ON A CLASS OF RANK SCORES TESTS FOR CENSORED DATA*

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Summary

It is well known that one-sample or c-sample (location) problems are special cases of the general linear regression model $Y_i=\beta_1 x_{1i}+\cdots+\beta_k x_{ki}+\varepsilon_i$, where we wish to test the hypothesis $H: \beta_1=\cdots=\beta_q=0$, $q\leqq k$. This problem has been considered by Hájek [5] and Srivastava [13], [14], and a class of asymptotically most powerful rank score tests has been proposed. In this paper, the above problem of testing H against a sequence of alternatives tending to H at a suitable rate has been considered for the *censored* data, i.e., when only the first r-ordered observations are available. A class of rank score tests has been proposed. It has been shown that the proposed test is superior to those proposed by Gastwirth [6], Sobel [10], [11] and Basu [1], [2], [3] in the sense defined in Section 4; no large sample comparison with Rao, Savage and Sobel [8] statistic is possible since its asymptotic distribution is not known.

The c-sample problem as a special case of the regression model has been considered in Section 3. In this case, however, the design matrix X_r becomes a random variable.

1. Introduction

Let $Y_1, Y_2, \cdots, Y_n$, $n\geqq k$ be n independent observations with

$$Y_n'=\beta' X_n+\varepsilon_n' , \tag{1.1}$$

where $Y_n'=(Y_1,\cdots,Y_n)$ is a row n-vector, β a k-vector, and $X_n=(x^{(1)},\cdots,x^{(n)})$, a $k\times n$, $k\leqq n$, matrix of known constants (design matrix); we assume that X_k is of full rank, so that e.g. $x^{(1)},\cdots,x^{(k)}$ can be taken as linearly independent vectors. $\varepsilon_n'=(\varepsilon_1,\cdots,\varepsilon_n)$ where the random variable ε_i obeys an unknown distribution function F such that

$$P_\beta(Y_i\leqq y)=P_\beta(\varepsilon_i\leqq y-\beta' x^{(i)})=F(y-\beta' x^{(i)})$$

where P_β denotes that the probability is being computed for the param-

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eter value $\boldsymbol{\beta}$. The form of F is not known but we shall assume that $F \in \mathcal{F}$ where

$$\text{(C1)} \qquad \mathcal{F} = \Big\{ \text{absolutely continuous } F: \\ \text{(i) } F'(x) = f(x) \text{ is absolutely continuous,} \\ \text{(ii) } \int_{-\infty}^{\infty} [f'(x)/f(x)]^2 f(x) dx < \infty \Big\} .$$

We will refer in the sequel to the above conditions on the class $\mathcal{F}$ of distribution functions as (C1). It may be noted that (i) and (ii) imply that

$$\int_{-\infty}^{\infty} f'(x) dx = 0 .$$

Let

$$\boldsymbol{\beta}' = (\boldsymbol{\beta}_1', \boldsymbol{\beta}_2') , \qquad \boldsymbol{\beta}_1' = (\beta_1, \cdots, \beta_q), \ \boldsymbol{\beta}_2' = (\beta_{q+1}, \cdots, \beta_k)$$

$$\text{(1.3)} \qquad X_n = ((x_{ij})) = (\boldsymbol{x}^{(1)}, \cdots, \boldsymbol{x}^{(n)}) = \begin{pmatrix} \boldsymbol{x}_1^{(1)}, \boldsymbol{x}_1^{(2)}, \cdots, \boldsymbol{x}_1^{(n)} \\ \boldsymbol{x}_2^{(1)}, \boldsymbol{x}_2^{(2)}, \cdots, \boldsymbol{x}_2^{(n)} \end{pmatrix} = \begin{pmatrix} X_n^{(1)} \\ X_n^{(2)} \end{pmatrix}$$

where $\boldsymbol{x}_1^{(i)}$'s are q-vectors and $\boldsymbol{x}_2^{(i)}$'s are $(k-q)$-vectors. Let R_i be the rank of Y_i in the ordered sample $V_1 < V_2 < \cdots < V_n$ i.e.,

$$\text{(1.4)} \qquad Y_i = V_{R_i} , \qquad 1 \leqq i \leqq n .$$

The problem of testing $H: \boldsymbol{\beta}_1 = \mathbf{0}$ when all the n observations are available has been considered by Srivastava [13], [14].

Sometimes we cannot observe the entire ordered sample $(V_1, \cdots, V_n)$ but only the first r $(<n)$, $r > k$. We call such data observation censored (right-censored) because the censoring point is determined by the sample—the rth largest observation. We assume that r/n approaches p $(0<p<1)$ as r and n tend to infinity.

The object of this paper is twofold. First to give a unified approach with the help of regression model for testing the hypothesis $H: \boldsymbol{\beta}_1 = \mathbf{0}$ for the above problem of censored data. From this one-sample and c-sample results follow as corollaries. Secondly to show that the efficiency is lost by giving any weight to the truncated portion unless p is small. Specifically we show that if $p \geqq 1/3$, the test proposed in this paper is superior to those proposed by Sobel [11], Basu [1], [2], [3] and Gastwirth [6] contrary to Gastwirth's claim that his is an asymptotically most powerful test. Thus the title and Theorem 3.1 of Gastwirth's paper appear misleading. Actually his Theorem 3.1 should be read in conjunction with his conditions (a), (b) and (c) given just above the theorem which require that a constant weight must be given to the unobserved data. This criteria is indeed arbitrary and one cannot hope to get asymptotically most powerful test in the general case.

It should be pointed out that the unobserved data does contain an information, the observed observations are smaller than the unobserved ones, and the loss of efficiency should be expected if one does not use this information. However, if this information is not properly used and is given a lot of weight, this will also lead to inefficiency as is the case with the results of Gastwirth. It may be pointed out that the proposed tests of this paper are not claimed to be asymptotically most powerful rank scores tests; the same should be true of the results of Gastwirth and Basu.

In order to find the asymptotic distribution of the proposed test statistic, we require Condition (C2)—that the maximum (in magnitude) of the elements in $T_r^{-1}X_r \to 0$ as $r \to \infty$, where T_r is the *unique* $k \times k$ *upper* triangular matrix such that

$$X_r = T_r L_r = \begin{pmatrix} T_r^{(1)} & T_r^{(12)} \\ 0 & T_r^{(2)} \end{pmatrix} \begin{pmatrix} L_r^{(1)} \\ L_r^{(2)} \end{pmatrix}, \tag{1.5}$$

where

$$L_r = ((l_{ij}(r))) = (\boldsymbol{l}_r^{(1)}, \cdots, \boldsymbol{l}_r^{(r)}) = T_r^{-1} X_r \tag{1.6}$$

is a $k \times r$, $k \leqq r$, semi-orthogonal matrix, $L_r L_r' = I_k$. $L_r^{(1)}$ and $L_2^{(2)}$ are semi-orthogonal matrices, orthogonal to each other;

$$L_r^{(1)} L_r^{(1)'} = I_q \,, \quad L_r^{(2)} L_r^{(2)'} = I_{k-q} \,, \quad L_r^{(1)} L_r^{(2)'} = 0 \,, \quad L_r^{(2)} L_r^{(1)'} = 0 \,.$$

The condition (C2) is equivalent to

$$\lim_{r \to \infty} \max_{1 \leqq i \leqq r} \boldsymbol{l}_r^{(i)'} \boldsymbol{l}_r^{(i)} = 0 \,. \tag{C2}$$

It may be pointed out that the (C2) condition is weaker than Hájek's [5] conditions (cf. Srivastava [12], [13], [14]).

2. Proposed Tests

Let

$$\phi^*(u) = -[g'(G^{-1}(u))/g(G^{-1}(u))] \,, \qquad 0 < u < 1 \tag{2.1}$$

where G^{-1} is the inverse of G and G (known) is the truncated (at $X_0 = G^{-1}(p)$) distribution function belonging to the class $\mathcal{F}$ (satisfying condition (C1)). The (2.1) function that corresponds to the truncated (unknown, the true distribution) distribution F (at $X_0^* = F^{-1}(p)$) is

(2.2) $$\phi^*(u) = -[f'(F^{-1}(u))/f(F^{-1}(u))] , \qquad 0<u<1 .$$

ψ^* is used in constructing rank scores tests and ϕ^* comes in the efficiency expression which depends on the unknown underlying distribution function F. We will consider only those $\mathcal{F}$ for which $\psi^*(u)$ and $\phi^*(u)$ are non-decreasing functions of u. We will refer to this condition in the sequel as condition (C3). It may be noted that the condition (C3) of monotonicity of ψ^* and ϕ^* is satisfied if the density functions f and g are strongly unimodal density functions. The two most commonly used scores functions Wilcoxon scores and Normal scores satisfy the condition (C3).

Also, we will consider the normalized (usually by adding a constant) $\psi^*(u)$ and $\phi^*(u)$ say $\psi(u)$ and $\phi(u)$ respectively such that

(2.3) $$0=\int_0^1 \psi(u)du=\int_0^1 \phi(u)du .$$

From (C1) it follows that

(2.4) $$\gamma^2=\int_0^1 \psi^2(u)du<\infty ,$$

(2.5) $$\gamma^{*2}=\int_0^1 \phi^2(u)du<\infty .$$

Consider a vector of statistics defined by

(2.6) $$Z_r'=(Z_1^{(r)},\cdots, Z_r^{(r)})=[\psi_r(R_1/r+1),\cdots, \psi_r(R_r/r+1)] ,$$

where R_i is the rank of Y_i as defined in (1.4),

(2.7) $$\psi_r(u)=\psi(j/r+1) , \qquad (j-1)/r<u\leqq j/r$$

and

(2.8) $$\lim_{r\to\infty} \int_0^1 [\psi_r(u)-\psi(u)]^2du=0$$

by Hájek [4] and condition (C3). Define

(2.9) $$\bar{M}_r=Z_r' L_r^{(1)\prime} L_r^{(1)} Z_r/\gamma^2$$

we propose $\bar{M}_r(\psi)$ as the class of test statistics for the hypothesis H: $\beta_1=\cdots=\beta_q=0$, $q\leqq k$ in the linear regression model (1.1a).

To every $G\in\mathcal{F}$, corresponds one test statistic $\bar{M}_r$. In particular if G is the truncated logistic distribution

$$G(x)=(1/p)\{1+e^{-c(x+k)}\}^{-1} , \qquad -\infty<x<X_0 ,$$

$X_0=-(1/c)\ln((1-p)/p)-k$, then $\psi(u)=2pc(u-1/2)$, which is the normalized $\psi^*(u)$. Following as in Srivastava [12], [13], it can be shown that under conditions (C1)–(C3)

$$\lim_{r\to\infty} \mathrm{P}_0\{\bar{M}_r \leqq y\} = \mathrm{P}\{\chi_q^2 \leqq y\} \tag{2.10}$$

where P_0 denotes that the probability is computed under $H: \beta_1 = \cdots = \beta_q = 0$ and χ_q^2 denotes the chi-square random variable with q degrees of freedom. $\bar{M}_r^{(n)}$ has a non-central chi-square χ_q^2 under the sequence of alternatives

$$\begin{aligned} K_r : \boldsymbol{\beta}_{1r}' &= r^{-1/2}(r^{-\alpha_1}b_1, \cdots, r^{-\alpha_q}b_q) \\ &= r^{-1/2}(b_1^{(r)}, \cdots, b_q^{(r)}) , \qquad \text{say} \\ &= r^{-1/2}b^{(r)\prime} , \end{aligned} \tag{2.11}$$

where $b_1, b_2, \cdots, b_q$ are real constants and $\max_{1\leqq j\leqq r} |x_{ij}| = O(r^{\alpha_i})$, $\alpha_i \geqq 0$. The non-centrality parameter is

$$\delta^2 = \lim_{r\to\infty} \boldsymbol{\beta}_{1r}'(T_r^{(1)}T_r^{(1)\prime})\boldsymbol{\beta}_{1r}B^2 \tag{2.12}$$

where

$$B^2 = p^2\left[\int_0^1 \phi(u)\psi(u)du\right]^2 \Big/ \left(\int_0^1 \psi^2(u)du\right) . \tag{2.13}$$

3. c-sample problem

Let

$$F_i(x) = F(x-\theta_i) , \qquad i = 1, 2, \cdots, c , \tag{3.1}$$

and let a sample of size n_i be taken from F_i. Let

$$\begin{aligned} n &= \sum_{i=1}^{c} n_i \\ \boldsymbol{\beta}_1' &= (\theta_1-\theta_c, \cdots, \theta_{c-1}-\theta_c) \\ \beta_2 &= \theta_c . \end{aligned} \tag{3.2}$$

On the basis of the first r ordered observations out of n, ordered from smallest to largest, we wish to test the hypothesis $H: \boldsymbol{\beta}_1 = \mathbf{0}$, which is equivalent to the hypothesis $H: \theta_1 = \cdots = \theta_c$. Let $n_i^{(r)}$ be the number of observations from the ith sample among the first r ordered observations. $n_i^{(r)}$, $i = 1, 2, \cdots, c-1$ are random variables with (under H as well as under K_r)

$$\begin{aligned} &\mathrm{E}(n_i^{(r)}) = rn_i/n \\ &\mathrm{Var}(n_i^{(r)}) = r(n_i/n)(1-n_i/n)(1-r(n-1)^{-1}) \\ &\mathrm{Cov}(n_i^{(r)}, n_j^{(r)}) = -r(n_i/n)(n_j/n)(1-r(n-1)^{-1}) . \end{aligned} \tag{3.3}$$

Let n_i, n and r tend to infinity in such a way that

$$\lim_{n\to\infty} r/n=p>0\ , \qquad \lim_{n\to\infty}(n_i/n)=\lambda_i\ , \tag{3.4}$$

where $0<\lambda_0\leqq\lambda_i\leqq 1-\lambda_0<1, i=1,2,\cdots,c$ and λ_0 is a constant not greater than $1/c$. Then the random variable $(n_1^{(r)}/r,\cdots,n_{c-1}^{(r)}/r)$ tends to a degenerate distribution at $(\lambda_1,\cdots,\lambda_{c-1})$. Let

$$X_r=((x_{i\alpha})) \tag{3.5}$$

where $x_{i\alpha}=1$ if $n_1^{(r)}+\cdots+n_{i-1}^{(r)}<\alpha\leqq n_1^{(r)}+\cdots+n_i^{(r)}$ and $x_{i\alpha}=0$ otherwise for $i=1,2,\cdots,c-1$, and $x_{c\alpha}=1$ for all α. Conventionally $n_0=0$. Hence

$$X_rX_r'=\begin{pmatrix} n_1^{(r)} & 0 & \cdots & 0 & n_1^{(r)} \\ 0 & n_2^{(r)} & \cdots & 0 & n_2^{(r)} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & n_{c-1}^{(r)} & n_{c-1}^{(r)} \\ n_1^{(r)} & n_2^{(r)} & \cdots & n_{c-1}^{(r)} & r \end{pmatrix} \tag{3.6}$$

and

$$r^{-1}X_rX_r'\to\begin{pmatrix} \lambda_1 & & 0 & \lambda_1 \\ & \ddots & & \vdots \\ 0 & & \lambda_{c-1} & \lambda_{c-1} \\ \lambda_1 & \cdots & \lambda_{c-1} & 1 \end{pmatrix} \tag{3.7}$$

in probability. Consequently the test statistic $\bar{M}_r$ has a chi-square distribution with $c-1$ degrees of freedom under the hypothesis H and non-central chi-square under the alternative (2.11) with the non-centrality parameter

$$\delta^2=\left[\sum_{i=1}^{c-1}\lambda_i b_i^2-(\sum\lambda_i b_i)^2\right]B^2\ . \tag{3.8}$$

4. Comparison with other tests (two-sample case)

Gastwirth [6] proposed a class of rank score statistics for the two-sample problem. His weight function for the Wilcoxon type test is

$$\phi_G(u)=\begin{cases} u-1/2\ , & 0\leqq u\leqq p \\ p/2\ , & p<u\leqq 1\ . \end{cases} \tag{4.1}$$

Sobel [10], [11] proposed two tests of Wilcoxon type. Basu [1], [2] recently considered one of the Sobel's statistic, and showed that the other Sobel's statistic is asymptotically equivalent to Gastwirth [6] statistic. Thus, we need to compare the proposed procedure with Basu and Gastwirth statistics. Basu's [1], [2] weight function is given by

$$\phi_B(u)=\begin{cases} u-p+p^2/2\ , & 0\leqq u\leqq p\ , \\ p^2/2\ , & p<u\leqq 1\ . \end{cases} \tag{4.2}$$

The weight function for the test proposed in this paper is given by

$$\phi(u)=2cp(u-1/2)\ , \qquad 0\leqq u\leqq 1\ . \tag{4.3}$$

In order to see how the weight functions (4.1) and (4.2) arise and how many more can be a candidate, we consider the following density function.

$$h(x)=\begin{cases} dce^{-c(x+k)}/[1+e^{-c(x+k)}]^2\ , & -\infty\leqq x\leqq X_0 \\ c^*e^{-c^*x}\ , & x>X_0\ , \end{cases} \tag{4.4}$$

where

$$d\geqq p \quad \text{and} \quad d\to 1 \quad \text{as} \quad p\to 1\ , \tag{4.5}$$

and

$$X_0=H^{-1}(p)=-(1/c)\ln((d-p)/p)-k\ . \tag{4.6}$$

It thus follows that

$$k=-(1/c)\ln((d-p)/p)+(1/c^*)\ln(1-p)\ , \tag{4.7}$$

and

$$\phi(u)=-\frac{h'(H^{-1}(u))}{h(H^{-1}(u))}=\begin{cases} (c/d)(2u-d)\ , & 0\leqq u\leqq p \\ c^*\ , & p<u<1\ . \end{cases} \tag{4.8}$$

We assume, without any loss of generality, that

$$\int_0^1 \phi(u)du=(c/d)(p^2-dp)+c^*(1-p)=0\ . \tag{4.9}$$

The density (4.4) involves four constants k, c, c^* and d. There are only two equations (4.7) and (4.9) and a restriction (4.6) to obtain them. Thus there can be infinitely many candidates as a weight function. For example if $d=1$, we get from (4.9)

$$c^*=pc\ . \tag{4.10}$$

If we assume further that $c=1/2$, we get Gastwirth [6] weight function (4.1). If $d=2(p-p^2/2)=p(2-p)$, we get from (4.9)

$$c^*=(c/d)p^2\ . \tag{4.11}$$

If we assume further that $2c=d$, we get Basu's weight function (4.2). The weight function (4.2) gives an impression that it does not depend upon c, the scale parameter. That it is so follows from (4.6) with $d=2c$; the restriction is $2c>p$. As is apparent the two weight functions (4.1) and (4.2) are obtained from the density function (4.4) by specializing on c. However Basu's weight function (4.2) gives less weight to the truncated portion as compared to the Gastwirth's weight function (4.1) and hence is likely to be more efficient than the latter. That this is so follows from (4.25) and (4.26).

The weight function (4.3) gives *zero* weight to the truncated portion, and hence is likely to be more efficient unless p is small. It may be remarked that although the weight function (4.3) appears to be giving some weight to the truncated portion this is *not* so. The efficiency is gained by redistributing the weights over the observed data. Naturally one cannot score from unobserved data, and any weight given to them would lead to inefficiency. This is precisely what is happening with the weight functions (4.1) and (4.2).

It appears that the multitudes of weight functions arise from the introduction of exponential density of *arbitrary* scale c^* in the truncated portion. The bigger the c^*, the less efficient the test should be expected. One way to obliterate this difficulty is to consider tests of the form proposed in this paper, i.e., taking

$$h(x)=\begin{cases}(1/p)ce^{-c(x+k)}/[1+e^{-c(x+k)}]^2\,, & -\infty<x\leqq X_0\\ (1/p)g(x)\,, & \text{say}\end{cases} \tag{4.12}$$

where

$$X_0=-(1/c)\ln((1-p)/p)-k=G^{-1}(p)\,. \tag{4.13}$$

This gives

$$\phi^*(u)=-\frac{h'[H^{-1}(u)]}{h[H^{-1}(u)]}=2c\left[pu-\frac{1}{2}\right]. \tag{4.14}$$

Without loss of generality we can normalize ϕ^* such that $\int_0^1\phi^*(u)du=0$. This gives us a normalized

$$\phi(u)=2pc[u-1/2]\,,\qquad 0\leqq u\leqq 1\,. \tag{4.15}$$

Hence

$$\int_0^1\phi^2(u)du=4p^2c^2/12\,. \tag{4.16}$$

As in Basu [1], [2] and Gastwirth [6], we assume that

$$\lim_{u\to 1 \text{ or } 0}\phi(u)f(F^{-1}(u))=0\,,$$

and $0<\lambda_0\leqq\lambda_i\leqq 1-\lambda_0<1$ for some $\lambda_0\leqq 1/2$ where

$$\lambda_i=\lim_{n\to\infty}\frac{n_i}{n}\,,\qquad n=n_1+n_2,\ i=1,2\,. \tag{4.17}$$

From (2.9) and (3.6) we find that the test statistic can be taken as

$$T_r=(1/n_1^{(r)})\sum_{i=1}^{n_1^{(r)}} Z_i^{(r)}, \tag{4.18}$$

where $n_1^{(r)}$ is the number of observations from the first sample among the first r ordered observations. Given $n_1^{(r)}$, T_r is conditionally asymptotically normally distributed with mean

$$\mu_r=\beta_r(n_2^{(r)}/r)\left(2c\int_0^{F^{-1}(p)} f^2(x)dx\right), \qquad \beta_r\to 0, \tag{4.19}$$

(β_r is the difference of the means, $\beta_r=r^{-1/2}b$), and variance

$$\sigma_r^2=(n_2^{(r)}/rn_1^{(r)})\int_0^1 \phi^2(u)du, \tag{4.20}$$

where f is any arbitrary non-truncated density. Hence, unconditionally (see Section 3), T_r is asymptotically normally distributed with mean

$$\mu_n=2p^{-1/2}c\beta_n(1-\lambda)\int_0^{F^{-1}(p)} f^2(x)dx, \tag{4.21}$$

and variance given by

$$\begin{aligned}\sigma_n^2&=\mathrm{E}\left(\frac{n_2^{(r)}}{rn_1^{(r)}}\right)\int_0^1\phi^2(u)du+\mathrm{Var}\,[\mathrm{E}\,(T_r\,|\,n_1^{(r)})]\\ &=[(1-\lambda_1)/12np\lambda_1]4p^2c^2+O(\beta_n^2).\end{aligned} \tag{4.22}$$

From Noether [7], the efficacy of a test T_n is defined by

$$e(T)=[d\,\mathrm{E}\,(T_n)/d\theta\,|\,\theta=\theta_0]^2/\sigma_0^2(T_n). \tag{4.23}$$

Hence, the efficacy of the proposed test is

$$e(T_r)=[12n\lambda_1(1-\lambda_1)/p^2]\left[\int_{-\infty}^{F^{-1}(p)} f^2(x)dx\right]^2. \tag{4.24}$$

Basu [1] has calculated the efficacy of his test and is given by

$$e(B_r^{(n)})=[12n\lambda_1(1-\lambda_1)/p^3(4-3p)]\left[\int_{-\infty}^{F^{-1}(p)} f^2(x)dx\right]^2. \tag{4.25}$$

The efficacy of Gastwirth [6] test is given by

$$e(G_r^{(n)})=[12n\lambda_1(1-\lambda_1)/p(p^2-3p+3)]\left[\int_{-\infty}^{F^{-1}(p)} f^2(x)dx\right]^2. \tag{4.26}$$

Hence, the efficiencies of Basu's and Gastwirth's tests with respect to the proposed tests are respectively given by

$$e(B_r^{(n)}, T_r)=1/(4-3p)p, \tag{4.27}$$

and

$$e(G_r^{(n)}, T_r)=p/(p^2-3p+3) . \tag{4.28}$$

The following table shows that the proposed test is superior to Basu's and Gastwirth's tests (and hence to Sobel's tests) for $p \geqq 1/3$.

p \ Efficiency	$e(B_r^{(n)}, T_r)$	$e(G_r^{(n)}, T_r)$
.34	1.00	.17
.50	.80	.29
.67	.75	.46
.75	.76	.57
1.00	1.00	1.00

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