NOTE ON AN INEQUALITY FOR TACTICAL CONFIGURATIONS

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1. Introduction and summary

Raghavarao [3] proved that if b is the number of blocks in a t-(v, k, λ_t) design with $k\neq v-1$, then $b\geqq(t-1)(v-t+2)$. Furthermore, Dey and Saha [1] has recently shown that for a t-(v, k, λ_t) design with $v\geqq k+t-1$, an inequality $b\geqq 2^{t-2}(v-t+2)$ holds. Moreover, they stated that for $t>3$, whenever $v\geqq k+t-1$[2)], Dey and Saha's inequality is an improvement of Raghavarao's one, and that when $v<k+t-1$, Raghavarao's inequality appears to be the best.

In this note we enlighten the existence of a more stringent inequality than Dey and Saha's one in almost all cases, and make a comparison of these inequalities from a combinatorial point of view of a design.

2. Statement

Wilson and Ray-Chaudhuri [4] demonstrated that for a t-(v, k, λ_t) design with $t=2s$ and $v\geqq k+s$, an inequality $b\geqq\binom{v}{s}$ holds. Furthermore, it is easily seen (cf. [2]) that for a t-(v, k, λ_t) design with $t=2s+1$ and $v\geqq k+s$, an inequality $b\geqq(v-s)\binom{v}{s}\Big/k$ holds. Then we can prove the following:

THEOREM. *For a t-(v, k, λ_t) design with $v\geqq k+t-1$, if $t=2s$, then*

$$b\geqq\binom{v}{s}\geqq 2^{2(s-1)}(v-2s+2)\ , \tag{1}$$

and if $t=2s+1$, then

$$b\geqq\max\left\{\frac{v-s}{k}\binom{v}{s},\ 2^{2s-1}(v-2s+1)\right\}\ . \tag{2}$$

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2) In the paper of Dey and Saha [1], the equality sign of $v\geqq k+t-1$ is carelessly omitted.

Consider $\binom{v}{s}-2^{2(s-1)}(v-2s+2)=\{v(v-1)\cdots(v-s+1)-2^{2(s-1)}(v-2s+2)s!\}/s!$. Some combinatorial calculations lead to $\{v(v-1)\cdots(v-s+1)-2^{2(s-1)}(v-2s+2)s!\}\geqq 0$. Since $v\geqq k+t-1\geqq k+s$, (1) follows from Wilson and Ray-Chaudhuri's inequality [4]. As examples of (2), take a 3-(10, 6, 5) design [2] and a 3-(17, 5, 1) design [2]. The inequalities, $b\geqq(v-s)\cdot\binom{v}{s}\Big/k$ and $b\geqq 2^{2s-1}(v-2s+1)$ become $30\geqq 15$ and $30\geqq 18$, respectively, for the former design, and $68\geqq 54.5$ and $68\geqq 32$, respectively, for the latter design. A 3-(20, 10, 4) design [2] attains the same value for the both bounds of (2). Note that the relation $b\geqq(v-s)\binom{v}{s}\Big/k\geqq 2^{2s-1}(v-2s+1)$ holds for almost all $(2s+1)$-(v, k, λ_{2s+1}) designs.

Thus, for a $2s$-design, Wilson and Ray-Chaudhuri's inequality is more stringent than Dey and Saha's one. Furthermore, in the statement, "when $v<k+t-1$, Raghavarao's inequality appears to be the best" ([1]), the condition $v<k+t-1$ should be changed into $k+2\leqq v<k+s$, since when $v\geqq k+s$ for $t=2s$ or $t=2s+1$, Wilson and Ray-Chaudhuri's inequality is more stringent than Raghavarao's one [2]. Note that since no non-trivial t-designs are known for $t\geqq 6$, the range, $k+2\leqq v<k+s$, may be essentially meaningless as yet. Further note that when $k+s\leqq v<k+t-1$, Wilson and Ray-Chaudhuri's bound is the most stringent among the inequalities described above.

For a t-(v, k, λ_t) design, when there exists the divisibility between v and k, or when there exists one block which appears many times, or when $\lambda_t=1$, we can have further improvements on the above inequalities. However, when there are no restrictions described above, we believe the inequalities given in Theorem to be the best for $t\geqq 3$. Furthermore, similar discussions for $t=2$ will appear in a later paper.

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