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VARIANCE OF SIGHTINGS IN THE SURVEY OF PATCHILY DISTRIBUTED OBJECTS

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Summary

In the field, animals, birds and fishes are often distributed inhomogeneously. In such situations, the variance of sightings of strip or line transect survey would be larger than when they are independently distributed one another. We formulate in this paper the structure of the patchy configuration, and deduce the variance of sightings of strip or line transect survey. From this, we see that it is larger when the patch size (the expected number of objects in each patch) is larger and the patch radius is smaller.

1. Introduction

Since 1979, the International Whaling Commission (IWC) has been conducting the sighting survey of minke whales in the Antarctic Ocean. The abundance is estimated by the line transect methodology with parallel ship experiments (Butterworth [7], Cooke [11], Hiby and Ward

Table 1. Number of sightings and its variance in the 1984/85 IWC/IDCR Antarctic mink whale assessment cruise in Area IV W. Variances are based on inter-transect variance weighted by transect length.

Strata vessel	Northern stratum K27		Intermediate stratum SM1		Southern stratum SM2	
Survey mode	Closing*	Passing**	Closing	Passing	Closing	Passing
No. of schools sighted (n)	37	23	26	44	94	78
Variance of n	31.2	68.9	42.9	187.3	570.1	554.9

* Closing mode; when a detection is done, the pods are closed to identify the species and confirm the school size (Butterworth et al. [10] and Kishino [17]).

** Passing mode; the usual survey mode passing through the pods and going straight when there is a detection.

Key words and phrases: Line transect, inhomogeneity, patch size, patch radius.

[14] and Ward and Hiby [19]). Contrary to the usual case, the detection probability on the track line cannot be assumed to be one in general, because whales spend most of their time below the water surface. So, the additional information about the proportion of duplicate sightings in the parallel ship experiment is used for estimation. Butterworth et al. [8]–[10] analysed the sighting error of distance and angular from the ship, the school size and the species. Butterworth [9] and Kishino [16] studied model robustness, and Buckland [4], [5] compared several models. The identification problem of duplicate sighting was discussed in Kishino [16].

Here we consider about the variance of sightings. Table 1 shows the number of sightings and its variance in the 1984/85 IWC/IDCR Antarctic minke whale assessment cruise in Area IV (Butterworth [10]). It is seen that the variance is usually larger than the number of sightings. Furthermore the ratio of the former to the latter is larger in the southern stratum, where whales are abundant and the distribution of them is considered to be highly patched. If the sighting probability does not change and the distribution of pods is free from after

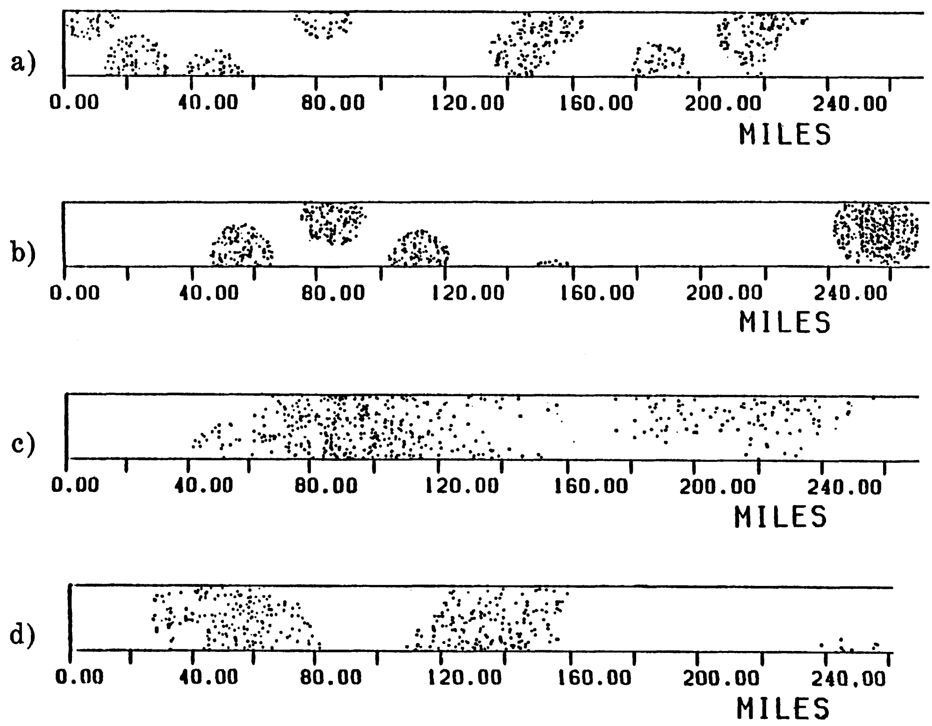
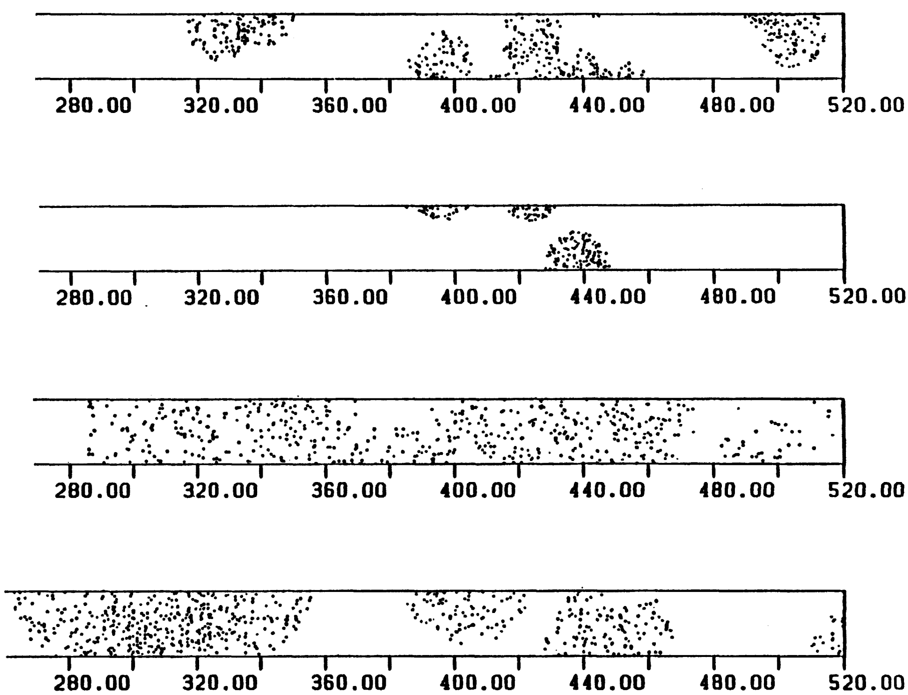


Fig. 1. Examples of patchy configurations. 1040 points are scattered in the rectan- First, centers of circles are located at random, and then the elements of $r=10$ n.miles, the number of elements of each patch $n=100$ pods per patch,

effect (Daley and Vere-Jones [12] and Matthes et al. [18]), then the variance of the number of sightings is the same as its mean. In other words, the difference between the variance and the mean reflects the daily fluctuation of sighting probability coming from the fluctuation of detection ability and the weather condition (Kasamatsu and Kishino [15]), and the patchily distribution of pods. In this paper, we discuss the latter factor.

In the field, animals, birds, fishes and so on are often distributed inhomogeneously. Several hierarchical structures can be found in this inhomogeneity, that is, from families, schools, patches to sub-populations. Usually the variance of sightings of objects distributed patchily is larger than when they are distributed independently one another, and there are lots of work concerning to this (e.g. Anscombe [1], Bliss and Fisher [2]).

Figure 1 and Table 2 are the results of simulation of line transect survey in various degrees of inhomogeneity (Kishino and Kasamatsu [17]). Here, inhomogeneity is realized by decomposing the scattering of points into two steps, that is, deciding the locations of patches and



gular whose area is $20\times520=10400$ n.miles^2. So the density is 0.1 pods per n.miles^2. each 'patch' are distributed uniformly in the circle. a) The radius of patches b) $r=10$, $n=200$, c) $r=20$, $n=100$, d) $r=20$, $n=200$.

Table 2. The means and standard deviations of sightings by line transect survey—the result of the simulation study by Kishino and Kasamatsu [17]. For each of twenty types of patchy distributions, we generate ten configurations and simulate the line transect survey. The table lists the mean and standard deviation (in parenthesis) of sightings of the ten trials for each type.

	Sightings (n)			
Radius \ Size	50	100	150	200
5.00	51.500 (12.545)	38.400 (22.446)	45.900 (21.053)	50.400 (19.225)
10.00	44.000 (7.587)	46.600 (6.603)	44.500 (12.140)	44.700 (10.100)
15.00	44.900 (6.244)	45.900 (7.651)	39.100 (3.695)	46.500 (5.967)
20.00	44.600 (5.739)	44.700 (7.718)	44.800 (5.051)	41.800 (8.337)
25.00	41.300 (6.038)	46.300 (7.197)	44.400 (6.132)	46.400 (9.082)

then scattering the elements of each patch. Since the density is set to the constant value 0.1/n.miles^2, it is highly inhomogeneous when the radius of patches is small and the number of elements of each patch (which we call the patch size) is large. It is seen that the variance becomes large according to inhomogeneity.

Therefore, we consider how the patch size and the patch radius influence the variance, in particular of strip or line transect. First, in the following section, we build a model representing the patch structure and from this deduce the variance of strip transect. Next, we modify the model for line transect in Section 3. And last, the case is treated when the patch size and the patch radius are random variables.

2. Model representation and deduction of variance

In this section, we formulate the patchy distribution and from this deduce the variance of sightings of strip transect.

As in the Introduction, we decompose the configuration of objects into two steps. The first is the configuration of patches and the second is that of elements within each patch. Patches are supposed to follow the Poisson random field $N_0(dz)$, $z \in \boldsymbol{R}^2$, with the intensity measure $\lambda_0(z)dz$ (we assume that $N_0(\cdot)$ is non atomic). For simplicity, the location of each patch is assumed to be decided by one particular point, and we shall call this the center of the patch. And within each patch, its elements follow the Poisson random field. That is, if there is a patch centered at $z \in \boldsymbol{R}^2$, the elements of the patch are distributed according to the Poisson random field with the intensity measure $\lambda_1^{(z)}(x)dx = \lambda_1(x-z)dx$. Here $N_0(\cdot)$, $N_1^{(z)}(\cdot)$; $z \in \boldsymbol{R}^2$ are supposed to be independent.

With the above model, the number of elements in the region A is obtained as follows;

$$N(A)=\int_A\int_{R^2} N_0(dz)N_1^{(z)}(dx)\,. \tag{1}$$

The mean is obvious;

$$\begin{aligned}\mathrm{E}[N(A)]&=\int_A dx\int_{R^2} dz\;\lambda_0(z)\lambda_1(x-z)\\&=\int_A \lambda_0*\lambda_1(x)dx\,.\end{aligned} \tag{2}$$

Here $*$ denotes the convolutions. In particular, when $\lambda_0(z)$ is a constant λ_0, it reduces to

$$\mathrm{E}[N(A)]=\lambda_0|A|\Lambda_1\,, \tag{3}$$

where $|A|$ is the area of A and Λ_1 is the mean number of objects in a patch, $\int \lambda_1(x)dx$.

The variance is obtained as follows:

PROPOSITION 1. *The variance of $N(A)$ is*

$$\mathrm{V}[N(A)]=\int_A \lambda_0*\lambda_1(x)dx+\int_{R^2}\lambda_0(z)dz\left\{\int_A \lambda_1(x-z)dx\right\}^2\,. \tag{4}$$

Remark 1.1. The first term on the right-hand side is equal to the mean $\mathrm{E}[N(A)]$. Noting that the variance is equal to the mean when $N(dz)$ is itself Poisson random field, that is, when each object is independent one another, the second term represents the increase of the variance coming from fluctuation of patch locations.

Remak 1.2. We consider the second term as a function of the patch size Λ_1 and the 'patch radius' σ. Let λ_1 be expressed as

$$\lambda_1^\sigma(y)=\frac{\Lambda_1}{\sigma^2}\rho(y/\sigma)\,,$$

where ρ is the probability density and write the second term as $f(\sigma,\Lambda_1)$. Because $\lambda_1^\sigma(\cdot)$ tends to $\Lambda_1\delta(\cdot)$ as $\sigma\downarrow 0$, where $\delta(\cdot)$ is Dirac's delta function, we get from (4)

$$f(+0,\Lambda_1)=\Lambda_1^2\int_A \lambda_0(z)dz\,. \tag{5}$$

On the other hand, noting that $f(\sigma,\Lambda_1)$ is rewritten as

$$f(\sigma,\Lambda_1)=\int_{R^2}\lambda_0(z)dz\left\{\int_{(A-z)/\sigma}\Lambda_1\rho(y)dy\right\}^2\,, \tag{6}$$

we get

$$(7)\qquad f(\infty, \Lambda_1)=0\,.$$

PROOF. Because $\mathrm{V}[N(A)]$ is expressed as

$$\mathrm{V}[N(A)]=\mathrm{E}[N(A)^2]-\{\mathrm{E}[N(A)]\}^2\,,$$

we will calculate $\mathrm{E}[N(A)^2]$. In the expression

$$(8)\qquad N(A)^2=\int_{A\times A}\int_{\boldsymbol{R}^2\times\boldsymbol{R}^2} N_0(dz_1)N_0(dz_2)N_1^{(z_1)}(dx_1)N_1^{(z_2)}(dx_2)\,,$$

we note the following;

(i) Since $N_0(\cdot)$ is the Poisson random field,

$$(9)\qquad \mathrm{E}[N_0(dz_1)N_0(dz_2)]=\lambda_0(z_1)\lambda_0(z_2)dz_1dz_2+\delta(z_1-z_2)\lambda_0(z_1)dz_1dz_2\,.$$

(ii) As for $N_1^{(z)}(dx)$, noting that $N_1^{(z_1)}(\cdot)$ and $N_1^{(z_2)}(\cdot)$ are independent to each other if $z_1\neq z_2$, we get

$$(10)\qquad \mathrm{E}[N_1^{(z_1)}(dx_1)N_1^{(z_2)}(dx_2)] =\begin{cases}\lambda_1(x_1-z_1)\lambda_1(x_2-z_2)dx_1dx_2+\delta(x_1-x_2)\lambda_1(x_1-z_1)dx_1dx_2\,, & \text{if } z_1=z_2\\ \lambda_1(x_1-z_1)\lambda_1(x_2-z_2)dx_1dx_2\,, & \text{if } z_1\neq z_2\,.\end{cases}$$

Therefore

$$(11)\qquad \begin{aligned}\mathrm{E}[N(A)^2]&=\int_{\boldsymbol{R}^2\times\boldsymbol{R}^2}\lambda_0(z_1)\lambda_0(z_2)dz_1dz_2\int_{A\times A}\lambda_1(x_1-z_1)\lambda_1(x_2-z_2)dx_1dx_2\\ &\quad+\int_{\boldsymbol{R}^2}\lambda_0(z)dz\left\{\int_{A\times A}\lambda_1(x_1-z)\lambda_1(x_2-z)dx_1dx_2\right.\\ &\qquad\left.+\int_A\lambda_1(x-z)dx\right\}\\ &=\left\{\int_A\lambda_0*\lambda_1(x)dx\right\}^2+\int_A\lambda_0*\lambda_1(x)dx\\ &\qquad+\int_{\boldsymbol{R}^2}\lambda_0(z)dz\left\{\int_A\lambda_1(x-z)dx\right\}^2,\end{aligned}$$

from which we get the conclusion.

Example 2.1. As an example, we calculate $f(\sigma, \Lambda)$ for the case when

$$(12)\qquad \begin{aligned}&\lambda_0(z)=\lambda_0\\ &\lambda_1^\sigma(y)=\frac{\Lambda_1}{2\pi\sigma^2}\exp\left\{-\frac{|y|^2}{2\sigma^2}\right\}.\end{aligned}$$

By Fubini's theorem, it is easily seen that

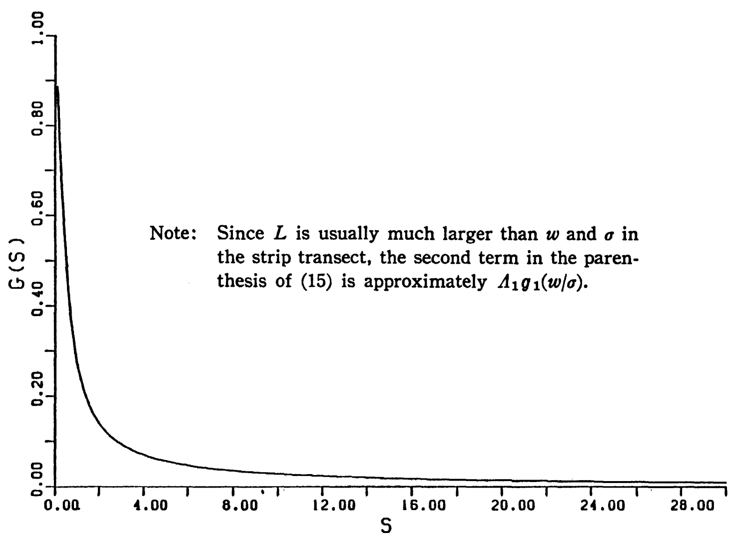


Fig. 2. Graph of $g_1(s)$.

(13) $$f(\sigma, \Lambda_1)=\frac{\lambda_0\Lambda_1^2}{4\pi\sigma^2}\int_{A\times A}\exp\left\{-\frac{|x_1-x_2|^2}{4\sigma^2}\right\}dx_1dx_2\,.$$

Let A be a rectangular $[0, w]\times[0, L]$, $w>0$, $L>0$. By

(14) $$\frac{1}{2\sqrt{\pi}\,\sigma}\int_0^w du\int_0^w dv\exp\left\{-\frac{(u-v)^2}{4\sigma^2}\right\}=\int_0^w\left\{2\Phi\left(\frac{x}{\sqrt{2}\,\sigma}\right)-1\right\}dx\,,$$

where $\Phi(\cdot)$ is the cumulative function of the standard Normal distribution. The variance of $N(A)$ is written as

(15) $$\mathrm{V}[N(A)]=\mathrm{E}[N(A)]\{1+\Lambda_1 g_1(\sigma/w)g_1(\sigma/L)\}\,.$$

Here the function $g_1(s)$ is defined as

(16) $$g_1(s)=2\int_0^1\Phi\left(\frac{x}{\sqrt{2}\,s}\right)dx-1\,.$$

Obviously, $g_1(s)$ is decreasing, so the second term in the parenthesis is proportional to Λ_1 and the decreasing function of the patch radius σ. Figure 2 gives the graph of $g_1(s)$.

3. Modification of model to line transect

While it is assumed in the strip transect that all objects in the strip are sighted, in the line transect the detection probability decreases in general with the distance from the course line (Buckland [3], Burnham et al. [6] and Hayes and Buckland [13]). So the number of sightings in the line transect is expressed as

$$N=\int_{R^2}\int_{R^2} N_0(dz)N_1^{(z)}(dx)K^{(x)} . \tag{17}$$

Here $\{K^{(x)}; x\in \boldsymbol{R}^2\}$ are random variables distributed as

$$K^{(x)}=\begin{cases} 1 & \text{with probability } \ c(x) \\ 0 & \text{with probability } \ 1-c(x) , \end{cases} \tag{18}$$

where $c(\cdot)$ is the detection function and assumed to be in $L^1(\boldsymbol{R}^2)$. $N_0(\cdot)$, $N_1(\cdot)$; $z\in \boldsymbol{R}^2$ and $K^{(x)}$; $x\in \boldsymbol{R}^2$ are independent one another.

As for the mean of N, (2) is modified as

$$\begin{aligned} \mathrm{E}[N]&=\int_{R^2} dz \int_{R^2} dx\ \lambda_0(z)\lambda_1(x-z)c(x) \\ &=\int_{R^2} \lambda_0 * \lambda_1(x)c(x)dx . \end{aligned} \tag{19}$$

In particular, when $\lambda_0(z)$ takes the constant value λ_0, (3) becomes

$$\mathrm{E}[N]=\lambda_0\Lambda_1 \int_{R^2} c(x)dx . \tag{20}$$

If we take as $c(x)$ the indicator function $I_A(x)$, the above equations are easily seen to include (2) and (3).

Now let us consider the variance of N.

PROPOSITION 2. *The variance* $\mathrm{V}[N]$ *of* N *is written as*

$$\mathrm{V}[N]=\mathrm{E}[N]+\int_{R^2} \lambda_0(z)dz\left\{\int_{R^2} \lambda_1(x-z)c(x)dx\right\}^2 . \tag{21}$$

Remark 2.1. Obviously, this is an extension of (4).

Remark 2.2. Corresponding to the Remark 1.2 of Proposition 1, we can say the following. Let $\lambda_1^\sigma(y)$ be as in Remark 1.2. We write the second term on the right-hand side of (21) as $f(\sigma, \Lambda_1)$,

$$f(+0, \Lambda_1)=\Lambda_1^2 \int_{R^2} \lambda_0(z)c(x)^2dz . \tag{22}$$

And, as $f(\sigma, \Lambda_1)$ can be rewritten as

$$f(\sigma, \Lambda_1)=\Lambda_1^2 \int_{R^2} \lambda_0(z)dz\left\{\int_{R^2} \rho(y)c(\sigma y+z)dy\right\}^2 , \tag{23}$$

$$f(\infty, \Lambda_1)=0 , \tag{24}$$

if $c(x)\downarrow 0$ as $|x|\to\infty$.

PROOF. Besides the comments in the proof of Proposition 1, we note the following;

(25) $$\mathrm{E}[K^{(x_1)}K^{(x_2)}]=\begin{cases} c(x_1) & \text{if } x_1=x_2 \\ c(x_1)c(x_2) & \text{if } x_1\neq x_2 . \end{cases}$$

Then we can calculate $\mathrm{E}[N^2]$ as

(26) $$\mathrm{E}[N^2]=\int_{R^2}\lambda_0(z)dz\left\{\int_{R^2}\lambda_1(x-z)c(x)dx + \int_{R^4}\lambda_1(x_1-z)\lambda_1(x_2-z)c(x_1)c(x_2)dx_1dx_2\right\} + \int_{R^4}\lambda_0(z_1)\lambda_0(z_2)dz_1dz_2\int_{R^4}\lambda_1(x_1-z_1)\lambda_1(x_2-z_2)c(x_1)c(x_2)dx_1dx_2 .$$

As the first and the third term of the three are equal to $\mathrm{E}[N]$ and $\{\mathrm{E}[N]\}^2$, respectively, the results follow.

We calculate (21) for two types of detection function in the line transect. One is the half normal model and the other is the negative exponential model.

Example 3.1. Let

(27) $$c(x)=a\exp\left(-\frac{x^2}{2v^2}\right)I_{[0,L]}(y) ,$$

where x and y are the coordinates of the vector $x\in \boldsymbol{R}^2$. λ_0 and λ_1 are as in Example 2.1. The effective width w is

(28) $$w=\sqrt{2\pi}\,av ,$$

so the mean of N is

(29) $$\mathrm{E}[N]=\sqrt{2\pi}\,\lambda_0\Lambda_1 avL .$$

It is easily seen that

(30) $$f(\sigma, \Lambda_1)=\lambda_0\Lambda_1^2a^2Lg_1(\sigma/L)\frac{1}{2\sqrt{\pi}\,\sigma}\int_{R^2}\exp\left\{-\frac{(x_1-x_2)^2}{4\sigma^2}-\frac{x_1^2+x_2^2}{2v^2}\right\}dx_1dx_2 ,$$

where $g_1(\cdot)$ is introduced in Example 2.1. Calculating the integral, we get

(31) $$\mathrm{V}[N]=\mathrm{E}[N]\{1+a\Lambda_1g_1(\sigma/L)g_2(\sigma/v)\} .$$

Here

(32) $$g_2(s)=\{2(1+s^2)\}^{-1/2} .$$

It is similar to Example 2.1 that the second term on the right-hand side of (31) is proportional to Λ_1 and decreasing with σ.

Example 3.2. λ_0 and λ_1 are as in the previous example. Let $c(x)$ be

(33) $$c(x, y)=a \exp(-|x|/v)I_{[0,L]}(y).$$

Then the effective width w is

(34) $$w=2av$$

and the mean of N is

(35) $$\mathrm{E}[N]=2\lambda_0\Lambda_1 avL.$$

$f(\sigma, \Lambda_1)$ is seen to be

(36) $$f(\sigma, \Lambda_1) = \lambda_0\Lambda_1^2 a^2 L g_1(\sigma/L)\frac{1}{2\sqrt{\pi}\,\sigma}\int_{R^2}\exp\left\{-\frac{(x_1-x_2)^2}{4\sigma^2}+\frac{|x_1|+|x_2|}{v}\right\}dx_1dx_2.$$

Calculating the integral, we obtain the variance of N as

(37) $$\mathrm{V}[N]=\mathrm{E}[N]\{1+a\Lambda_1 g_1(\sigma/L)g_3(\sigma/v)\}.$$

Here the function $g_3(s)$ is defined as

(38) $$g_3(s)=\frac{1}{\sqrt{\pi}}\left\{s+(1-2s^2)\int_s^\infty \exp(s^2-t^2)dt\right\}.$$

Figure 3 shows its graph. Comparing Examples 2.1, 3.1 and 3.2, the latters are decreasing with the radius of patches more slowly than the formers. This is because the tail of the detection curve drags longer for the latter one.

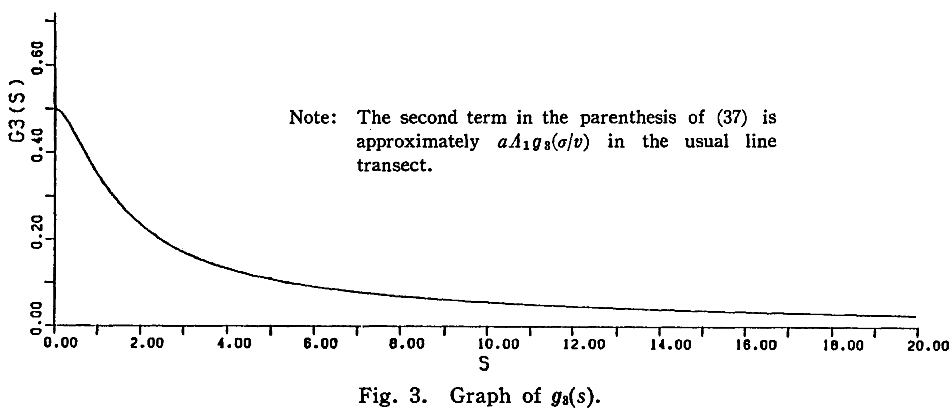


Fig. 3. Graph of $g_3(s)$.

4. Randomness of Λ_1 and σ

In the preceding sections, the intensity measure $\lambda_1^{(z)}(\cdot)$ of $N_1^{(z)}(\cdot)$ is assumed to be deterministic. But the patch size and the patch radius are likely to vary from patch to patch. So in this section, we take into account the randomness of $\lambda_1^{(z)}(\cdot)$.

We assume that $\lambda_1^{(z)}$, $z\in \boldsymbol{R}^2$ are independent one another and to $N_0(\cdot)$, $N_1(\cdot)$; $z\in \boldsymbol{R}^2$. Then the mean and the variance are expressed as follows;

$$\text{(39)} \qquad \mathrm{E}[N]=\int_{\boldsymbol{R}^2}\lambda_0(z)dz\int_{\boldsymbol{R}^2}\mathrm{E}[\lambda_1(x-z)]c(x)dx$$

and

$$\text{(40)} \qquad \mathrm{V}[N]=\mathrm{E}[N]+\int_{\boldsymbol{R}^2}\lambda_0(z)dz\int_{\boldsymbol{R}^4}\mathrm{E}[\lambda_1(x_1-z)\lambda_1(x_2-z)]c(x_1)c(x_2)dx_1dx_2\,.$$

Here, the expectation in the right-hand side of (39) and the second term of (40) are for patch size Λ_1 and patch radius σ.

5. Concluding remark

We have considered how the variance of sightings of strip or line transect relates the patch size and the patch radius. Fixing the density, the configuration is more inhomogeneous when the patch size is larger and the patch radius is smaller. When the objects are distributed patchily, the variance of sightings is larger than when they are independent one another. The part of variance coming from patchness is proportional to the patch size and decreases with the patch radius. If we combine the distribution of perpendicular distance of sightings, the variance estimate based on the inter-transect variance and the process of the time intervals between sightings, we will be able to obtain the information about the patch condition and the variability of sighting probability. This is left for our future study.

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