(\delta \mid n,t) &= E_\Lambda E_{M,\boldsymbol{Y}(M,t)|\Lambda}\{\delta(M,\boldsymbol{Y}(M,t) \mid n,t)[h(\Lambda) - C_3]\} \\ &= E_{M,\boldsymbol{Y}(M,t)} E_{\Lambda|M,\boldsymbol{Y}(M,t)}\{\delta(M,\boldsymbol{Y}(M,t) \mid n,t)[h(\Lambda) - C_3]\} \\ &= \sum_{m=0}^{n} \int_{\boldsymbol{y}(m,t)} \cdots \int \delta(m,\boldsymbol{y}(m,t) \mid n,t) \{E_{\Lambda|m,\boldsymbol{y}(m,t)}[h(\Lambda) - C_3]\} \\ &\quad \cdot f(m,\boldsymbol{y}(m,t)) d\boldsymbol{y}(m,t) \end{aligned}$$

and

$$\begin{aligned} (2.7)\quad E_{\Lambda|m,\boldsymbol{y}(m,t)}[h(\Lambda) - C_3] &= \int_0^\infty h(\lambda) g(\lambda \mid m,\boldsymbol{y}(m,t)) d\lambda - C_3 \\ &= \varphi_g(m, y(n,t,m)) - C_3 \end{aligned}$$

where $\varphi_g(m,y(n,t,m)) = \int_0^\infty h(\lambda) g(\lambda \mid m,\boldsymbol{y}(m,t)) d\lambda$, the posterior expectation of $h(\Lambda)$ given $(M,\boldsymbol{Y}(M,t)) = (m,\boldsymbol{y}(m,t))$.

Therefore, for the fixed sample size n and the censoring time t, the Bayes decision function $\delta_B(\ \mid n,t)$, which minimizes $r_1(\delta \mid n,t)$ among all decision function $\delta(\ \mid n,t)$ is given by:

$$\delta_B(m,\boldsymbol{y}(m,t) \mid n,t) = \begin{cases} 1, & \text{if } \varphi_g(m,y(n,t,m)) \le C_3, \\ 0, & \text{otherwise.} \end{cases} \tag{2.8}$$

In the following, we investigate the monotinicity of the Bayes decision function $\delta_B(\ \mid n,t)$ as (n,t) being fixed.

LEMMA 2.1. *Let* $0 \le m^*$, $m \le n$ *and* $y = y(n,t,m)$, $y^* = y(n,t,m^*)$. *Consider the likelihood ratio*

$$\ell(\lambda \mid (m,y),(m^*,y^*)) = g(\lambda \mid m^*,y^*)/g(\lambda \mid m,y) \qquad if \quad g(\lambda \mid m,y) \neq 0.$$

Then,

(a) *As* $m = m^*$ *and* $y < y^*$, $\ell(\lambda \mid (m,y),(m^*,y^*))$ *is nonincreasing in* λ.

(b) *As* $y = y^*$ *and* $m < m^*$, $\ell(\lambda \mid (m,y),(m^*,y^*))$ *is nondecreasing in* λ.

The following theorem is a direct consequence of Lemma 2.1.

THEOREM 2.1. *Let* $h(\lambda)$ *be a positive and increasing function of* λ *for* $\lambda > 0$. *Then,*

(a) $\varphi_g(m,y) = \int_0^\infty h(\lambda)g(\lambda \mid m,y)d\lambda$ *is nonincreasing in* y *and nondecreasing in* m.

(b) $\delta_B(m,\boldsymbol{y}(m,t) \mid n,t)$ *is nondecreasing in* $y(n,t,m)$ *and nonincreasing in* m.

Derivation of a Bayesian sampling plan. We consider the following scheme to derive a Bayesian sampling plan.

Scheme A.

Step 1: For fixed (n,t), derive the Bayes decision function $\delta_B(\ \mid n,t)$ to minimize the risks $r_1(\delta \mid n,t)$ among all decision functions $\delta(\ \mid n,t)$.

Step 2: For fixed n, derive the censoring time $t_B(n)$, which minimizes $tC_2 + r_1(\delta_B \mid n,t)$ among all $t > 0$. That is, $t_B(n)$ is such that $t_B(n)C_2 + r_1(\delta_B \mid n, t_B(n)) = \inf\{tC_2 + r_1(\delta_B \mid n,t) \mid t > 0\}$.

Step 3: Find the sample size n_B which minimizes $r(n,t_B(n),\delta_B(\ \mid n,t_B(n)))$ among all $n = 0,1,2,\ldots$.

We denote the sampling plan derived through the Scheme A by (n_B,t_B,δ_B). The sampling plan (n_B,t_B,δ_B) possesses the following optimality.

THEOREM 2.2. (n_B,t_B,δ_B) *is a Bayesian sampling plan in the sense that* $r(n_B, t_B(n_B), \delta_B(\ \mid n_B,t_B(n_B))) = \inf r(n,t,\delta)$ *among the class of all sampling plans.*

PROOF. For any sampling plan (n,t,δ),

$$\begin{aligned}(2.9)\qquad & r(n,t,\delta) - r(n_B,t_B(n_B),\delta_B(\ \mid n_B,t_B(n_B))) \\ &= [r(n,t,\delta) - r(n,t,\delta_B(\ \mid n,t))] \\ &\quad + [r(n,t,\delta_B(\ \mid n,t)) - r(n,t_B(n),\delta_B(\ \mid n,t_B(n)))] \\ &\quad + [r(n,t_B(n),\delta_B(\ \mid n,t_B(n))) - r(n_B,t_B(n_B),\delta_B(\ \mid n_B,t_B(n_B)))].\end{aligned}$$

By Scheme A, it follows that

$$(2.10)\qquad \begin{cases} r(n,t,\delta) - r(n,t,\delta_B(\ \mid n,t)) \ge 0, \\ r(n,t,\delta_B(\ \mid n,t)) - r(n,t_B(n),\delta_B(\ \mid n,t_B(n))) \ge 0, \\ r(n,t_B(n),\delta_B(\ \mid n,t_B(n))) - r(n_B,t_B(n_B),\delta_B(\ \mid n_B,t_B(n_B))) \ge 0. \end{cases}$$

Combining (2.9)–(2.10) concludes that (n_B,t_B,δ_B) is a Bayesian samping plan. The following theorem guarantees the finiteness of the optimal sample size n_B and the optimal censoring time $t_B(n_B)$.

THEOREM 2.3. *Let n_B and $t_B(n_B)$ be the optimal sample size and censoring time derived through Scheme A. Then,*

$$n_B \leq \min\left(\frac{\varphi_g(0,0)}{C_1}, \frac{C_3}{C_1}\right),$$

and

$$t_B(n_B) \leq \min\left(\frac{\varphi_g(0,0)}{C_2}, \frac{C_3}{C_2}\right),$$

where $\varphi_g(0,0) = \int_0^\infty h(\lambda)g(\lambda)d\lambda < \infty$ by assumption.

PROOF. Let $(0,0,\delta_B(\ \mid 0,0))$ denote the sampling plan for which no data is observed and

$$\delta_B(\ \mid 0,0) = \begin{cases} 1, & \text{if } \varphi_g(0,0) \leq C_3, \\ 0, & \text{otherwise.} \end{cases}$$

Then, $r(0,0,\delta_B) = \min(\varphi_g(0,0), C_3)$.

Since (n_B, t_B, δ_B) is the Bayesian sampling plan, we have: $r(n_B, t_B, \delta_B) \leq r(0,0,\delta_B)$. That is,

$$n_B C_1 + t_B(n_B)C_2 + C_3 + r_1(\delta_B \mid n_B, t_B(n_B)) \leq \min(\varphi_g(0,0), C_3). \tag{2.11}$$

Note that $r_1(\delta_B \mid n_B, t_B(n_B)) \geq 0$ and $C_3 \geq 0$. From (2.11), it follows that

$$n_B \leq \min\left(\frac{\varphi_g(0,0)}{C_1}, \frac{C_3}{C_1}\right)$$

and

$$t_B(n_B) \leq \min\left(\frac{\varphi_g(0,0)}{C_2}, \frac{C_3}{C_2}\right).$$

Hence, the proof of the theorem is complete.

3. Bayes sampling plan for quadratic loss

To look deep into the Bayesian sampling plan (n_B, t_B, δ_B), for simplicity, we assume $h(\lambda)$ to be a quadratic function $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ where a_0, a_1 and a_2 are positive coefficient. We consider a conjugate prior distribution, namely, $g(\lambda)$ is the probability density of a $\Gamma(\alpha,\beta)$ distribution given by

$$g(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\lambda\beta}, \qquad \lambda > 0.$$

A straightfoward computation shows that: Given $(M, \boldsymbol{Y}(M,t)) = (m, \boldsymbol{y}(m,t))$, the posterior probability density of Λ is:

$$g(\lambda \mid m, y(n,t,m)) \sim \Gamma(m+\alpha, y(n,t,m)+\beta).$$

We have:

$$\begin{aligned}\varphi_g(m, y(n,t,m)) &= \int_0^\infty h(\lambda)g(\lambda \mid m, y(n,t,m))d\lambda \\ &= a_0 + \frac{a_1(m+\alpha)}{y(n,t,m)+\beta} + \frac{a_2(m+\alpha)(m+\alpha+1)}{[y(n,t,m)+\beta]^2},\end{aligned} \tag{3.1}$$

and

$$\delta_B(m,\boldsymbol{y}(m,t)\mid n,t)=\begin{cases}1, & \text{if}\quad \varphi_g(m,y(n,t,m))\le C_3,\\ 0, & \text{otherwise.}\end{cases} \tag{3.2}$$

Note that if $C_3 \le a_0$, then $\varphi_g(m,y(n,t,m)) > C_3$ for all $(m,\boldsymbol{y}(m,t))$. Therefore $\delta_B(m,\boldsymbol{y}(m,t)\mid n,t)\equiv 0$. To avoid this extreme case, we assume that $C_3 > a_0$.

From (3.1)–(3.2) it follows that $\delta_B(m,\boldsymbol{y}(m,t)\mid n,t)=1$ if, and only if

$$(C_3-a_0)[y(n,t,m)+\beta]^2-a_1(m+\alpha)[y(n,t,m)+\beta]-a_2(m+\alpha)(m+\alpha+1)\ge 0,$$

which is equivalent to

$$\begin{aligned} y(n,t,m)+\beta &\ge \frac{a_1(m+\alpha)+\sqrt{a_1^2(m+\alpha)^2+4(C_3-a_0)a_2(m+\alpha)(m+\alpha+1)}}{2(C_3-a_0)}\\ &\equiv D_n(m).\end{aligned}$$

Thus, the Bayes decision function $\delta_B(\ \mid n,t)$ can be expressed as

$$\delta_B(m,\boldsymbol{y}(m,t)\mid n,t)=\begin{cases}1, & \text{if}\quad y(n,t,m)\ge D_n(m)-\beta,\\ 0, & \text{otherwise.}\end{cases} \tag{3.3}$$

Let $\mu_1=E_g[\Lambda]$, $\mu_2=E_g[\Lambda^2]$. Thus, $\int_0^\infty h(\lambda)g(\lambda)d\lambda=a_0+a_1\mu_1+a_2\mu_2$. Also, let

$$\triangle_m(n,t,\beta)=\left\{(y_1,\ldots,y_m)\mid 0<y_1<\cdots<y_m<t,\ \sum_{j=1}^m y_j<D_n(m)-\beta-(n-m)t\right\},$$

and

$$H(n,t,\beta,m)=\int_{\triangle_m(n,t,\beta)}\cdots\int \lambda^m \exp\left\{-\lambda\sum_{j=1}^m y_j\right\}dy_1\cdots dy_m.$$

Then, the Bayes risk of the sampling plan $(n,t,\delta_B(\ \mid n,t))$ is:

$$\begin{aligned} r(n,t,\delta_B) &= nC_1+tC_2+\int_0^\infty h(\lambda)g(\lambda)d\lambda\\ &\quad +E\{[C_3-h(\Lambda)][1-\delta_B(M,\boldsymbol{Y}(M,t)\mid n,t)]\}\\ &= nC_1+tC_2+a_0+a_1\mu_1+a_2\mu_2\\ &\quad +\int_0^\infty [C_3-h(\lambda)]P\{\delta_B(M,\boldsymbol{Y}(M,t)\mid n,t)=0\mid\lambda\}g(\lambda)d\lambda \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} &P\{\delta_B(M,\boldsymbol{Y}(M,t)\mid n,t)=0\mid\lambda\}\\ &\quad =P\left\{\sum_{j=1}^M Y_j+(n-M)t<D_n(M)-\beta\mid\lambda\right\} \end{aligned} \tag{3.5}$$

$$= P\left\{\sum_{j=1}^{M} Y_j < D_n(M) - \beta - (n-M)t \mid \lambda\right\}$$
$$= P\{M = 0 \mid \lambda\} I(nt < D_n(0) - \beta)$$
$$+ \sum_{m=1}^{n} \int_{\triangle_m(n,t,\beta)} \cdots \int \frac{n!}{(n-m)!} \lambda^m$$
$$\cdot \exp\left\{-\lambda\left\{\sum_{j=1}^{m} y_j + (n-m)t\right\}\right\} dy_1 \cdots dy_m$$
$$= P\{M = 0 \mid \lambda\} I(nt < D_n(0) - \beta)$$
$$+ \sum_{m=1}^{n} \frac{n!}{(n-m)!} \exp\{-\lambda(n-m)t\} H(n,t,\beta,m).$$

Let [x] denote the largest integer not exceeding x. Let $\ell^* \equiv \ell^*(n,t,\beta,m) = [\frac{D_n(m)-\beta-(n-m)t}{t}]$. From Theorem A of Lam (1994), it follows that

(3.6) $H(n,t,\beta,m)$

$$= \begin{cases} 0, & \text{if } D_n(m) - \beta - (n-m)t \le 0, \\ \dfrac{\lambda^m}{m!(m-1)!} \displaystyle\sum_{j=0}^{\ell^*} \int_0^{D_n(m)-\beta-(n-m)t-jt} (-1)^j \binom{m}{j} \mu^{m-1} e^{[-\lambda(\mu+jt)]} d\mu, & \\ \quad \text{if } 0 < D_n(m) - \beta - (n-m)t \le mt, & \\ \dfrac{1}{m!}[1 - e^{(-\lambda t)}]^m, & \text{if } D_n(m) - \beta - (n-m)t > mt. \end{cases}$$

Let $I_n = \{1, \ldots, n\}$ and let

$A \equiv A(n,t,\beta) = \{m \in I_n \mid D_n(m) - \beta - (n-m)t \le 0\}$,
$B \equiv B(n,t,\beta) = \{m \in I_n \mid 0 < D_n(m) - \beta - (n-m)t \le mt\}$,
$C \equiv C(n,t,\beta) = \{m \in I_n \mid D_n(m) - \beta - (n-m)t > mt\}$

By the definition of $D_n(m)$, we see that both $D_n(m)-\beta-nt$ and $D_n(m)-\beta-(n-m)t$ are increasing in m for $m \in I_n$. Suppose that A, B and C are nonempty. Then, for any m_1 in A, m_2 in B and m_3 in C, we must have $m_1 < m_2 < m_3$. According to (3.6), for $m \in A$, $H(n,t,\beta,m) = 0$. For $m \in C$, $H(n,t,\beta,m) = \frac{1}{m!}[1 - \exp(-\lambda t)]^m$, and for $m \in B$,

$$H(n,t,\beta,m) = \frac{\lambda^m}{m!(m-1)!} \sum_{j=0}^{\ell^*} \int_0^{D_n(m)-\beta-(n-m)t-jt} (-1)^j \binom{m}{j} \mu^{m-1} \exp[-\lambda(\mu+jt)] d\mu.$$

Therefore, the Bayes risk $r(n,t,\delta_B(\ \mid n,t))$ can be expressed as:

(3.7) $r(n,t,\delta_B(\ \mid n,t))$

$$= [nC_1 + tC_2 + a_0 + a_1\mu_1 + a_2\mu_2]$$
$$+ \int_0^\infty [C_3 - h(\lambda)] P\{M = 0 \mid \lambda\} I(nt < D_n(0) - \beta) g(\lambda) d\lambda$$
$$+ \sum_{m\in B} \int_0^\infty [C_3 - h(\lambda)] \frac{n!}{(n-m)!} \exp\{-\lambda(n-m)t\} H(n,t,\beta,m) g(\lambda) d\lambda$$

$$+\sum_{m\in C}\int_0^\infty [C_3-h(\lambda)]\frac{n!}{(n-m)!}\exp\{-\lambda(n-m)t\}$$
$$\cdot\frac{1}{m!}[1-\exp(-\lambda t)]^m g(\lambda)d\lambda$$
$$\equiv r_1+r_2+r_3+r_4.$$

where $r_1 = nC_1 + tC_2 + a_0 + a_1\mu_1 + a_2\mu_2$.

Note that $P\{M = 0 \mid \lambda\} = \exp\{-\lambda nt\}$. A stragihtforward computation shows that

$$\text{(3.8)}\quad r_2 = I(nt < D_n(0)-\beta)\int_0^\infty [C_3-a_0-a_1\lambda-a_2\lambda^2]e^{-\lambda nt}\frac{\beta^\alpha\lambda^{\alpha-1}}{\Gamma(\alpha)}e^{-\lambda\beta}d\lambda$$
$$= I(nt < D_n(0)-\beta)\left\{\frac{(C_3-a_0)\beta^\alpha}{(nt+\beta)^\alpha}-\frac{a_1\alpha\beta^\alpha}{(nt+\beta)^{\alpha+1}}-\frac{a_2\alpha(\alpha+1)\beta^\alpha}{(nt+\beta)^{\alpha+2}}\right\}.$$

Following a discussion analogous to (2.17)–(2.19) of Lam (1994), we can obtain

$$\text{(3.9)}\quad r_3 = \sum_{m\in B}\int_0^\infty [C_3-h(\lambda)]\frac{n!}{(n-m)!}\exp\{-\lambda(n-m)t\}H(n,t,\beta,m)g(\lambda)d\lambda$$
$$= \sum_{m\in B}\sum_{j=0}^{\ell^*}\binom{n}{m}\binom{m}{j}(-1)^j[(m-1)!]^{-1}\beta^\alpha(\Gamma(\alpha))^{-1}[(n-m)t+\beta+jt]^{-(\alpha+2)}$$
$$\times[(C_3-a_0)\Gamma(m+\alpha)[(n-m)t+\beta+jt]^2\beta_y(m,\alpha)$$
$$-a_1\Gamma(m+\alpha+1)[(n-m)t+\beta+jt]\beta_y(m,\alpha+1)$$
$$-a_2\Gamma(m+\alpha+2)\beta_y(m,\alpha+2)]$$

where $\beta_y(u,v) = \int_0^y x^{u-1}(1-x)^{v-1}dx$, $0 < y < 1$, is the incomplete beta function and $y = [D_n(m)-\beta-(n-m)t-jt]/D_n(m)$.

Analogous to (2.20) of Lam (1994), we have

$$\text{(3.10)}\quad r_4 = \sum_{m\in C}\int_0^\infty [C_3-h(\lambda)]\frac{n!}{(n-m)!}\exp\{-\lambda(n-m)t\}\frac{1}{m!}[1-\exp(-\lambda t)]^m g(\lambda)d\lambda$$
$$= \sum_{m\in C}\sum_{j=0}^{m}\binom{n}{m}\binom{m}{j}(-1)^j\beta^\alpha$$
$$\times\left\{\frac{(C_3-a_0)((n-m)t+\beta+jt)^2-a_1\alpha((n-m)t+\beta+jt)-a_2\alpha(\alpha+1)}{[(n-m)t+\beta+jt]^{\alpha+2}}\right\}.$$

Combining (3.7)–(3.10) it provides an explicit presentation of the Bayes risk of the sampling plan (n, t, δ_B).

Algorithm for finding n_B and $t_B(n_B)$. In the following, we shall consider the algorithm to derive the Bayes sample size n_B and the Bayes censoring time $t_B(n_B)$. Let $n^* = \min(a_0 + a_1\mu_1 + a_2\mu_2, C_3)/C_1$ and $t^* = \min(a_0 + a_1\mu_1 + a_2\mu_2, C_3)/C_2$. Note that both n^* and t^* are finite. According to Theorem 2.3,

$$\text{(3.11)}\qquad 0 \le n_B \le n^* \quad\text{and}\quad 0 \le t_B(n_B) \le t^*.$$

Table 1. Bayes risks of the sampling plans $r(n_B, t_B, \delta_B)$ for $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, $C_1 = 0.5$, $C_2 = 0.5$, $C_3 = 30$ as α or β varies.

α	β	n_B	t_B	$r(n_B, t_B, \delta_B)$	α	β	n_B	t_B	$r(n_B, t_B, \delta_B)$
0.1	0.2	2	0.4038	6.18324	2.5	0.8	3	0.6750	25.28678
0.2	2.0	0	0.0000	2.32000	2.5	1.0	4	0.6738	22.08527
1.0	0.2	3	0.8250	24.89661	2.5	1.2	0	0.0000	18.31944
1.5	0.8	4	0.5677	16.99461	3.0	0.8	3	0.8250	28.00886
1.5	2.0	0	0.0000	5.37500	3.5	0.8	2	0.8250	29.71367

Based on the explicit expression of the Bayes risk $r(n, t, \delta_B(\ \mid n, t))$, the upper bounds given in (3.11) and following the steps of Scheme A, we propose the following algorithm to determine the values of n_B and $t_B(n_B)$.

Algorithm B. (1) Start with $n = 0$, compute $r(0, 0, \delta_B)$.

(2) For each $n = 1, \ldots, n^*$, compute $r(n, t, \delta_B(\ \mid n, t))$ and minimize $r(n, t, \delta_B(\ \mid n, t))$ with respect to t. We denote the minimizer by $t_B(n)$.

(3) Compare the risks among $r(0, 0, \delta_B)$ and $r(n, t_B(n), \delta_B(\ \mid n, t_B(n)))$. Let $S = \{n \in I_{n^*} \mid r(n, t_B(n), \delta_B(\ \mid n, t_B(n))) < r(0, 0, \delta_B)\}$. Then, n_B is determined as:

$$n_B = \begin{cases} 0, & \text{if } S = \phi, \\ \min\{n \mid n \in S\}, & \text{if } S \neq \phi. \end{cases} \tag{3.12}$$

The expression of $r(n, t, \delta_B(\ \mid n, t))$ is very complicated because $H(n, t, \beta, m)$ is not a continuous function of t; see (3.6). Thus, many numerical optimization methods such as Newton-Gauss, steepest descent and conjugate gradient methods, are not applicable for finding the $t_B(n)$. In the following, an alternative method is applied to provide a numerical approximation to the value of $t_B(n)$.

Numerical approximation C. First divide the interval $[0, t^*]$ into N subintervals of equal length and let $L(N, t^*) = \frac{t^*}{N}$ denote the length of these subintervals. Let

$$t_j \equiv t_j(N, t^*) = (j - 0.5)L(N, t^*), \qquad j = 1, \ldots, N.$$

For each n, compute $r(n, t_j, \delta_B(\ \mid n, t_j))$, $j = 1, \ldots, N$, and let

$$t_B^*(n) = \min\{t_i \mid i \in I_N, r(n, t_i, \delta_B(\ \mid n, t_i)) = \min_{1 \le j \le N} r(n, t_j, \delta_B(\ \mid n, t_j))\}.$$

We may use $t_B^*(n)$ to approximate the optimal censoring time $t_B(n)$ for n being fixed. According to the choice of the length of the subintervals, the distance between $t_B(n)$ and $t_B^*(n)$ will not be greater than $\frac{2t^*}{N}$. Thus, we may choose a large N to have a better approximation.

Numerical results. To illustrate the model, the Scheme A, the Algorithm B and the Numerical approximation C, certain numerical results are prepared and tabulated in Tables 1–3. The values $\alpha = 2.5$, $\beta = 0.8$, $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, $C_1 = 0.5$, $C_2 = 0.5$, $C_3 = 30$ and N=400 are used for comparison. In each table, only two parameters or one coefficient may vary and the others are fixed. For example, in the first column of Table

Table 2. Bayes risks of the sampling plans $r(n_B,t_B,\delta_B)$ for $\alpha = 2.5$, $\beta = 0.8$, $C_1 = 0.5$, $C_2 = 0.5$, $C_3 = 30$ as a_0, a_1, or a_2 varies.

a_0	n_B	t_B	$r(n_B,t_B,\delta_B)$	a_1	n_B	t_B	$r(n_B,t_B,\delta_B)$	a_2	n_B	t_B	$r(n_B,t_B,\delta_B)$
0.05	4	0.8250	24.24753	0.05	4	0.6638	22.95417	0.05	0	0.0000	8.93359
0.1	4	0.8250	24.27502	0.1	4	0.6673	23.02205	0.1	0	0.0000	9.61719
0.5	4	0.8250	24.49384	0.5	4	0.6750	23.55444	0.5	0	0.0000	15.08594
1.0	4	0.8250	24.76458	1.0	4	0.8250	24.18191	1.0	5	0.6029	21.33966
1.5	3	0.6750	25.03009	1.5	3	0.6750	24.75236	1.5	4	0.6471	23.63835
2.0	3	0.6750	25.28678	2.0	3	0.6750	25.28678	2.0	3	0.6750	25.28678
3.0	3	0.8250	25.78700	3.0	3	0.8250	26.22373	3.0	3	0.9750	27.28157
5.0	3	0.8250	26.71756	5.0	3	0.9750	27.74081	5.0	2	1.1250	29.20082
10.0	2	0.8250	28.70745	10.0	2	1.1250	29.85795	10.0	0	0.0000	30.00000

Table 3. Bayes risks of the sampling plans $r(n_B,t_B,\delta_B)$ for $\alpha = 2.5$, $\beta = 0.8$, $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, as C_1, C_2 or C_3 varies.

C_1	n_B	t_B	$r(n_B,t_B,\delta_B)$	C_2	n_B	t_B	$r(n_B,t_B,\delta_B)$	C_3	n_B	t_B	$r(n_B,t_B,\delta_B)$
0.3	6	0.6750	24.45479	0.3	4	0.8750	25.12706	10	0	0.0000	10.00000
0.4	4	0.8250	24.89679	0.4	4	0.8438	25.21314	15	0	0.0000	15.00000
0.5	3	0.6750	25.28678	0.5	3	0.6750	25.28678	20	2	0.8500	19.33027
0.6	3	0.6750	25.58678	0.6	3	0.6875	25.35124	30	3	0.6750	25.28678
0.8	3	0.6750	26.18678	0.8	3	0.7031	25.49009	40	4	0.6230	29.35520
1.0	3	0.6750	26.78678	1.0	3	0.6375	25.62906	50	5	0.6229	32.21752
2.0	3	0.6750	29.78678	2.0	3	0.4313	26.24363	100	0	0.0000	35.59375

2, a_0 varies while $a_1 = a_2 = 2$ keeps fixed. In the entries of the tables, $r(n_B,t_B,\delta_B)$ is used to denote the minimum Bayes risk while $(n_B,t_B(n_B),\ \delta_B(\ \mid n_B,t_B(n_B)))$ is the Bayesian sampling plan.

We summarize the results in optimal Bayes risks and optimal sampling plans against parameters in Tables 1–3. (n_B,t_B,δ_B) and $r(n_B,t_B,\delta_B)$ are the optimal plan and the corresponding optimal Bayes risk respectively, in which $(0,0,\delta_B)$ denotes the sampling plan of accepting the batch without sampling. For the parameters $(\alpha,\beta,a_0,a_1,a_2,C_1,C_2,C_3) = (2.5,0.8,2,2,2,0.5,0.5,30)$, the optimal sampling plan $(3,0.6750,\delta_B)$ shows that we take 3 items from the batch and the censoring time is 0.6750. The corrsponding minimum Bayes risk is $r(n_B,t_B,\delta_B) = 25.28678$.

In Table 1, when $(\alpha,\beta) = (0.2,2.0)$ and $(\alpha,\beta) = (1.5,2.0)$, the associated n_B and t_B of the proposed Bayes rule are both 0. This means, in each situation, the Bayes sampling plan δ_B suggests to accept the batch without taking any sample. In Table 2–3s, the value of the minimum Bayes risk increases initially with respect to a_0, a_1, a_2, C_1, C_2 and C_3.

4. A case when $C_2 = 0$

We continue the statistical model setup in Section 3 except assuming that there is no cost for the time used for life testing. That is, we let $C_2 = 0$. This is the situation on which Lam (1994) developed the sampling plan $(n_0,t_0,\delta^L_{T_0})$. Lam (1994) claimed that $(n_0,t_0,\delta^L_{T_0})$ was a Bayesian sampling plan for the statistical model given in Section 3 with $C_2 = 0$. Since there is no cost for the time used for life testing, we may let the censoring time to be infinite so that we can observe the complete lifetime data of the components put on life testing. Based on the complete lifetime data, we may make a more suitable decision to reduce the overall Bayes risks of the sampling plans. We denote a sampling plan with $t = \infty$ by (n, ∞, δ). We shall derive a Bayesian sampling plan in the following.

For each fixed n, we first derive its corresponding Bayes decision function. For $t = \infty$, $M \equiv n$. Hence, $y(n,t,n) = \sum_{j=1}^{n} y_j$,

$$g(\lambda \mid n, y(n,t,n)) \sim \Gamma\left(n+\alpha, \sum_{j=1}^{n} y_j + \beta\right),$$

$$\varphi_g(n, y(n,t,n)) = a_0 + \frac{a_1(n+\alpha)}{\sum_{j=1}^{n} y_j + \beta} + \frac{a_2(n+\alpha)(n+\alpha+1)}{(\sum_{j=1}^{n} y_j + \beta)^2}.$$

Hence the Bayes decision function is:

$$\delta_B\left(n, \sum_{j=1}^{n} y_j\right) = \begin{cases} 1, & \text{if } \sum_{j=1}^{n} y_j \geq D_n(n) - \beta, \\ 0, & \text{otherwise.} \end{cases} \tag{4.1}$$

The Bayes risk of the sampling plan $(n, \infty, \delta_B(\ \mid n, \infty))$ is:

$$\begin{aligned} &r(n, \infty, \delta_B(\ \mid n, \infty)) \\ &= C_3 + nC_1 + \int_0^\infty [h(\lambda) - C_3] P\left\{\delta_B\left(n, \sum_{j=1}^{n} Y_j\right) = 1 \mid \lambda\right\} g(\lambda) d\lambda \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} P&\left\{\delta_B\left(n, \sum_{j=1}^{n} Y_j\right) = 1 \mid \lambda\right\} \\ &= P\left\{\sum_{j=1}^{n} Y_j \geq D_n(n) - \beta \mid \lambda\right\} \\ &= I(D_n(n) - \beta > 0) \sum_{\ell=0}^{n-1} e^{-\lambda[D_n(n)-\beta]} \frac{[\lambda(D_n(n)-\beta)]^\ell}{\ell!} + I(D_n(n) - \beta \leq 0), \end{aligned} \tag{4.3}$$

by noting that $\sum_{j=1}^{n} Y_j \mid \lambda \sim \Gamma(n, \lambda)$.

Replacing (4.3) into (4.2), and carrying an integration, we obtain:

$$\begin{aligned} r(n, \infty, \delta_B(\ \mid n, \infty)) &= C_3 + nC_1 + [a_0 + a_1\mu_1 + a_2\mu_2 - C_3] I(D_n(n) - \beta \leq 0) \\ &+ I(D_n(n) - \beta > 0) \frac{\beta^\alpha}{\Gamma(\alpha)} \sum_{\ell=0}^{n-1} \frac{[D_n(n) - \beta]^\ell \Gamma(\alpha + \ell)}{\ell! [D_n(n)]^{\ell+\alpha}} \\ &\times \left\{a_0 - C_3 + \frac{a_1(\alpha+\ell)}{D_n(n)} + \frac{a_2(\alpha+\ell)(\alpha+\ell+1)}{D_n^2(n)}\right\}. \end{aligned}$$

Then, following Algorithm B with $t=\infty$, we obtain an integer, say n_B^*, $0 \le n_B^* \le n^*$ such that $r(n_B^*, \infty, \delta_B(\ \mid n_B^*, \infty)) = \inf_{n\in I_{n^*}} r(n, \infty, \delta_B(\ \mid n, \infty))$.

Consider the sampling plan $(n_0, \infty, \delta_{T^*}^L)$, which is described in Section 1, where n_0 is the "optimal" sample size determined by Lam (1994) and reported in Tables 1–6 of Lam (1994), and T^* is a positive value satisfying $r(n_0, \infty, \delta_{T^*}^L) = \inf_{T>0} r(n_0, \infty, \delta_T^L)$. Note that when $n_B^* = n_0$, $\delta_B(\ \mid n_B^*, \infty)$ is essentially the same as $\delta_{T^*}^L$. That is, $T^* = (D_{n_0}(n_0) - \beta)/n_0$. However, in general, n_0 may not be the Bayes sample size which minimizes $r(n, \infty, \delta_B(\ \mid n, \infty))$ among all n in I_{n^*}. Thus, we have: $r(n_B^*, \infty, \delta_B(\ \mid n_B^*, \infty)) \le r(n_0, \infty, \delta_{T^*}^L)$.

Some numerical computations show that $r(n_B^*, \infty, \delta_B) \le r(n_0, \infty, \delta_{T^*}^L) \le r(n_0, t_0, \delta_{T_0}^L)$ with strict inequalities in many cases. Therefore, in this sense, this confirms again that the Lam's "optimal" sampling plan $(n_0, t_0, \delta_{T_0}^L)$ is not a Bayes solution and $(n_B^*, \infty, \delta_B)$ is the best among the three sampling plans.

5. Conclusion

In this paper, we reconsider the Lam's (1994) model under a general Bayes set up. We take the censoring time t into consideration as one of main factors and introduce cost of unit time in loss function. A Bayes sampling plan has been proposed under general setting and an explicit Bayesian sampling plan has been derived while a quadratic loss is used. Usual discretization method and some algorithms (Scheme A and Algorithm B) to find the optimal Bayes sampling plan are also addressed. Some optimal Bayes plans and its Bayes risks are tabulated (Tables 1–3). When $t=\infty$, i.e. under complete date, an alternative explicit Bayes risk is also derived. Two competitors of sampling plans $(n_0, \infty, \delta_{T^*}^L)$ and $(n_B^*, \infty, \delta_B)$ against the Lam's sampling plan are also proposed and it is found that $(n_B^*, \infty, \delta_B)$ dominates $(n_0, \infty, \delta_{T^*}^L)$ and the latter dominates the Lam's $(n_0, t_0, \delta_{T_0}^L)$ nontrivially.

It is to be noted that the proposed competitiors $(n_0, \infty, \delta_{T^*}^L)$ and (n_B, ∞, δ_B) are just for the theoretical study and comparisons against the Lam's (n_0, t_0, δ_T^L), since for practical applications the two former sampling plans need complete data and this is not the situation of our primary interest.

It should be pointed that the Bayes risk is not a smooth function of those variables. Therefore numerical computations for finding optimal n_B and t_B should take a special care. To strengthen its accuracy of the numerical approximation, we take $N = 400$ for the division of $[0, t^*]$, which is much bigger than the Lam's case.

Finally, to extend the life model under consideration, it is natural to consider Weibull, IFR or more general life distribution. For the cases of Weibull, the Bayes rule proposed in (3.2) can be analogously applied though some computations may become laborious. These may become our next goal.

Acknowledgements

We are grateful to the Editor-in-chief and two referees for some helpful comments which lead to improvement of present presentation.

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