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AN APPLICATION OF BAYESIAN (ABIC) SMOOTHING METHODS TO ESTIMATING SPACE AND TIME VARIATIONS IN THE MAGNITUDE DISTRIBUTIONS OF EARTHQUAKES*

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Abstract. On the basis of Akaike's Bayesian information criterion (ABIC), a new method of estimating temporal and spacial variations in the magnitude-frequency relation (b value) is developed. The space-time volume studied is divided into a number of segments having equal volume, for which b values are estimated. The smoothness of the estimated b value is guaranteed by the introduction of a prior distribution. The parameters of the prior distribution are chosen in such a way as to minimize the ABIC. This method is employed to study the temporal and spatial variations of b values for microearthquake activity in the Kanto, Tokai and Tottori areas, where seven earthquakes having magnitudes of M6.0 and larger have recently taken place. States of temporal variations of b values are classified into three groups; increase, decrease and no-change. The space-time volume of decrease accounts for approximately 10% of the total volume. For a period before the occurrence of earthquakes, the decrease appears more frequently than might be expected. It is concluded that the decrease in the b value is a promising candidate to act as a precursory phenomena.

Key words and phrases: ABIC, b value, earthquake prediction, magnitude-frequency, microearthquakes, space time.

1. Introduction

It is well known that the number of earthquakes of smaller magnitudes is greater than the number of larger ones. The number (N) of occurrences for earthquakes of magnitude M and larger in a region during a certain period is related to M by the equation, $\log_{10} N = a - bM$, where a and b are constants. This equation, which has been derived empirically, is usually called the Gutenberg-Richter formula. Precursory changes in b values before major shocks have often been reported and are supported by evidence from laboratory studies of rock deformation

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(Scholz (1968)). It may be relevant for earthquake prediction to carry out a study of b values in space and time. Such studies have been made by several authors (e.g. Li *et al.* (1978), Ma (1978, 1982), Smith (1986)), using running mean techniques. The length of the running mean in each of these studies, however, appears to be selected rather arbitrarily.

Ishiguro and Sakamoto (1983) have proposed a practical procedure to estimate the binary response curve with a single parameter using Akaike's Bayesian information criterion (ABIC; Akaike (1980)), which is a criterion for selecting the best prior distribution. Using this method, Imoto and Ishiguro (1986) treated the magnitude-frequency relation as a binary response problem, where earthquakes are classified and the number of shocks in one class is compared with that in another. They reported a significant time variation of the ratio between two such numbers associated with recent major shocks in central Japan. Imoto (1987*a*) has developed a procedure to estimate variations of the multinomial response with two parameters. This procedure can be applied to obtain a space-time variation of the magnitude-frequency relation with an exponential distribution (Gutenberg-Richter relation).

In the present paper, space-time variations of b values in the Kanto, Tokai and Tottori areas are studied by applying this method to the compiled data of microearthquakes for the respective areas, where the Gutenberg-Richter relation is assumed for magnitude distributions.

2. Method

A certain domain in time and space is assumed, where earthquakes having a magnitude between m_1 and m_2 are detected. To estimate the variation of the magnitude distribution in this domain, the time and space continuum is divided into λ and μ segments, respectively, where each time segment is of duration Δt and each space segment is of volume Δs. It is not unreasonable to assume that the b value is the same throughout each segment as long as both Δt and Δs are sufficiently small. The range of magnitude is also divided into ν segments, each containing an increment of Δm. This results in the time, space, and range of magnitude being divided into $\lambda \times \mu \times \nu$ segments. All shocks in these segments are classified and the number of shocks in each segment is obtained as n_{ijk} ($i = 1, 2, \ldots, \lambda$; $j = 1, 2, \ldots, \mu$; $k = 1, 2, \ldots, \nu$). The distribution of magnitude in the (i, j)-th segment is defined by the probability p_{ijk} ($k = 1, 2, \ldots, \nu$) that a shock occurring in the (i, j)-th segment is classified as a shock in the k-th segment of the range of magnitude. The likelihood of the parameter p_{ijk} is expressed by

$$L(\boldsymbol{p}) = \prod_{i=1}^{\lambda} \prod_{j=1}^{\mu} \prod_{k=1}^{\nu} p_{ijk}^{n_{ijk}}, \tag{2.1}$$

where

$$\sum_{k=1}^{\nu} p_{ijk} = 1 \qquad (i = 1, 2, \ldots, \lambda;\ j = 1, 2, \ldots, \mu). \tag{2.2}$$

Hereafter, the Gutenberg-Richter relation is assumed for magnitude distributions. Under this assumption, it is deduced that p_{ijk} obeys the following relations:

$$\frac{p_{ij1}}{p_{ij2}} = \frac{p_{ij2}}{p_{ij3}} = \cdots = \frac{p_{ij(\nu-1)}}{p_{ij\nu}} = \alpha_{ij}. \tag{2.3}$$

Substituting this equation into (2.2),

$$p_{ij\nu}(1 + \alpha_{ij} + \alpha_{ij}^2 + \cdots + \alpha_{ij}^{\nu-1}) = 1 \tag{2.4}$$

is obtained. Thus, one independent variable, α_{ij}, is obtained which is related to the b value as

$$\ln \alpha_{ij} = b_{ij} \Delta m \ln 10, \tag{2.5}$$

where ln refers to the natural logarithm. For the sake of calculation, the vector $\boldsymbol{q}$ is introduced with its m-th component being $q_{ij}(m = (i-1) \times \mu + j)$, which is defined by

$$q_{ij} = \ln \alpha_{ij}. \tag{2.6}$$

Substituting (2.3), (2.4) and (2.6) into (2.1), the likelihood function for q_{ij} is given by

$$L(\boldsymbol{q}) = \prod_{i=1}^{\lambda} \prod_{j=1}^{\mu} \prod_{k=1}^{\nu} \left[\exp\{q_{ij}(\nu - k)\} \Big/ \sum_{l=1}^{\nu} \exp\{q_{ij}(\nu - l)\} \right]^{n_{ijk}}. \tag{2.7}$$

The maximum likelihood estimate, $\tilde{q}_{ij}$, is given by the mode of the likelihood function. When the number of earthquakes for a certain space-time segment is small, the respective $\tilde{q}_{ij}$ becomes unstable and unreliable. In order to keep the estimate stable and reliable, a large number of samples is required for each segment. However, it is not always possible to satisfy this requirement. The Bayesian modeling method which was recently developed by Ishiguro and Sakamoto (1983), Imoto and Ishiguro (1986) and Imoto (1987a) is applied to overcome this difficulty.

Hereafter, it is assumed that the space-time variation of q_{ij} is so smooth that the difference in q_{ij} between adjacent segments in the space-time domain is small. This assumption is formulated by introducing the prior distribution of q_{ij} with the hyper-parameters u, v and q_0 as:

$$\pi(\boldsymbol{q} \mid \boldsymbol{r}, u, v) = C\pi_1(\boldsymbol{q} \mid \boldsymbol{r}, u)\pi_2(\boldsymbol{q} \mid v), \tag{2.8}$$

$$\pi_1(\boldsymbol{q} \mid \boldsymbol{r}, u) = \exp\left[\frac{1}{2u^2} \left\{ (q_0 - q_{11})^2 + \sum_{i=2}^{\lambda} \sum_{j=1}^{\mu} w_{ij}(q_{(i-1)j} - q_{ij})^2 \right\} \right] \tag{2.9}$$

and

$$\pi_2(\boldsymbol{q} \mid v) = \exp\left[\frac{1}{2v^2} \left\{ \sum_{i=1}^{\lambda} \sum_{j=1}^{\mu} \sum_{\eta} (q_{ij} - q_{i\eta})^2 \right\} \right] \tag{2.10}$$

(η: segments adjacent to j in space, $j < \eta \leq \mu$).

(2.9) and (2.10) can be regarded either as penalty terms or as terms arising from Gaussian prior densities for q_{ij} of the form

$$\pi_1(\boldsymbol{q} \mid \boldsymbol{r}, u) = \exp\left\{-\frac{1}{2u^2}(D_1\boldsymbol{q} + \boldsymbol{r})^T(D_1\boldsymbol{q} + \boldsymbol{r})\right\}, \tag{2.11}$$

$$\pi_2(\boldsymbol{q} \mid v) = \exp\left\{-\frac{1}{2v^2}(D_2\boldsymbol{q})^T(D_2\boldsymbol{q})\right\} \tag{2.12}$$

where D_1 and D_2 are square roots of the positive definite matrices corresponding to the quadratic forms in (2.9) and (2.10), and

$$\boldsymbol{r} = (-q_0, 0, 0, \ldots, 0)^T. \tag{2.13}$$

Here q_0, u and v play the role of hyperparameters. The value of w_{ij} is usually set to be 1.0 except for cases where breaks in the time variation are introduced (details are given in the last of this section). The values of u, v and q_0 are chosen in such a way so that

$$\mathcal{L}(u, v, q_0) = -2\ln\int L(\boldsymbol{q})\pi(\boldsymbol{q} \mid \boldsymbol{r}, u, v)d\boldsymbol{q} \tag{2.14}$$

is minimized. The criterion ABIC is defined by min $\mathcal{L}(u, v, q_0) + 2 \times 3$, where 3 stands for the number of the hyperparameters. The best estimate of $\boldsymbol{q}$ is then obtained as the mode of the posterior distribution of $\boldsymbol{q}$, which is proportional to $L(\boldsymbol{q})\pi(\boldsymbol{q} \mid \boldsymbol{r}, u, v)$. The integral can be well approximated by

$$\int \exp\{T(\boldsymbol{q})\}d\boldsymbol{q}, \tag{2.15}$$

where $T(\boldsymbol{q})$ denotes the Taylor expansion of the logarithm of the integrand in (2.14), up to the second order term. If the $\log\{L(\boldsymbol{q})\pi(\boldsymbol{q})\}$ attains its maximum at the point $\hat{\boldsymbol{q}}$, $T(\boldsymbol{q})$ can be expressed by the quadratic form

$$T(\boldsymbol{q}) \cong T(\hat{\boldsymbol{q}}) - 1/2(\boldsymbol{q} - \hat{\boldsymbol{q}})^T G(\boldsymbol{q} - \hat{\boldsymbol{q}}). \tag{2.16}$$

Here G can be expressed by

$$G = H + \frac{1}{u^2}D_1^T D_1 + \frac{1}{v^2}D_2^T D_2, \tag{2.17}$$

where H is a diagonal matrix, with each component given by

$$H_{mm} = n_{ij}\sum_{k=1}^{\nu-1}\sum_{l=1}^{\nu-1}\{(\nu - k)(\nu - l) \times (\delta_{kl}p_{ijk} - p_{ijk}p_{ijl})\}. \tag{2.18}$$

The constant C is counted in $T(\hat{\boldsymbol{q}})$. Since the positive definitness of G is easily proved by taking (2.9), (2.10), (2.17) and (2.18) into consideration, the numerical minimization of ABIC by using the Newton approximation method can be successfully carried out (Ishiguro and Sakamoto (1983), Imoto and Ishiguro (1986), Imoto (1987*b*)). A good approximation to the value of ABIC is given by

$$\text{ABIC} = -2T(\hat{\boldsymbol{q}}) + \log\{\det(G)\} - \lambda\mu\log 2\pi, \tag{2.19}$$

where the correction term for the number of hyperparameters is ignored. The optimization of (2.19) with respect to u and v can be accomplished by the grid search technique.

It is sometimes useful to estimate the error of q_{ij} for each segment. The error distribution of the parameters at the point $\hat{\boldsymbol{q}}$ is approximated by the multivariate normal $N(\hat{\boldsymbol{q}}, G^{-1})$, where G^{-1} is the inverse of the matrix in (2.17) (Ogata and Katsura (1988)).

The weights w_{ij} in (2.9) were assigned according to the following considerations. It is well known that usually many aftershocks occur after large earthquakes. In fact, earthquakes for the present study were followed by a large number of aftershocks. In such cases, b values just before the main shocks may be strongly affected by those of the aftershocks since the prior distribution was introduced in order to keep the smooth variation of the b value. The main purpose of the present study is to examine the behavior of the b value before large earthquakes. For this reason, an effort has been made to reduce the effects of aftershock activity on estimates of b values before large earthquakes. The smoothness of b values in the time-domain is controlled by (2.9). If the w_{ij} of a special segment is a smaller value (0.1 in the present study), the smoothness of the b value between the respective two segments, $(i-1, j)$ and (i, j), can become weakened (Akaike and Ishiguro (1983)). Many selections for such segments can be assumed, among which the best prior distribution is selected according to the ABIC. In an actual case, four different ways to select segments, or four models of D_1 are compared: (1) $w_{ij} = 1$, for all; (2) $w_{ij} = 1$ $(i \neq i', j \neq j')$, $w_{i'j'} = 0.1$ (the segment (i', j') includes the time and the hypocenter of the earthquake listed in Table 3); (3) the same as in Model (2) but for the segment (i', j') which includes the hypocenter of the aftershock area; (4) the same as in Model (2) but for the segment (i', j') which is the segment of the hypocenter or one of the eight segments surrounding it.

3. Data

Spatial and temporal variations of b values of microearthquake activity are studied for data from the Kanto, Tokai and Tottori regions in central and western Japan (see Fig. 1). The values of the parameters, as shown in Tables 1 and 2, are suitably chosen, taking into account the detection capabilities of the respective networks (Matsumura (1984), Shibutani (1989)), the number of observed shocks, and other factors such as interest in the relationship between large earthquakes and changes in the b value.

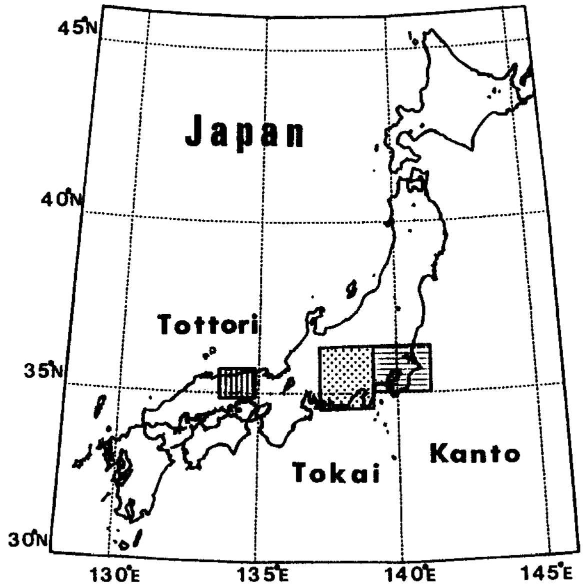

Fig. 1. Map of Japan showing the three areas for study, Kanto, Tokai and Tottori.

Table 1. Catalogue data; Space-time, and magnitude ranges of shocks studied are listed for each area.

Name of area	Period	Spatial extent EW (km)	NS (km)	Depth (km)	Size	Number of shocks
Kanto	Jan. 1981–Dec. 1987	175 ×	150	$H \le 100.$	$2. \le M < 4.$	8986
Tokai	Jan. 1981–Dec. 1987	175 ×	200	$H \le 50.$	$2. \le M < 4.$	3180
Tottori	July 1976–June 1987	125 ×	100	$H \le 30.$	$1.5 \le M < 4.$	1776

Table 2. Terms for segmentation; The size and numbers of segments in time and space are listed. The total number of segments and the number of shocks per segment are shown in the last two columns.

Name of area	Time Δt(mon)	λ	Space Δs(km^3)	μ	Number of segments	N/seg
Kanto	2	42	25 × 25 × 100	42	1764	5.1
Tokai	2	42	25 × 25 × 50	56	2352	1.4
Tottori	2	66	25 × 25 × 30	20	1320	1.3

3.1 *The Kanto region*

Figure 2 shows the segmentation of the grid in space, together with the epicenters of shocks which occurred during the period of study and which had magnitudes of 3.0 and larger, based on the Kanto-Tokai Network catalogue of the National Research Institute for Earth Science and Disaster Prevention. As far as earthquakes shallower than 100 km are concerned, the depth of high seismicity varies from place to place, mainly due to the subducting slabs of the Pacific and Phillippine Sea plates. The depth range of high seismicity at a certain point, however, is about 30 km wide with its depth depending on the location. With respect to spatial dimension, two-dimensional analysis of b values was carried out under restrictions of computer power, with results being interpreted as showing b values at the representative depth of the respective segment.

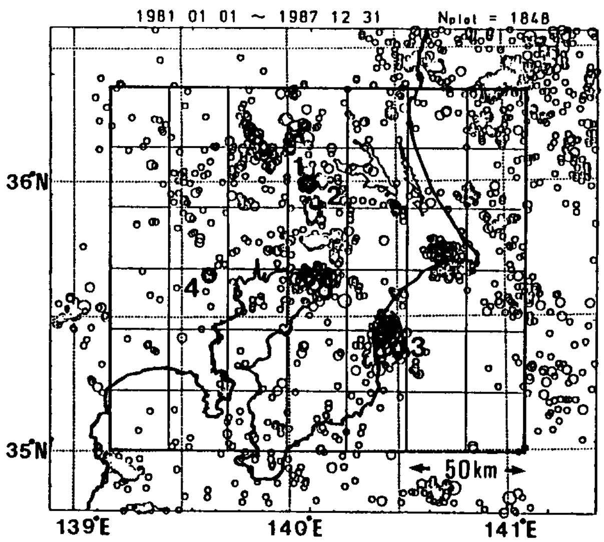

Fig. 2. Epicenters of microearthquakes ($M \geq 3.0$) in the Kanto area and partitioning of the area. The epicenters of the Ibaraki (1), southern Ibaraki (2), Chiba (3), and Tokyo (4) earthquakes and their aftershock areas are indicated by asterisks and dashed lines, respectively.

Three earthquakes having magnitudes of 6.0 and above, the southwestern Ibaraki (February 1983, M6.0), the southern Ibaraki (October 1985, M6.1) and the Chiba (December 1987, M6.7) earthquakes, occurred within the studied space-time domain. Another earthquake, the Tokyo earthquake (March 1988, M6.0) occurred within the study space, however, about 3 months after the end of the study period. The epicenters of these earthquakes are plotted in Fig. 2 and listed in Table 3. The minimum ABIC for each of the four models is listed in Table 4. As was discussed by Sakamoto *et al.* (1983), if the difference of ABIC between two models is greater than a value of order 1, their difference is significant. Since only the difference of ABIC between Model (4) and the others is significant, we may select any one among the three models, Model (1)–(3). However, the ABIC of Model (3) is the smallest and the difference between Model (3) and Model (2) (the second smallest ABIC) is 0.5. Therefore, it was concluded that Model (3) had the best prior distribution and hereafter the results of Model (3) are used.

Table 3. Hypocentral parameters of large earthquakes with magnitudes 6.0 and larger.

No.	Origin time (JST)			M	Location			
	Y	M	D		Lat.	Long.	Dep.	
Kanto								
1	1983	Feb.	27	6.0	35.993	140.105	67.0	Ibaraki
2	1985	Oct.	4	6.1	35.910	140.112	71.6	Southern Ibaraki
3	1987	Dec.	17	6.7	35.372	140.519	47.3	Chiba
4	1988	Mar.	18	6.0	35.668	139.626	90.5	Tokyo
Tokai								
5	1983	Aug.	8	6.0	35.536	139.046	18.1	Yamanashi
6	1984	Sep.	14	6.8	35.807	137.554	1.1	Nagano
Tottori								
7	1983	Oct.	31	6.2	35.413	133.927	15.0	Tottori

Table 4. Four different prior distributions and their ABIC's for the Kanto region.

Model	ABIC
1	28602.3
2	28602.1
3	28601.6
4	28614.2

The Chiba earthquake

Figure 3 shows the time variations of the b values for the segment of the Chiba earthquake and its adjoining segments in four directions. In the region of the earthquake (indicated by the middle set), the b value remains at the nearly constant value of 0.8 during the period from the beginning of 1981 to the middle of 1982. The following decrease for about one year may be related to a swarm activity which occurred in May 1983, including a large earthquake with a magnitude of 5.1. After 1983, b values increase until the end of 1984, when they begin to decrease. This decrease continues for about three years until the occurrence of the Chiba earthquake. Only one b value for the aftershock activity, which also includes activity before the earthquake, was obtained since the occurrence of this earthquake was almost at the end of the study period and the time interval was chosen as 2 months. Since the *b* values for the aftershocks' activity are stepped, with a larger value by about 0.05 than that of the previous period, it is concluded that the decrease in *b* before the earthquake is not related to the aftershock activity. For the surrounding four areas, the variation in the temporal pattern differs for each segment. This means that the decrease is rather restricted to a small volume and can not be caused by a man-made change in seismicity. It should be noted that the steps in the *b* value at the occurrence of M6.7 in the region of the earthquake and in the north segment are due to the prior distribution in Model (3) where areas of aftershock activity are taken into consideration.

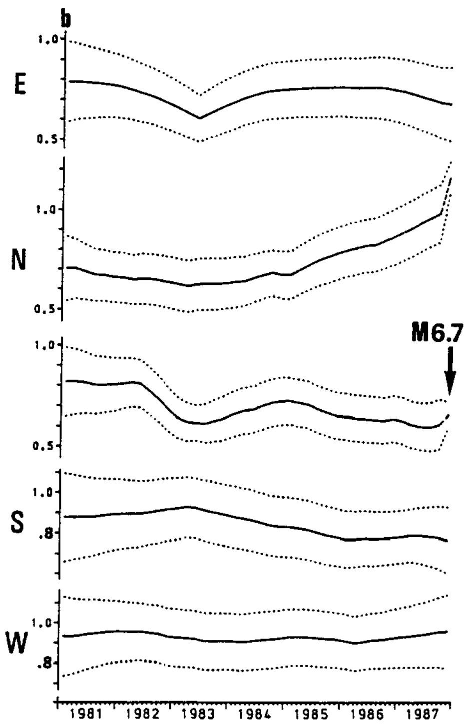

Fig. 3. Time variations of the *b* value for segments of the Chiba earthquake and its neighboring parts. E: east; N: north; S: south; W: west. Upper and lower error bands are indicated by dotted lines. Dashed lines just before the earthquake correspond to weak bounds introduced into the prior distribution.

The Ibaraki and southern Ibaraki earthquakes

In the south-western Ibaraki Prefecture, two earthquakes (Number 1 and 2 in Table 3) occurred some ten kilometers apart and within a two and a half year interval. The former during February 1983, and the latter in October 1985, are interpreted as an inter plate earthquake between the Philippine Sea and the Pacific plate and an intra-plate earthquake of the Pacific plate, respectively (Ohtake

(1986), Hori (1986)). Although these earthquakes are sorted in the same region with the present segmentation, the aftershock area of the latter spreads out to the south. As will be shown later, such situations are considered in constructing the prior distribution of Model (3). In the same manner as Fig. 3, the time variations of the b values for the earthquake segment and its adjoining segments are shown in Fig. 4. Two steps in the b value of more than 0.1 can be clearly seen in the middle set, which again demonstrates that interactions of b values just prior to and immediately after earthquakes are completely eliminated. Patterns showing a decrease of the b value for about one year before the occurrence of earthquakes are commonly found except for a few months just prior to the events. A similar time variation is obtained for the segment south of the source area. On the contrary, the variation for the east segment exhibits a rather constant pattern of b. Similarities in the variations may be related to the north-south trend of high seismicity, the main part of which is caused by the collision between the Philippine Sea and Pacific plates.

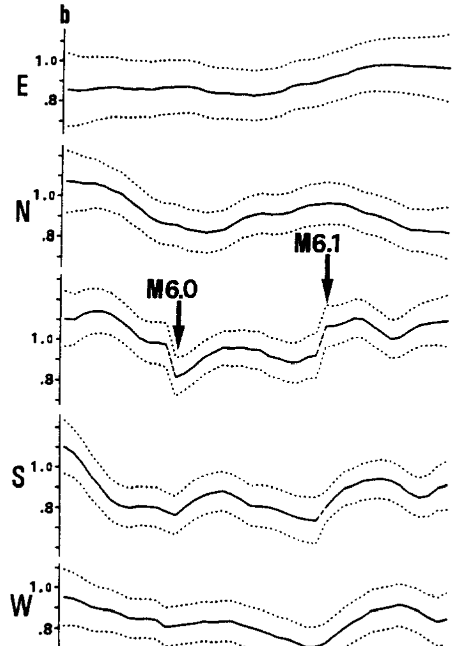

Fig. 4. Same as Fig. 3 but for the Ibaraki and the southern Ibaraki earthquakes.

The Tokyo earthquake

The fourth earthquake, the Tokyo earthquake, occurred on March 18, 1988, 5/2 months after the end of the period under investigation. Accordingly, examination of this earthquake may give proper examples of the time variation of the b

value based on the activity before the earthquake. As is shown by the middle set of Fig. 5, the b value is fairly constant at a value of approximately 1.1, and exhibits a decrease for the last two years. For the north segment, the decrease in the last 3/2 year is very clear in contrast with the constant patterns of the previous $4 \sim 5$ year period. These decreases may be precursors of the Tokyo earthquake. Since this earthquake occurred on the border of the north segment, the precursors could appear in both segments. Comparing the variations in the north segment with those of the south and west segments, it is concluded that the decrease is rather restricted to a small volume and can not be caused by artificial means.

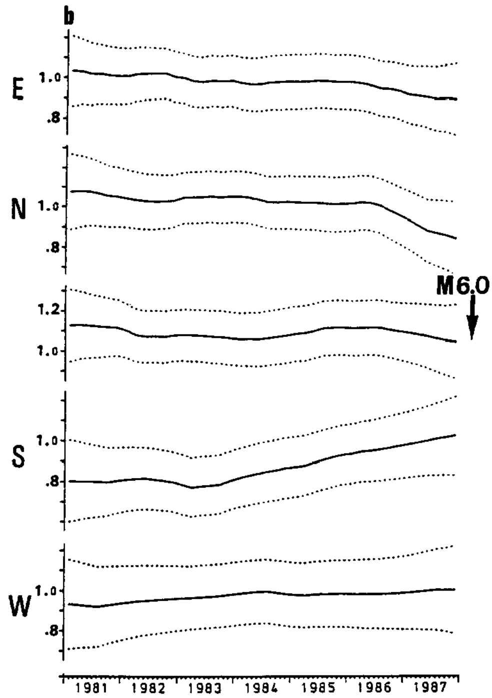

Fig. 5. Same as Fig. 3 but for the Tokyo earthquake.

3.2 *The Tokai region*

Figure 6 shows epicenters of microearthquakes ($2.0 < M$) in the Tokai region, and the segmentation of the grid used for this study, similar to Fig. 2. In this area, the Philippine Sea plate is subducting beneath the Eurasian plate in a northwest direction. The microearthquakes are, for the most part, located at a depth of less than 50 km, only rarely occurring deeper than this. Two earthquakes having magnitudes of 6.0 and larger occurred at shallower depths, one in the western part of Nagano and the other in the eastern part of Yamanashi. The epicenters of these earthquakes are indicated in Fig. 6 along with their aftershock areas. In a similar manner to that used for the Kanto area, space-time variations of b values are analyzed.

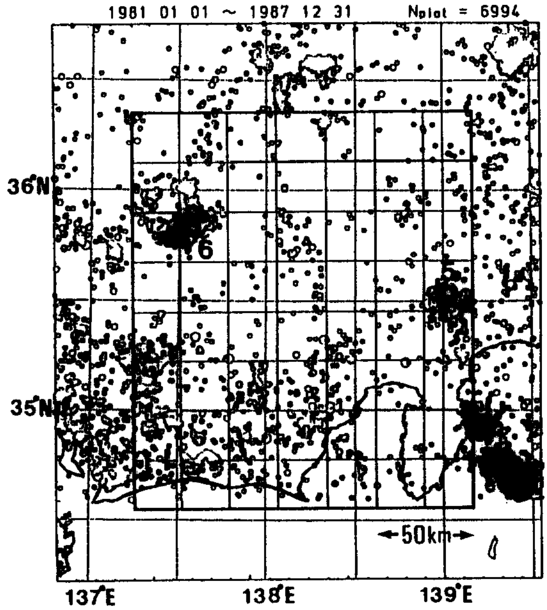

Fig. 6. Epicenters of microearthquakes in the Tokai area and partitioning of the area. The epicenters of the Yamanashi (5) and Nagano (6) earthquakes and their aftershock areas are indicated by asterisks and dashed lines, respectively.

The Nagano earthquake

Figure 7 shows the time variations of the b value for the segment of the Nagano earthquake and its adjoining segments in four directions. In the segment of the earthquake (indicated by the middle set), the b value starts at a value greater than 1.05, decreasing to a value of less than 1.0 during the early months of 1982 and remaining at this value with a slight fluctuation until the occurrence of the Nagano earthquake. It then exhibits steps, down to values of 0.9 immediately after the occurrence, recovers gradually to reach the level of the 1981 value in the early part of 1986. Contrary to this, variations of the b values for the east and north segments show no remarkable changes, remaining very constant. The segment just west of the source exhibits a similar temporal variation to that of the source. This confirms that the change in the b value before this earthquake is due to a natural phenomenon.

The Yamanashi earthquake

Time variations for segments of the Yamanashi earthquake show stable features of the b value except for the slight increase at the end of 1981 in the source region (Fig. 8). This is the only case among the seven earthquakes studied in that no clear decrease in the b value appears in the region of the earthquake during the period of a few years before the occurrence. This result is contrary to that of previous work by Imoto and Ishiguro (1986). This inconsistency probably originated

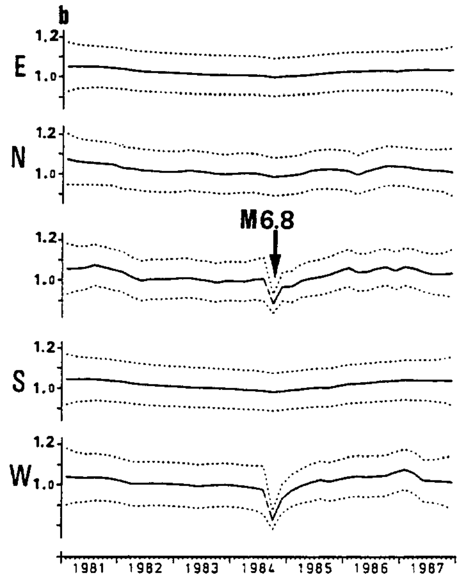

Fig. 7. Same as Fig. 3 but for the Nagano earthquake.

from differences between the data sets used for these studied, since the lower limit of the range of magnitude used for the present study is 0.5 larger than that of the previous work.

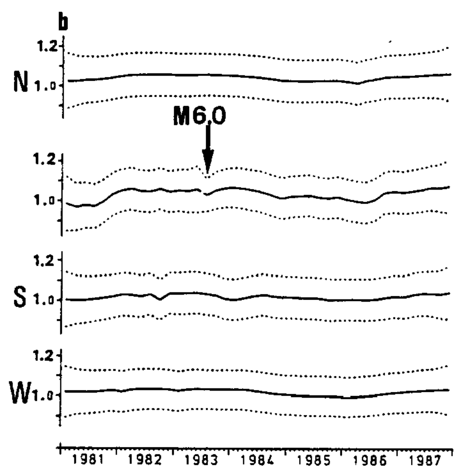

Fig. 8. Same as Fig. 3 but for the Yamanashi earthquake.

3.3 *The Tottori region*

Figure 9 shows epicenters of microearthquakes ($1.5 < M$) in the Tottori region and the segmentation of the grid used in the present study, similar to Fig. 2. In this area, microearthquakes, for the most part, are located at depths of less than 25 km. The Tottori earthquake of October 1983, having a magnitude of 6.2 is the greatest event in the region since the Tottori earthquake of 1943 which was measured as M7.4. Catalog data of microearthquakes for a period of more than 7 years before the earthquake of 1983 is available from the network of the Tottori Observatory of Kyoto University. Therefore, it is possible to detect precursors for longer time periods than those in the previous cases. Space-time variations of b values are analyzed using the same method mentioned previously.

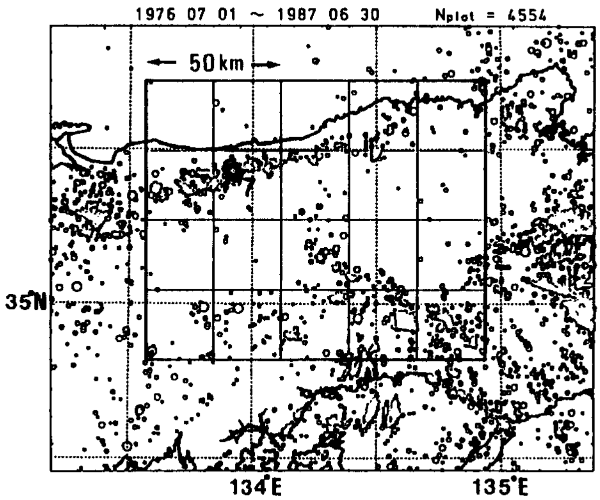

Fig. 9. Epicenters of microearthquakes in the Tottori area and partitioning of the area. The epicenter of the Tottori (7) earthquake and its aftershock area are indicated by the asterisk and the dashed line, respectively.

The Tottori earthquake

Figure 10 shows the time variations of the b values for the segment of the Tottori earthquake and its surrounding segments. For the segment of the earthquake (the middle set), an increase in the b value appears during the period 1977 through 1981, after which the b value decreases until the occurrence of the earthquake of October 1983. After the earthquake, the b value shows no rapid recovery, attaining the value that occurred just prior to the earthquake in the middle of 1987. Those of the surrounding segments show a pattern more or less similar to those at the source region with a few exceptions. First, the occurrence of the peak b values for the segments east, north and south of the source coincide with that of the source region while that for the west segment occurs 3/2 years later. Second, variations in the former three segments have lower amplitudes and occur in a narrower range than those at the source. It is inferred that the variations of these three areas are strongly affected by those of the source area according to the prior distribution. On the other hand, the variations for the west segment do not appear to be as strongly affected as those of the previous three areas. Therefore, it is not unreasonable to conclude that the decrease in the b value before the Tottori earthquake is due to a natural phenomenon rather than to a man-made change.

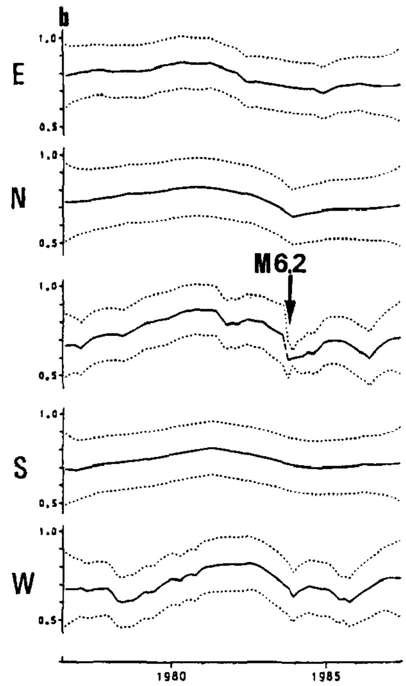

Fig. 10. Same as Fig. 3 but for the Tottori earthquake.

4. Decrease in the *b* value as a precursor

Examining b variations of the seven earthquakes, it is commonly found with the exception of the Yamanashi earthquake, that a decrease in the b value appears a few years prior to the earthquakes. The decrease is, however, not identified quantitatively, since the patterns differ from earthquake to earthquake. A quantitative definition is required to determine whether the decrease in the b value is a precursor or merely coincident with earthquakes, since a statistical approach based on an exact definition may be beneficial.

Using a sequence of b values for a certain region, a segment is defined as a 'Decrease' if it satisfies the following conditions:

CONDITION. The first segment of 'Decrease' occurs where the b value is smaller, by a threshold value, than the previous maximum b value.

In the case where some segments defined as 'Decrease' or 'Increase' appear before the given segment,

(i) Following a 'Decrease', the b value of the given segment is smaller than that of the last segment of 'Decrease', or

(ii) Following an 'Increase', the b value of the given segment is smaller, by a threshold value, than that of the last segment of 'Increase'.

'Increase' can be defined in the same manner as 'Decrease' but for the inverse. Segments defined as neither 'Decrease' (D) nor 'Increase' (I) are classified as 'No-change'. A tentative threshold value is set at half the standard error of the b values being compared.

The number of segments of the three categories are listed in Table 5 for each network. Roughly speaking, the number of 'D' occurring are only about 10 percent of the totals, slightly less than those of 'I'. Table 6 shows the sequences of 'D' or 'I' over two years prior to the seven earthquakes for the respective regions. For example, for the four earthquakes in the Kanto region, 12 segments of 'D' are found out of the 24 segments (6 segments prior to each of the four earthquakes), whereas 248 segments of 'D' are observed among the total 1764 segments. If 'D' appears independently and identically with a probability of $p = 248/1764$, the probability that 'D' appears more than twelve times in 24 random segments of the 1764 total segments would be $\sum_{i=12}^{24} {}_{24}C_i p^i (1-p)^{24-i} \fallingdotseq 3 \times 10^{-5}$. In a similar way, if we focus our attention on the point that there appears no 'I' in the 24 segments, such a probability is given by $(1454/1764)^{24} \fallingdotseq 0.005$. These probabilities imply a certain relationship between appearances of 'D' or 'I' and occurrences of earthquakes. The present case, however, does not appear to satisfy the condition of independence since the prior distribution was introduced for the calculations of the b value. Taking such conditions into consideration, it is rather difficult to estimate the probability that large earthquakes occur coincidently with appearances of 'D'. Therefore, it can not be conclusive, but only inferred that the appearance of 'D' before a large earthquake can not be explained by chance alone but is ascribed to a certain physical relationship between them.

Table 5. Numbers of segments classified into three groups: Increase, Decrease, and No-change, depending on b time variations. The threshold level used for classification is set to be half that of the standard error at the reference point.

Area	Increase	Decrease	No-change	Total
Kanto	310	248	1206	1764
Tokai	108	72	2172	2352
Tottori	200	196	924	1320

Table 6. The b value classification for a period 2 years before the earthquakes. 'D' or 'I' indicate b decreasing or increasing, respectively.

		(months before event)											
No.	Location	24	22	20	18	16	14	12	10	8	6	4	2
1	Ibaraki								D	D	D	D	D
2	Southern Ibaraki	I	I		I				D	D	D		
3	Chiba	D	D	D	D	D	D			D	D	D	
4	Tokyo											D	
5	Yamanashi			I	I	I							
6	Nagano						D	D					
7	Tottori	D								D	D	D	D

5. Discussion and conclusions

In the present study, decreases in b values have been found which appear a few years prior to large earthquakes. These findings agree with results of previous studies of variations in b values such as Ma (1978) and Smith (1986). Ma (1978) found decreases in b values before earthquakes in north China for smaller areas around their epicenters and also found that, for larger areas, peak values of b appear immediately before the earthquake. This result implies that space-time variations of b values are very complicated and shows an example of how the length of the increment plays an important role in the calculation of b values. Concerning this, it should be pointed out that given the spatial variability in b values, temporal variations in b values are possibly caused by seismicity migration (e.g. Udias (1977)). With the ABIC method used in the present study, two hyperparameters u and v, which are related to the smoothness in time and space variations of b values, respectively, are determined by the minimization of ABIC. Whether, therefore, the above apparent temporal variations are due to seismic migration can not be determined conclusively by this method.

From the standpoint of earthquake prediction research, the re-occurrence of the suggested precursor and the evaluation of efficiency are important points. Although such a consideration involves much more effort, it is very important for the establishment of a credible prediction scheme based on the precursory phenomenon. Regarding variations of b values, it has been reported that not all temporal decreases in b values are followed by a significant earthquake (e.g. Wyss and Lee (1973), Robinson (1978)). This implies that a prediction scheme based on temporal decreases in b values can not make a complete success of forecasting earthquakes. Therefore, it becomes important to make quantitative discussion on the efficiency of monitoring the b value at the present stage where many authors (e.g. Ma (1978), Smith (1986), Imoto (1987b)) have reported temporal decreases in b values before large earthquakes. Our results suggest that decreases in b values before large earthquakes cannot be explained by chance alone.

In summary, the method proposed in the present study is a powerful method for estimating space-time variations of b values, where the smoothness of the estimated values is guaranteed by the introduction of a prior distribution. From results of the analyses, it was found that decreases in b values are likely to appear a few years before earthquakes having a magnitude of 6.0 and larger. The appearence of decrease in the b value should be related to the occurrence of a large earthquake with a certain physical mechanism, which suggests that further studies of the b value as a promising parameter for earthquake prediction should be carried out.

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