Ann. Inst. Statist. Math.
Vol. 42, No. 1, 163–171 (1990)

PAIRWISE-BALANCED, VARIANCE-BALANCED AND RESISTANT INCOMPLETE BLOCK DESIGNS REVISITED*

JERZY K. BAKSALARY[1**] AND P. D. PURI[2]

[1]*Department of Mathematical and Statistical Methods, Academy of Agriculture in Poznań, ul. Wojska Polskiego 28, 60-637 Poznań, Poland*
[2]*Department of Mathematics and Statistics, Harayana Agricultural University, Hisar 125004, India*

(Received April 13, 1987; revised June 2, 1988)

Abstract. A general solution is derived to the problem of characterizing block designs that are simultaneously pairwise-balanced and variance-balanced. Applications of the characterizations obtained to some problems concerned with the local resistance of BIB designs are presented.

Key words and phrases: Block design, pairwise balance, variance balance, resistance, BIB design.

1. Introduction

Let D denote a block design in which v distinct treatments, $T_1,\dots, T_v$, are allocated on n experimental units arranged in b blocks, the allocation being described by a $v \times b$ incidence matrix $\boldsymbol{N} = (n_{ij})$, $i = 1,\dots, v, j = 1,\dots, b$. The vector of treatment replications and the vector of block sizes of D are $\boldsymbol{N}\mathbf{1}_b = \boldsymbol{r} = (r_i)$ and $\boldsymbol{N}'\mathbf{1}_v = \boldsymbol{k} = (k_j)$, where $\mathbf{1}_a$ denotes the $a \times 1$ vector of ones and $\boldsymbol{N}'$ is the transpose of $\boldsymbol{N}$. A block design is said to be *equireplicated* if $\boldsymbol{r} = r\mathbf{1}_v$ for some positive integer r, and to be *proper* if $\boldsymbol{k} = k\mathbf{1}_b$ for some positive integer k. Moreover, D is said to be *pairwise-balanced* if $\boldsymbol{NN}' \in \Delta_v$, where Δ_v denotes the set of $v \times v$ matrices with the off-diagonal elements all equal; D is said to be *variance-balanced* if every normalized estimable treatment contrast is estimated with the same variance; and D is said to be *efficiency-balanced* if every estimable treatment contrast is estimated with the same efficiency (cf., e.g., Hedayat and Federer (1974) and Puri and Nigam (1977)). All designs considered in this paper are assumed to be connected; i.e., $\operatorname{rank}(\boldsymbol{R} - \boldsymbol{NK}^{-1}\boldsymbol{N}') = v - 1$, where $\boldsymbol{R} = \operatorname{diag}(r_1,\dots, r_v)$ and $\boldsymbol{K} = \operatorname{diag}(k_1,\dots, k_b)$. It is known (cf., Puri and Nigam (1975)) that a

*This research was begun while the second author was visiting the Department of Mathematical and Statistical Methods, Academy of Agriculture in Poznań, Poland.

**The work of this author was partially supported by the Polish Academy of Sciences Grant No. CPBP 01.01.2/2.

connected D is variance-balanced if and only if $\boldsymbol{NK}^{-1}\boldsymbol{N}' \in \Delta_v$ and is efficiency-balanced if and only if every off-diagonal element of $\boldsymbol{NK}^{-1}\boldsymbol{N}'$ is proportionate to the product of the two relevant replications.

An interesting problem concerned with the different notions of balance is that of characterizing the class of block designs which, being balanced in one sense, are also balanced in another sense. A solution to one version of this problem, referring to the notions of efficiency balance and variance balance, is well known (cf., Puri and Nigam (1975) and Williams (1975)), and asserts that if D is connected and $v \geq 3$, then any two of the conditions:

(a) D is efficiency-balanced,
(b) D is variance-balanced,
(c) D is equireplicated,

imply the third condition. In the present paper, this problem is considered with reference to pairwise balance and variance balance. Hedayat and Federer (1974) showed, through counterexamples, that pairwise balance is neither necessary nor sufficient for variance balance. On the other hand, it is clear that these two notions of balance coincide for all proper designs, which is a (very) partial solution to the problem stated above. Our purpose is to derive a general solution and discuss its applicability to certain problems concerned with the concept of local resistance of BIB designs, introduced by Hedayat and John (1974).

2. Pairwise and variance balance

A solution to the problem of characterizing improper block designs that are simultaneously pairwise-balanced and variance-balanced will be given in two parts. The first part deals with designs having blocks of exactly two different sizes.

THEOREM 2.1. *Let D be a connected block design with v treatments and $b = b_1 + b_2$ blocks: b_1 blocks of size κ_1 and b_2 blocks of size κ_2, where $2 \leq \kappa_1 < \kappa_2$. Further, let D_h denote the subdesign of D comprising all the blocks of size h, $h = \kappa_1, \kappa_2$. Then D is simultaneously pairwise-balanced and variance-balanced if and only if both D_{κ_1} and D_{κ_2} are pairwise-balanced* (*and then variance-balanced as well*).

PROOF. Let $\boldsymbol{N}_h$ denote the incidence matrix of D_h. It is clear that if

$$\boldsymbol{N}_{\kappa_1}\boldsymbol{N}'_{\kappa_1} + \boldsymbol{N}_{\kappa_2}\boldsymbol{N}'_{\kappa_2} \in \Delta_v \tag{2.1}$$

and

$$(1/\kappa_1)\boldsymbol{N}_{\kappa_1}\boldsymbol{N}'_{\kappa_1} + (1/\kappa_2)\boldsymbol{N}_{\kappa_2}\boldsymbol{N}'_{\kappa_2} \in \Delta_v\ , \tag{2.2}$$

then $(1-\kappa_2/\kappa_1)N_{\kappa_1}N'_{\kappa_1} \in \Delta_v$ and $(1-\kappa_1/\kappa_2)N_{\kappa_2}N'_{\kappa_2} \in \Delta_v$. Hence, in view of $\kappa_1 < \kappa_2$, it follows that

$$N_h N'_h \in \Delta_v, \qquad h = \kappa_1, \kappa_2 . \tag{2.3}$$

Since (2.3) obviously implies (2.1) and (2.2), the proof is complete. □

Since a binary and proper variance-balanced block design is necessarily a BIB design (cf., Tocher (1952), p. 61 and Rao (1958)), it is clear that under the additional assumption that $n_{ij} = 1$ or 0 for every $i = 1,\ldots,v$, $j = 1,\ldots,b$, a necessary and sufficient condition for D in Theorem 2.1 to be simultaneously pairwise-balanced and variance-balanced is that both D_{κ_1} and D_{κ_2} be BIB designs. Consequently, both D_{κ_1} and D_{κ_2} are equireplicated, and hence, in view of the result of Puri and Nigam (1975) and Williams (1975) quoted in Section 1, it follows that if a binary block design with two different block sizes is simultaneously pairwise-balanced and variance-balanced, then it is necessarily efficiency-balanced as well.

The restriction in Theorem 2.1 to the case of two different block sizes is essential, because in general the balance properties of D are not inherited by its subdesigns. As an example consider the design with 6 treatments and 76 blocks specified as

$$D = D_2 \cup D_3 \cup D_4 = (3D_2^* \cup 4D_2^\#) \cup (3D_3^*) \cup (2D_4^*) ,$$

where each symbol of the type cD_a denotes c copies of a design D_a, and where D_2^*, $D_2^\#$, D_3^* and D_4^* are as specified below:

(i) D_2^* consists of the following 8 blocks: (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5) and (2, 6),

(ii) $D_2^\#$ consists of the following 6 blocks: (3, 4), (3, 5), (3, 6), (4, 5), (4, 6) and (5, 6),

(iii) D_3^* consists of the following 4 blocks: (1, 2, 3), (1, 2, 4), (1, 2, 5) and (1, 2, 6),

(iv) D_4^* consists of the following 8 blocks: (1, 3, 4, 5), (1, 3, 4, 6), (1, 3, 5, 6), (1, 4, 5, 6), (2, 3, 4, 5), (2, 3, 4, 6), (2, 3, 5, 6) and (2, 4, 5, 6).

It can be verified that D is simultaneously pairwise-balanced and variance-balanced (with 12 and 4 as the unique off-diagonal elements of NN' and $NK^{-1}N'$, respectively), while each of the subdesigns D_2, D_3 and D_4 is not balanced. On the other hand, however, the designs $(2D_2) \cup D_3$ and $D_3 \cup (2D_4)$ are variance-balanced. According to Theorem 2.2 below, this is just a necessary and sufficient condition for a design with blocks of sizes two, three and four to be simultaneously pairwise-balanced and variance-balanced.

THEOREM 2.2. *Let D be a connected block design with v treatments* *and $b = b_1 + \cdots + b_u$ blocks: b_1 blocks of size $\kappa_1, \ldots, b_u$ blocks of size κ_u, where $u \geq 3$ and $2 \leq \kappa_1 < \cdots < \kappa_u$. Further, let D_h denote the subdesign of D comprising all the blocks of size h, $h = \kappa_1, \ldots, \kappa_u$. Then any two of the conditions:*

(a) $\bigcup_{m=1}^{u-1} (\kappa_u - \kappa_m) D_{\kappa_m}$ *is variance-balanced,*

(b) *D is pairwise-balanced,*

(c) *D is variance-balanced,*

imply the third condition. Moreover, if (a) *holds along with* (b) *or* (c), *then* $\bigcup_{m=2}^{u} (\kappa_m - \kappa_1) D_{\kappa_m}$ *is also variance-balanced.*

PROOF. Notice that the conditions (a), (b) and (c) are equivalent to the relations

$$\kappa_u \sum_{m=1}^{u-1} (1/\kappa_m) \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} - \sum_{m=1}^{u-1} \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} \in \Delta_v ,$$

$$\sum_{m=1}^{u} \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} \in \Delta_v ,$$

$$\kappa_u \sum_{m=1}^{u-1} (1/\kappa_m) \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} + \boldsymbol{N}_{\kappa_u} \boldsymbol{N}'_{\kappa_u} \in \Delta_v ,$$

respectively. Hence the first part of Theorem 2.2 is a consequence of the fact that any two of the relations $\boldsymbol{A} \in \Delta_v$, $\boldsymbol{B} \in \Delta_v$ and $\boldsymbol{A} + \boldsymbol{B} \in \Delta_v$ imply the third relation. The second part follows by observing that $\bigcup_{m=2}^{u} (\kappa_m - \kappa_1) D_{\kappa_m}$ is variance-balanced if and only if

$$\sum_{m=2}^{u} \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} - \kappa_1 \sum_{m=2}^{u} (1/\kappa_m) \boldsymbol{N}_{\kappa_m} \boldsymbol{N}'_{\kappa_m} \in \Delta_v . \qquad \square$$

A design D is called a *linked block design* (cf., Youden (1951)) if any two of its blocks have the same number of treatments in common; i.e., if $\boldsymbol{N}'\boldsymbol{N} \in \Delta_b$. It is obvious that the dual of a variance-balanced design D is a linked block design if and only if D is also pairwise-balanced. A characterization of such a design is therefore obtainable directly from Theorems 2.1 and 2.2. Consequently, Theorem 2.1(ii) of Nigam and Puri (1982), stating that the dual of a variance-balanced design D is a linked block design if and only if D is proper, seems to be incorrect; the condition given by them is sufficient but not necessary. The same remark applies to their Theorem 2.1(iii).

3. Local resistance of BIB designs

Let D be a BIB (v,b,r,k,λ) design, let $\mathscr{T}_0=\{T_{i_1},\ldots,T_{i_s}\}$ be a subset of $\{T_1,\ldots,T_v\}$, and let $D_\#$ denote the design obtained by deleting from D all the experimental units assigned to the treatments in $\mathscr{T}_0$ and ignoring empty blocks if such emerge. Then, according to Hedayat and John (1974) and Most (1975), D is said to be *locally resistant of degree s with respect to* $\mathscr{T}_0$ if $D_\#$ is variance-balanced. Since, on the other hand, $D_\#$ is pairwise-balanced irrespective of the choice and cardinality of $\mathscr{T}_0$, it is clear that the problem of examining the resistance of BIB designs is very closely connected to the problem of characterizing binary block designs which are simultaneously pairwise-balanced and variance-balanced.

The incidence matrix of a BIB (v,b,r,k,λ) design may always be written in the form

$$N=\begin{pmatrix} \mathbf{1}'_r & \mathbf{0}'_{b-r} \\ N_{k-1} & N_k \end{pmatrix},$$

where N_{k-1} and N_k correspond to the subdesigns D_{k-1} and D_k comprising blocks of size $k-1$ and k, respectively. Consequently, a simple corollary to Theorem 2.1 asserts that a BIB design is locally resistant of degree one with respect to a given treatment if and only if the corresponding subdesign D_{k-1} is a BIB design (if and only if the corresponding subdesign D_k is a BIB design), which was originally established by Hedayat and John (1974), Theorem 4.1 (see also Kageyama (1987)).

For examining the local resistance of degree two it is useful to partition N as

$$N=\begin{pmatrix} \mathbf{1}'_\lambda & \mathbf{1}'_{r-\lambda} & \mathbf{0}'_{r-\lambda} & \mathbf{0}'_{b-2r+\lambda} \\ \mathbf{1}'_\lambda & \mathbf{0}'_{r-\lambda} & \mathbf{1}'_{r-\lambda} & \mathbf{0}'_{b-2r+\lambda} \\ N_{k-2} & N^*_{k-1} & N^\#_{k-1} & N_k \end{pmatrix}, \tag{3.1}$$

where N_{k-2}, $N_{k-1}=(N^*_{k-1}:N^\#_{k-1})$ and N_k correspond to the subdesigns D_{k-2}, D_{k-1} and D_k comprising blocks of size $k-2$, $k-1$ and k, respectively. As an immediate consequence of Theorem 2.2 we get the following.

THEOREM 3.1. *A* BIB (v,b,r,k,λ) *design, in which* $b>2r-\lambda$ *and* $k>3$, *is locally resistant of degree two with respect to a given pair of treatments if and only if the design* $(2D_{k-2})\cup D_{k-1}$ *corresponding to this pair of treatments is variance-balanced* (*if and only if the design* $D_{k-1}\cup(2D_k)$ *corresponding to this pair of treatments is variance-balanced*).

Theorem 3.1 is to be supplemented by noting that if $b = 2r - \lambda$, then the submatrix $\boldsymbol{N}_k$ in (3.1) is absent, and thus $D_\#$ contains blocks of two different sizes only. Consequently, we may apply Theorem 2.1 to conclude that a necessary and sufficient condition for the local resistance of degree two is then the requirement that each of the subdesigns D_{k-2} and D_{k-1} be a BIB design. For a discussion of BIB designs satisfying the condition $b = 2r - \lambda$ see, e.g., Kageyama and Mohan (1984), Section 3.

It can be verified that the designs $(2D_{k-2}) \cup D_{k-1}$ and $D_{k-1} \cup (2D_k)$ in Theorem 3.1 are equireplicated (with the replications equal to 2λ and $2(r-\lambda)$, respectively), and thus, being variance-balanced, they are efficiency-balanced as well. Therefore, the question arises as to how to utilize this additional property in investigating local resistance of degree two of BIB designs. Notice that the design given as an example before Theorem 2.2 is not equireplicated, having $\boldsymbol{r}' = (32, 32, 33, 33, 33, 33)$.

THEOREM 3.2. *The following conditions are necessary for a* BIB (v, b, r, k, λ) *design to be locally resistant of degree two*:

(a) $\lambda(k-2)/(v-2)$ *is a positive integer*,

(b) *there exist integers* x, y, z *satisfying the equations*

$$2(k-1)x + (k-2)y = \frac{2(k-1)(k-2)[\lambda(v-2) - r]}{(v-2)(v-3)} \tag{3.2}$$

and

$$x + y + z = \lambda\,. \tag{3.3}$$

PROOF. Let D_{k-2}, D_{k-1} and D_k be subdesigns of BIB (v, b, r, k, λ) with the incidence matrices $\boldsymbol{N}_{k-2}, \boldsymbol{N}_{k-1} = (\boldsymbol{N}^*_{k-1} : \boldsymbol{N}^\#_{k-1})$ and $\boldsymbol{N}_k$, respectively, as specified in (3.1). In view of the equality of treatment replications in $(2D_{k-2}) \cup D_{k-1}$, comparing the diagonal elements on the two sides of

$$\begin{aligned} &\frac{2}{k-2}\boldsymbol{N}_{k-2}\boldsymbol{N}'_{k-2} + \frac{1}{k-1}\boldsymbol{N}_{k-1}\boldsymbol{N}'_{k-1} \\ &\quad = \frac{2(r-\lambda)}{v-3}\boldsymbol{I}_{v-2} + \frac{2[\lambda(v-2)-r]}{(v-2)(v-3)}\mathbf{1}_{v-2}\mathbf{1}'_{v-2} \end{aligned} \tag{3.4}$$

shows that D_{k-2}, D_{k-1} and, consequently, also D_k are equireplicated, with the replications equal to

$$\frac{\lambda(k-2)}{v-2}, \quad \frac{2\lambda(v-k)}{v-2} \quad \text{and} \quad \frac{r(v-2k)+\lambda k}{v-2}, \tag{3.5}$$

respectively. Hence we get the condition (a), which implies that also the second and third quantities in (3.5) are integers, whereas the condition (b) is obtained by comparing the off-diagonal elements in (3.4). □

The necessity of the condition (a) in Theorem 3.2 was established by Kageyama and Saha ((1987), Corollary 3.4), who also pointed out that another necessary condition is that

$$\frac{\lambda(k-2)(k-3)(v-1)}{(v-2)(v-3)} \quad \text{is an integer .} \tag{3.6}$$

Notice that condition (b) in Theorem 3.2 requires that

$$\frac{2(k-1)(k-2)[\lambda(v-2)-r]}{(v-2)(v-3)} \quad \text{is an integer ,}$$

and since

$$\begin{aligned}
&2(k-1)(k-2)[\lambda(v-2)-r] \\
&\quad = 2(k-2)[\lambda(k-1)(v-2)-r(k-1)] \\
&\quad = 2(k-2)[\lambda(k-1)(v-2)-\lambda(v-1)] \\
&\quad = \lambda(k-2)(k-3)(v-1)+\lambda(k-1)(k-2)(v-3) ,
\end{aligned}$$

it follows that, under (a), the condition (3.6) is implied by (b). It also follows that if there is only one solution to the equations (3.2) and (3.3), then each of the subdesigns D_{k-2}, D_{k-1} and D_k is pairwise-balanced and, being proper, is variance-balanced as well. But a proper variance-balanced block design is necessarily a BIB design, and hence we conclude that if there is exactly one solution to (3.2) and (3.3), then D_{k-2}, D_{k-1} and D_k are all BIB designs.

Condition (b) in Theorem 3.2 proves to be quite useful. For example, Kageyama and Saha (1987) verified that within the class of BIB designs with $v = 2k+1$, $k \geq 4$ and $r \leq 30$, only BIB$(11, 33, 15, 5, 6)$ and BIB$(11, 66, 30, 5, 12)$ designs fulfil the necessary conditions for the local resistance of degree two given in their Corollary 3.4, i.e., (a) in Theorem 3.2 and (3.6). In these two cases, the equations (3.2) and (3.3) take the forms

$$8x+3y=13, \qquad x+y+z=6 , \tag{3.7}$$

and

$$(3.8) \qquad 8x + 3y = 26, \qquad x + y + z = 12\,,$$

respectively. It is seen that there is no integer solution to the equations (3.7) and that there is a unique solution, viz. $x = 1$, $y = 6$ and $z = 5$, to the equations (3.8). The former observation implies that there does not exist a BIB (11, 33, 15, 5, 6) design which would be locally resistant of degree two, thus answering in the negative the question of Kageyama and Saha ((1987), p. 89). On the other hand, the latter observation implies that, as originally asserted by Kageyama and Saha (1987), there does exist a BIB (11, 66, 30, 5, 12) locally resistant of degree two. Its subdesigns D_3, D_4 and D_5 are BIB (9, 12, 4, 3, 1), BIB (9, 36, 16, 4, 6) and BIB (9, 18, 10, 5, 5) designs, respectively.

The last part of this paper is concerned with problem (c) of Hedayat and John ((1974), p. 157). They asserted, in their Theorem 5.5, that if $\mathscr{T}_0$ consists of k treatments occurring in any one block, then a sufficient condition for a BIB (v, b, r, k, λ) design to be locally resistant of degree k with respect to $\mathscr{T}_0$ is that it is symmetric, i.e., that $v = b$. Moreover, they posed the question whether this property is also a necessary condition. This question was answered in the negative by Chandak (1980), and perhaps the simplest example is the unreduced BIB $\left(v, \binom{v}{k}, \binom{v-1}{k-1}, k, \binom{v-2}{k-2}\right)$ design, which is known (cf., Kageyama (1987), Lemma 2.1), to be resistant with respect to every subset of treatments whose cardinality does not exceed k.

However, the symmetry of a block design becomes both necessary and sufficient for the local resistance of degree k with respect to $\mathscr{T}_0$ consisting of treatments which occur in the same block when a modified definition is adopted, according to which D is said to be *locally strongly resistant of degree s with respect to* $\mathscr{T}_0$ if $D_\#$ is a BIB design.

THEOREM 3.3. *Let D be a* BIB (v, b, r, k, λ) *design, and let* $\mathscr{T}_0$ *consist of k treatments occurring in one block which is not repeated in D. Then D is locally strongly resistant of degree k with respect to* $\mathscr{T}_0$ *if and only if* $v = b$.

PROOF. As pointed out by Hedayat and John (1974) in the proof of their Theorem 5.5, the sufficiency is an immediate consequence of the fact that if $v = b$, then $D_\#$ is a BIB $(v - k,\ v - 1,\ r,\ k - \lambda,\ \lambda)$ design. Now assume that $D_\#$ is a BIB $(v_\#, b_\#, r_\#, k_\#, \lambda_\#)$ design. Then $\lambda_\# = \lambda$ by the definition of $D_\#$, and hence, since $v_\# = v - k$ and $r_\# = r$, it follows that $(k_\# - 1)/(v - k - 1) = (k-1)/(v-1)$. Consequently, $k_\# = k(v-k)/(v-1)$, while on the other hand, $k_\# = v_\# r_\# / b_\# = r(v-k)/(b-1)$. Comparing the two expressions for $k_\#$ yields $r = k$, or, equivalently, $v = b$. □

Acknowledgement

The authors are grateful to the referee for a number of helpful and constructive comments.

REFERENCES

Chandak, M. L. (1980). On the theory of resistant block designs, *Calcutta Statist. Assoc. Bull.*, **29**, 27–34.

Hedayat, A. and Federer, W. T. (1974). Pairwise and variance balanced incomplete block designs, *Ann. Inst. Statist. Math.*, **26**, 331–338.

Hedayat, A. and John, P. W. M. (1974). Resistant and susceptible BIB designs, *Ann. Statist.*, **2**, 148–158.

Kageyama, S. (1987). Some characterization of locally resistant BIB designs of degree one, *Ann. Inst. Statist. Math.*, **39**, 661–669.

Kageyama, S. and Mohan, R. N. (1984). On the construction of group divisible partially balanced incomplete block designs, *Bull. Fac. School Ed. Hiroshima Univ.*, **7**, 57–60.

Kageyama, S. and Saha, G. M. (1987). On resistant t-designs, *Ars Combin.*, **23**, 81–92.

Most, B. M. (1975). Resistance of balanced incomplete block designs, *Ann. Statist.*, **3**, 1149–1162.

Nigam, A. K. and Puri, P. D. (1982). On partially efficiency—balanced designs—II, *Comm. Statist. A—Theory Methods*, **11**, 2817–2830.

Puri, P. D. and Nigam, A. K. (1975). On patterns of efficiency balanced designs, *J. Roy. Statist. Soc. Ser. B*, **37**, 457–458.

Puri, P. D. and Nigam, A. K. (1977). Balanced block designs, *Comm. Statist. A—Theory Methods*, **6**, 1171–1179.

Rao, V. R. (1958). A note on balanced designs, *Ann. Math. Statist.*, **29**, 290–294.

Tocher, K. D. (1952). The design and analysis of block experiments (with discussion), *J. Roy. Statist. Soc. Ser. B*, **14**, 45–100.

Williams, E. R. (1975). Efficiency-balanced designs, *Biometrika*, **62**, 686–689.

Youden, W. G. (1951). Linked blocks: A new class of incomplete block designs (abstract), *Biometrics*, **7**, 124.