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TESTING SIGNIFICANCE OF A MEAN VECTOR—A POSSIBLE ALTERNATIVE TO HOTELLING'S T^2*

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Abstract

In problems involving multivariate measurements experimental considerations often indicate grouping of variables into subsets ordered according to their importance. In such situations, the problems such as comparison of two mean vectors and profile analysis may be treated by Hotelling's T^2-test adapted along the lines of the step-wise procedure of J. Roy [10], or the well known test for additional information due to Rao [9]. In this paper we study a modification of the step-wise procedure obtained by combining the component tests. The exact Bahadur slopes of resulting procedures are computed and it is shown that the procedure based upon Fisher's combination method is asymptotically equivalent to Hotelling's T^2. A Monte Carlo study suggests that even in small samples the power functions of the new method and Hotelling's T^2-test are practically equivalent.

1. Introduction

Hotelling's T^2-test which involves all the variables symmetrically is the most common method of testing multivariate hypotheses such as equality of two mean vectors or similarity of the profiles of two groups. This may not be appropriate if the variables are of unequal importance as in many biological experiments where the measurements are often associated with biological processes and are grouped into subsets which can be ordered according to their biological significance. A grouping of variables in two subsets occurs naturally in most investigations, the first group comprising the variables of primary interest and the second being the group of less relevant variables obtainable

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at little additional cost. A common procedure for testing the hypothesis in such situation is the step-wise procedure related to the well known test for additional information (J. Roy [10], Rao [9]). In this procedure the hypothesis on the means of the variables in the first group is tested by the usual T^2-test, but the hypotheses on the means of the subsequent groups are tested by T^2-tests in which the previous groups are regarded as concomitants. An advantage of this procedure consists in the independence of the T^2-statistics under the overall null hypothesis which makes the control of type I error manageable. Customarily the individual tests are conducted at suitable levels and the overall null hypothesis is rejected if at least one of the component tests is significant. Alternatively, one may summarize the step-wise procedure by reporting the P-values of the component tests and combining (e.g. Oosterhoff [8]) them to obtain the overall significance probability.

In Section 2 we present variations of the step-wise procedure related to several combination methods and discuss an underlying invariance structure leading to a canonical form. In Section 3 the exact slopes of these variations are obtained for computing Bahadur's ARE's. In Section 4, a Monte Carlo study and its conclusions are presented. The modified step-wise procedure based upon Fisher's combination method is seen to be an asymptotic equivalent of Hotelling's T^2. This observation is supported by the simulation study.

2. Some modifications of the step-wise procedure

Let $\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_n$ be n independent observations on a random vector $\boldsymbol{X}$ having a p-variate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, and consider the problem of testing H_0: $\boldsymbol{\mu}=\mathbf{0}$. If $\bar{\boldsymbol{X}}=(1/n)\sum \boldsymbol{X}_i$, $\boldsymbol{S}=\sum(\boldsymbol{X}_i-\bar{\boldsymbol{X}})(\boldsymbol{X}_i-\bar{\boldsymbol{X}})'$, then Hotelling's test rejects H_0 for large values of $T^2=n(n-1)\bar{\boldsymbol{X}}'\boldsymbol{S}^{-1}\bar{\boldsymbol{X}}$, where under H_0, $(n-p)T^2/[p(n-1)]=(n-p)n\bar{\boldsymbol{X}}'\boldsymbol{S}^{-1}\bar{\boldsymbol{X}}/p$ is distributed as F-variable with p and $(n-p)$ degrees of freedom. In case the variables have an a priori order, and T_i^2 denote the Hotelling's T^2-statistics for the first i variates, $i=1, 2, \cdots, p$, then the step-down procedure for MANOVA specialized to this problem consists of p tests based upon statistics

$$F_i=(n-i)[T_i^2-T_{i-1}^2]/[(n-1)+T_{i-1}^2] \tag{2.1}$$

$i=1, 2, \cdots, p$, and rejects H_0 if any of the component tests is significant. The type I error control of the procedure uses the fact that under H_0, F_i are independently distributed as $F(1, n-i)$ variates, $i=1, \cdots, p$.

The logic of the step-down procedure extends as well to the more general and common situation where the variables are grouped into k subsets and the subsets are ordered according to their importance. Let the number of variates in the ith subset be p_i, $q_i=\sum_{j=1}^{i} p_j$, and $q_k=\sum_{i=1}^{k} p_i = p$, $i=1,\cdots,k$. The random vector $\boldsymbol{X}$ and the parameters of its distribution may then be partitioned as

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}_1\\ \boldsymbol{X}_2\\ \vdots\\ \boldsymbol{X}_k\end{bmatrix}, \quad \boldsymbol{\mu}=\begin{bmatrix}\boldsymbol{\mu}_1\\ \boldsymbol{\mu}_2\\ \vdots\\ \boldsymbol{\mu}_k\end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma}=\begin{bmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \cdots & \boldsymbol{\Sigma}_{1k}\\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} & \cdots & \boldsymbol{\Sigma}_{2k}\\ \vdots & \vdots & & \vdots\\ \boldsymbol{\Sigma}_{k1} & \boldsymbol{\Sigma}_{k2} & \cdots & \boldsymbol{\Sigma}_{kk}\end{bmatrix}.$$

Then

$$\text{(2.2)} \qquad \mathrm{E}\,(\boldsymbol{X}_i\,|\,\boldsymbol{X}_1,\cdots,\boldsymbol{X}_{i-1})=\boldsymbol{\theta}_i+\boldsymbol{\beta}_{i1}\boldsymbol{X}_1+\boldsymbol{\beta}_{i2}\boldsymbol{X}_2+\cdots+\boldsymbol{\beta}_{i\,i-1}\boldsymbol{X}_{i-1}$$

where $\boldsymbol{\Sigma}_{i-1}$ denotes the first principal minor containing the first q_{i-1} rows and q_{i-1} columns of $\boldsymbol{\Sigma}$, $\boldsymbol{\beta}_i'=(\boldsymbol{\beta}_{i1}\,\boldsymbol{\beta}_{i2}\cdots\boldsymbol{\beta}_{i\,i-1})=(\boldsymbol{\Sigma}_{i1}\,\boldsymbol{\Sigma}_{i2}\cdots\boldsymbol{\Sigma}_{i\,i-1})\,\boldsymbol{\Sigma}_{i-1}^{-1}$, and $\boldsymbol{\theta}_i=\boldsymbol{\mu}_i-\boldsymbol{\beta}_{i1}\boldsymbol{\mu}_1-\boldsymbol{\beta}_{i2}\boldsymbol{\mu}_2-\cdots-\boldsymbol{\beta}_{i\,i-1}\boldsymbol{\mu}_{i-1}$. It may be noted that H_0: $\boldsymbol{\mu}=\boldsymbol{0}$ is equivalent to the conjunction of H_{0i}: $\boldsymbol{\theta}_i=\boldsymbol{0}$, i.e., $H_0=\bigcap_{i=1}^{k} H_{0i}$. Now, H_{0i}, which may be referred to as the hypothesis concerning the "additional information" provided by the ith subset, can be tested using

$$\text{(2.3)} \qquad F_i=(n-q_i)[T^2_{q_i}-T^2_{q_{i-1}}]/\{[(n-1)+T^2_{q_{i-1}}]p_i\}\ .$$

If $H_0=\bigcap_{i=1}^{k} H_{0i}$ is true then the F_i's are independently distributed as F-variables with $(p_i, n-q_i)$ d.f. The step-wise procedure, in this case consists of k tests with critical regions $F_i \geqq F(p_i, n-q_i; \alpha_i)$, where $F(p_i, n-q_i; \alpha_i)$ denotes the $(1-\alpha_i)$ 100th percentile of the $F(p_i, n-q_i)$ variate, $i=1,\cdots,k$, and rejects H_0 at level $\alpha=1-\prod_{i=1}^{k}(1-\alpha_i)$ if at least one of the tests is significant.

An alternative to comparing F_i with $F(p_i, n-q_i; \alpha_i)$ and labelling it as significant or insignificant is to report the P-value, $P_i=\Pr(F(p_i, n-q_i)\geqq F_i\,|\,H_0)$ associated with it as the summary of the test of H_{0i}. In addition to avoiding the problem of having to select the levels of the component tests, this approach permits an assessment of the overall significance of the data by combining the P-values, because under H_0 the P-values are independently uniformly distributed. A combination statistic is usually a simple function $\psi(P_1,\cdots,P_k)$ of the P-values with a simple null distribution (Oosterhoff [8]). In the sequel we investigate two such statistics $\psi_F=-2\sum_i \log P_i$ and $\psi_T=\min_i P_i$, where large values of ψ_F and small values of ψ_T indicate significance. Under H_0, ψ_F is distributed as χ^2 with $2k$ degrees of freedom and the distribution of ψ_T is given by $\Pr(\psi_T \leqq c | H_0) = 1-(1-c)^k$. A summary of the modified step-down procedure consists of the k P-values together with P-value of the combination statistic.

Now we present an invariance reduction of the problem leading to a canonical form which permits investigation of the properties of the modified step-down procedures. It is well known that Hotelling's T^2 is a maximal invariant under the group G of nonsingular transformation of the p variables, and is UMP invariant for testing the hypothesis $H_0: \boldsymbol{\mu}=\mathbf{0}$ vs. $H_1: \boldsymbol{\mu} \neq \mathbf{0}$. The power of the T^2-test involves only the noncentrality parameter $\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$. If all the variables can be arranged in a strictly decreasing order of importance, it is known that the step-down statistics F_i, $i=1, \cdots, p$ given in (2.1) are maximal invariants under the group of lower triangular transformation of the p variables (Subbaiah and Mudholkar [11]). In the following theorem this invariance reduction is extended to the case of block-structure.

THEOREM 2.1. *Let $\mathcal{T}$ be the group of nonsingular lower block triangular matrices $\boldsymbol{L}=(\boldsymbol{L}_{ij})$, where $\boldsymbol{L}_{ij}$ is of order $p_i \times p_j$ and $\boldsymbol{L}_{ij}=\mathbf{0}$ for $i<j$, $j=1, 2, \cdots, k$. Then the problem of testing $H_0: \boldsymbol{\mu}=\mathbf{0}$ vs. $H_1: \boldsymbol{\mu}\neq\mathbf{0}$ is invariant under transformation $\bar{\boldsymbol{X}} \to \boldsymbol{L}\bar{\boldsymbol{X}}$, $\boldsymbol{S} \to \boldsymbol{L}\boldsymbol{S}\boldsymbol{L}'$, and the step-down statistics F_i defined in* (2.3) *are maximal invariants.*

PROOF. The invariance of $F_1, F_2, \cdots, F_k$ follows trivially from the invariance of $T^2_{q_i}$, $i=1, 2, \cdots, k$. The maximal invariant character of the step-down statistics can be established by a straight forward extension of the proof of Theorem 1 in Subbaiah and Mudholkar [11].

THEOREM 2.2. *The power function of any test of H_0 vs. H_1 invariant under $\mathcal{T}$, depends upon parameters $\delta_i = \boldsymbol{\eta}_i'\boldsymbol{\eta}_i$, $i=1, \cdots, k$, where $\boldsymbol{\eta} = \boldsymbol{B}^{-1}\boldsymbol{\mu} = (\boldsymbol{\eta}_1', \cdots, \boldsymbol{\eta}_k')'$ and $\boldsymbol{B}$ is a lower block triangular matrix such that $\boldsymbol{\Sigma}=\boldsymbol{B}\boldsymbol{B}'$.*

PROOF. The theorem follows from Theorem 2.1, replacing $\bar{\boldsymbol{X}}$ by $\boldsymbol{\mu}$ and $\boldsymbol{S}$ by $\boldsymbol{\Sigma}$, and noting that δ_i's are maximal invariants in the parametric space under the induced group of transformations (Lehmann [5]).

3. Bahadur ARE's of the modified step-down procedures

Let T_n be a statistic used for testing a null hypothesis $H_0: \theta \in \Theta_0$ vs. an alternative $H_1: \theta \in \Theta_1$, where large values of T_n indicate significance. Also let $F_n(t) = \Pr(T_n < t | \theta)$ for all $\theta \in H_0$, $-\infty < t < \infty$. Then the rate of decrease to zero of the P-value $L_n(t_n) = \Pr(T_n \geqq t_n | H_0)$, evaluated at $t_n = T_n$, as n increases is taken as a measure of efficiency of the test. The following Definition 3.1 and Theorem 3.2 summarize the concept of exact slope and a useful method for its computation.

DEFINITION 3.1. The exact slope $c(\theta)$ of $\{T_n\}$ is given by

$$-\frac{1}{2}c(\theta)=\lim_{n\to\infty} n^{-1}\log L_n\ , \tag{3.1}$$

providing that the (a.s.) limit exists.

THEOREM 3.1 (Bahadur [1], p. 27). *Suppose that*

$$\lim_{n\to\infty} n^{-1/2}T_n=b(\theta)\ ,\qquad a.s. \tag{3.2}$$

for each $\theta\in\Theta_1$, *where* $-\infty<b(\theta)<\infty$, *and that*

$$\lim_{n\to\infty} n^{-1}\log[1-F_n(\sqrt{n}\,t)]=-f(t)\ , \tag{3.3}$$

for each t *in an open interval* I, *where* f *is a continuous function on* I, *and* $\{b(\theta):\ \theta\in H_1\}\subset I$. *Then* (3.1) *holds with* $c(\theta)=2f(b(\theta))$ *for each* $\theta\in\Theta_1$.

Now consider the problem of testing the multivariate hypothesis H_0: $\boldsymbol{\mu}=\mathbf{0}$ vs. H_1: $\boldsymbol{\mu}\neq\mathbf{0}$ described in Section 2. The following theorem gives the exact slope of the T^2-test for this problem. The result of this theorem was earlier noted by Gleser ([4], p. 167) with an observation that its " verification is difficult and lengthy ".

THEOREM 3.2. *The exact slope of Hotelling's* T^2 *test is given by*

$$c_H=\log\left(1+\sum_{i=1}^{k}\boldsymbol{\eta}_i'\boldsymbol{\eta}_i\right)\ . \tag{3.4}$$

PROOF. If we denote $T_n=[(n-p)n\bar{\boldsymbol{X}}'\boldsymbol{S}^{-1}\bar{\boldsymbol{X}}/p]^{1/2}$, then $T_n/\sqrt{n}\xrightarrow{\text{a.s.}}(\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}/p)^{1/2}=\left(\sum_{i=1}^{k}\boldsymbol{\eta}_i'\boldsymbol{\eta}_i/p\right)^{1/2}$, and $\lim_{n\to\infty} n^{-1}\log[1-F_n(\sqrt{n}\,t)]=\lim_{n\to\infty} n^{-1}\log[\Pr(n\bar{\boldsymbol{X}}'\boldsymbol{S}^{-1}\bar{\boldsymbol{X}}\geqq npt^2/(n-p)]=(-1/2)\log(1+pt^2)$ (follows from Bahadur ([1], p. 13)). Hence the theorem.

Now we obtain the exact slopes of the modified step-down procedure based upon Fisher's and Tippett's methods.

LEMMA 3.1. *The exact slope of the ith component test is given by*

$$c_i(\theta)=\log\left(1+\frac{\boldsymbol{\eta}_i'\boldsymbol{\eta}_i}{1+\sum_{j=1}^{i-1}\boldsymbol{\eta}_j'\boldsymbol{\eta}_j}\right)\ . \tag{3.5}$$

PROOF. The theorem follows easily by noting that for $T_n=\sqrt{F_i(n)}$,

$$\lim_{n\to\infty} n^{-1/2}T_n=\left\{\boldsymbol{\eta}_i'\boldsymbol{\eta}_i\Big/\left[\left(1+\sum_{j=1}^{i-1}\boldsymbol{\eta}_j'\boldsymbol{\eta}_j\right)p_i\right]\right\}^{1/2}\ ,\qquad \text{a.s.}$$

and

$$\begin{aligned}\lim_{n\to\infty} n^{-1}\log[1-F_n(\sqrt{n}\,t)] &= \lim_{n\to\infty} n^{-1}\log\Pr(F_i \geqq nt^2)\\ &= \lim_{n\to\infty} n^{-1}\log\Pr\left[\frac{\chi^2(p_i)}{\chi^2(n-q_i)} \geqq \frac{nt^2p_i}{(n-q_i)}\right]\\ &= -\frac{1}{2}\log(1+t^2p_i)\end{aligned}$$

(follows from Bahadur ([1], p. 13, equation 5.7)).

THEOREM 3.3. *The exact slopes c_F, c_T of Fisher's* [3] *method and Tippett's* [12] *method of combining the step-down P_i's are given by*

$$c_F = \sum_{i=1}^{k} c_i = \log\left(1+\sum_{i=1}^{k} \eta_i'\eta_i\right)$$

$$c_T = \max_i c_i = \max_i \log\left[1+\frac{\eta_i'\eta_i}{1+\sum_{j=1}^{i-1}\eta_j'\eta_j}\right].$$

PROOF. This theorem follows from the results due to Littell and Folks ([6], pp. 803–804).

A comparison of Theorem 3.3 and Theorem 3.5 shows that the modified step-down procedure based upon Fisher's method of combination of tests is an asymptotic equivalent of Hotelling's T^2 in the sense of Bahadur ARE. On the other hand, in this sense the modified step-down procedure based upon Tippett's combination method is in general less effective and never more effective than the T^2-test.

4. Power functions of the modified step-down procedure based upon a simulation study

In this section we summarize a simulation experiment conducted in order to understand the moderate-size sample behavior of the modified step-wise methods in relation to Hotelling's T^2. In this experiment we study the special case $p_1=p_2=\cdots=p_k=1$. Our objective is to obtain a relatively detailed profile of the power function when $p=2$, and an indication of its general behavior in certain directions when $p=3, 4$.

In view of the invariance structure in the problem we take without any loss of generality $\boldsymbol{\Sigma}=\boldsymbol{I}$, in which case the power functions of the modified step-down procedures as well as that of Hotelling's T^2-test depend only upon the noncentrality parameters $\eta_i^2=\mu_i^2$, $i=1, 2, \cdots, p$.

For the case $p=2$, the power functions of the methods are estimated over the entire plane (μ_1, μ_2), whereas for $p=3, 4$, the estimates are obtained for certain directions only, namely, the equiangular line $\mu_1=\mu_2=\cdots=\mu_p$ and along a coordinate axis i.e., for alternatives $(\mu, 0, \cdots, 0)$.

Monte Carlo experiment. The standard normal deviates are generated on the IBM 360/365 computer at the University of Rochester using "McGill University random number package" based upon the technique of Marsaglia [7] for generating standard normal deviates. A random observation from a p-variate normal population $N_p(\boldsymbol{\mu}, \boldsymbol{I}_p)$ is obtained by drawing p random observations from a standard univariate normal population and adding $\mu_1, \mu_2, \cdots, \mu_p$ to them respectively. When $p=2$, the deviates are generated for the values of $\mu_1=0.0(0.1)1.6$ and $\mu_2=0.0(0.1)1.9$. When $p=3, 4$, they are obtained for $(\mu, \mu, \cdots, \mu)$ and $(\mu, 0, \cdots, 0)$, for $\mu=0.0(0.1)1.0$.

For each of the parameter values 3000 samples of size $n=20$ are obtained, and are then used to estimate the power functions of various tests of $H_0: \boldsymbol{\mu}=\boldsymbol{0}$. Specifically, for each of the samples we compute Hotelling's T^2 statistic ψ_F and ψ_T the statistics for the modified step-down procedures related to Fisher's and Tippett's combination methods. The IMSL routine MDFD is used for obtaining the P-values needed in the computation of ψ_T, ψ_F. The values of the statistics for each sample are compared with the corresponding critical constants for $\alpha=.01$, .05 and .10. The power of a procedure with a given $\boldsymbol{\mu}$ and α is estimated by the proportion $\hat{p}$ of times H_0 is rejected in the 3000 trials; the s.e. of the estimate being $(\hat{p}(1-\hat{p})/3000)^{1/2}\leqq .009$. The exact power of Hotelling's T^2 is also computed using the FORTRAN routine by Bargmann and Ghosh [2] for computing the c.d.f. of noncentral F distribution.

Results. A selection of the results of the simulation study is given in Tables 1 through 5. Tables 1, 2, 3 contain a relatively detailed profile of the power functions for $p=2$. In Tables 4, 5 we give the empirical power functions of the three tests and the exact power of the

Table 1 Exact power function of Hotelling's T^2 test procedure

		$\alpha=.05$, $p=2$							
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	.995	.996	.996	.999	1.00	1.00	1.00	1.00
	1.0	.965	.966	.970	.989	.998	1.00	1.00	1.00
	0.8	.845	.851	.867	.943	.990	.998	1.00	1.00
	0.5	.436	.451	.495	.742	.943	.989	.999	1.00
μ_2	0.2	.105	.118	.165	.495	.867	.970	.996	1.00
	0.1	.063	.076	.119	.451	.851	.966	.996	1.00
	0.0	.050	.063	.105	.436	.845	.965	.995	1.00
		0.0	0.1	0.2	0.5	0.8	1.0	1.2	1.5
					μ_1				

Table 2 Power function of Fisher's combination of the step-down tests estimated from the Monte Carlo experiment*

$\alpha=.05,\ p=2$

μ_2								
1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	.994	.993	.995	1.00	1.00	1.00	1.00	1.00
1.0	.954	.960	.966	.988	.999	1.00	1.00	1.00
0.8	.824	.838	.848	.947	.991	.999	1.00	1.00
0.5	.416	.441	.467	.764	.945	.989	.998	1.00
0.2	.100	.112	.168	.510	.864	.965	.996	1.00
0.1	.064	.087	.115	.422	.838	.964	.995	1.00
0.0	.050	.063	.116	.434	.842	.963	.997	1.00
	0.0	0.1	0.2	0.5	0.8	1.0	1.2	1.5
				μ_1				

* Each estimate is based upon 3000 trials.

Table 3 Power function of Tippett's combination of the step-down tests estimated from the Monte Carlo experiment*

$\alpha=.05,\ p=2$

μ_2								
1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	.997	.996	.993	.997	.998	1.00	1.00	1.00
1.0	.966	.969	.964	.969	.990	.998	1.00	1.00
0.8	.848	.849	.850	.890	.970	.993	.999	1.00
0.5	.441	.446	.453	.674	.916	.981	.998	1.00
0.2	.101	.114	.139	.474	.884	.969	.997	1.00
0.1	.063	.078	.109	.435	.856	.974	.997	1.00
0.0	.052	.060	.106	.457	.869	.975	.998	1.00
	0.0	0.1	0.2	0.5	0.8	1.0	1.2	1.5
				μ_1				

* Each estimate is based upon 3000 trials.

Table 4 Power functions of T^2-test and modified step-down tests*

$\alpha=.05,\ p=3$

Configuration	Tests	μ						
		0.0	0.1	0.2	0.4	0.6	0.8	1.0
$\begin{pmatrix}\mu\\ \mu\\ \mu\end{pmatrix}$	Fisher	.050	.079	.195	.645	.953	.998	1.00
	Tippett	.046	.082	.174	.496	.844	.982	.999
	T^2	.049	.078	.192	.629	.948	.998	1.00
	T^2-exact	.050	.080	.186	.631	.948	.998	1.00
$\begin{pmatrix}\mu\\ 0\\ 0\end{pmatrix}$	Fisher	.052	.059	.095	.239	.510	.763	.917
	Tippett	.048	.056	.090	.254	.559	.828	.958
	T^2	.050	.058	.091	.234	.510	.772	.926
	T^2-exact	.050	.060	.091	.238	.498	.768	.931

* Each estimate is based upon 3000 trials.

T^2 test for $p=3, 4$, and $\alpha=.05$ corresponding to the two configurations in the parametric space, viz., the equiangular configuration $(\mu, \mu, \cdots, \mu)$ and the extreme configuration $(\mu, 0, \cdots, 0)$. The results of the study seem generally supportive of the conclusions drawn from Theorems 3.3

Table 5 Power functions of T^2-test and modified step-down tests*

$\alpha = .05, \quad p = 4$

Configuration	Tests	μ						
		0.0	0.1	0.2	0.4	0.6	0.8	1.0
$(\mu, \mu, \mu, \mu)'$	Fisher	.045	.086	.201	.735	.979	1.00	1.00
	Tippett	.047	.084	.164	.508	.837	.982	.999
	T^2	.046	.085	.197	.712	.973	.999	1.00
	T^2-exact	.050	.082	.202	.695	.973	1.00	1.00
$(\mu, 0, 0, 0)'$	Fisher	.053	.064	.096	.209	.436	.694	.880
	Tippett	.048	.060	.084	.229	.513	.779	.945
	T^2	.055	.061	.090	.203	.427	.696	.885
	T^2-exact	.050	.058	.082	.202	.428	.695	.888

* Each estimate is based upon 3000 trials.

and 3.5. Specifically, (1) the power functions of the T^2-test and the modified step-wise test based on Fisher's method appear indistinguishable for the cases simulated, (2) the modified test based upon Tippett's method seems to have an advantage over the T^2-test along the coordinate axis. Along the equiangular line the T^2-test dominates it.

Conclusion. If there is an a priori ordering among the response variables, then the step-down procedure is a reasonable alternative to Hotelling's T^2-test. The results of the component tests of this alternative may be summarized in terms of their P-values instead of accepting or rejecting the step-down hypotheses at predetermined levels. In the modified step-down procedure consisting of combining these P-values in addition to considering them individually, the problem of distributing the type I error among component tests is avoided. If Fisher's method is used for combining the P-values, then the analytical and the Monte Carlo results in this paper suggest that the overall test of the modified step-down method is as good as Hotelling's T^2-test.

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