ON THE EVALUATION OF THE RISK INDEX OF THE RAILROAD CROSSING*

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(Received Jan. 13, 1959)

1. Introduction

Because of the continuous increasing of the traffic volume such as automobils, the number of the accidents at the railroad crossings is growing more and more. This is one of the serious social problems and the protection for these accidents is very urgent in these days.

As the cause of the accidents there are many factors, namely the psychological factors such as the traffic morality of pedestrians and drivers of automobils, the traffic volume, the equipments of crossings, the weather and so on.

We have investigated the traffic volume of the railroad crossings which belong to Japanese National Railways and studied the method to calculate the risk index of crossings by the so-called quantification.

2. Survey of the railroad crossings

There are three kinds of railroad crossings in Japanese National Railways (JNR), namely the crossings of the 1st kind which are equipped by the crossing gate and the guarders, the ones of the 3rd kind which are equipped by the alarm bells and switching signal lamps and the ones of the 4th kind which are equipped only by the crossing mark. The crossings of the 2nd kind, the roles of which are same to the 1st kind in the daytime but to the 3rd kind at night, are very few when we have studied. So we have ommitted them from our objects because of the lack of data.

The number of crossings with no accident and that of crossings with accident belonged to JNR in 1955 were given as following table.

* This study was carried by the committee of the standard making for the consolidation of the railroad crossings under the auspices of Signal Society in Japan in 1954, and also under the auspices of Japan Railway Engineering Association in 1957.

Table 2.1. Crossings (population).

Crossings	No accident	With accident	Total
1st kind	2,870	107	2,977
3rd kind	1,605	281	1,886
4th kind	34,319	1,885	36,204
Total	38,794	2,273	41,067

We used the sample of size about 4000 which consisted of nearly equal number of the crossings with no accident and the ones with accident in 1954, but we have used in 1957 the sample of size presented in the following table.

Table 2.2. Crossings (Sample).

Crossings	No accident	With Accident	Total
1st kind	200	104	304
3rd kind	300	237	537
4th kind	677	820	1,497
Total	1,177	1,161	2,338

As the characters of the railroad crossings we have picked up the factors as follows.

A. The characters for all crossings.

1. kind of crossing
2. number of rails
3. grade of railway
4. traffic volume of automobils and pedestrians
5. number of interceptions of gate (only for the 1st kind)
6. number of passing trains
7. kind of road
8. width of road
9. gradient of road
10. angle of intersection between road and railway
11. distance of visibility
12. kind of pavement
13. method of interception (only for the 1st kind)
14. alarm bell
15. bell which indicates the approach of trains (only for the 1st kind)

16. bell equipped between a crossing and the next followed crossing (only for the 1st kind)
17. crossing mark
18. mark of whistling
19. illumination at the crossing
20. circumferences arround the crossing
21. coefficient of the regional trait
22. reduced traffic volume
23. coefficient of congestion

B. The characters only for the crossings where the accident happened in that year.

1. date of the accident
2. weather
3. aspect of the accident
4. collided things
5. age, sex and experience of the drivers or the pedestrians
6. psychological condition of the drivers or the pedestrians
7. cause of the accident

In the following we should like to mention some characters. *The grade of railroad* consists of special, 1st, 2nd, 3rd and simple or private only which are distinguished by the speed of trains, the number of passing trains, the kind of the construction of trains and so forth.

As *the distance of visibility* we may have four distances, namely the distances of the left hand side and the right hand one from the points which are 5 meter apart from up train side and down train side of the rails. We have adopted only the minimum distance among them. *Circumferences* around the crossing are distinguished into 6 sorts, that is, factory districts, business ones, residential ones, farm, fishing, mountain villege ones, school ones, and harbour ones. We also defined *the coefficient of the regional trait* by the ratio of the number of accident crossings of the 4th kind to the total number of crossings of the 4th kind in the Railway Controlling Department to which the crossing belongs. It was used in place of the different reduced traffic values of an object, which are defined afterwards, in different districts. *The reduced traffic value* is the equivalent volume reduced to the number of pedestrians by using a sort of quantification. Let, for example, the number of accidents by the motor trucks of small size be n, and the total number of motor trucks of small size passing through be N in all crossings in that year. Here N is estimated by the investigation of traffic volume. Similarly let those of pedestrians be m and M respectively. Then the reduced traffic value of the motor trucks of small size is defined by

$$\frac{n}{N}\bigg/\frac{m}{M}. \tag{2.1}$$

Thus *the reduced traffic volume* at the crossing is the sum of these reduced traffic value multiplied by the number of objects passed through.

Table 2.3. Traffic volume.

Sorts of crossing objects	At the crossings of the 4th kind in 1955		
	Estimated total traffic volume in a day	Number of accidents in that year	Reduced traffic value
Three-wheeler motor van	599,971	396	21.3
Motor truck of small size	245,357	64	8.4
Motor truck of large size	447,095	167	12.1
Passenger car	416,710	148	11.5
Bicycles, small wagon	6,008,756	395	2.1
Wagon, cart, rear car	1,091,045	135	4.0
Autobicycle, scooter	1,227,312	353	9.3
Pedestrian	10,542,599	326	1.0

The coefficient of congestion I is a similar concept as the reduced traffic volume. That is the ratio of the total sum S of the reduced area of section of the objects to the width D of the crossing. Namely

$$I=\frac{S}{D}=\frac{\sum_i N_i S_i}{D} \tag{2.2}$$

where S_i is the reduced area of section of the i-th object to that of person and N_i is the number of these objects passed through in a day.

For a factor influencing to the danger at crossings we may use the difference of the width between the crossing and the road, but it is difficult to define this difference rigorously because of the complexity of the road at the end of the crossing. Therefore we had to use the coefficient I instead of the difference of the width between them. However there are many ways to define I, such as the one at the

Table 2.4. Reduced area of section.

Sorts of crossing objects	Reduced area of section
Three-wheeler motor van	19.5
Motor truck of small size	20.6
Motor truck of large size	46.3
Passenger car of small size	20.6
Passenger car of large size	64.0
Bicycle, small wagon	3.1
Wagon, cart, rear car	30.6
Autobicycle, scooter	4.3
Pedestrian	1.0

most congested hour or in all day long. The correlation coefficient between the coefficient from 7 to 8 a.m. and the coefficient in a day for the crossings of the 1st kind is 0.890, so we have taken the former mentioned definition, namely I in a day.

In order to study the relation between this coefficient of congestion and the velocity of objects passing through, we have investigated 4 crossings in the Tokyo Railway Controlling Department. The effect of the I to the congestion measured by the depletion of the velocity was only seen at the crossing of the 1st kind and only in the most congested hour.

3. Selection of factors effective to the danger at crossings

We have compared the distribution of the crossings with no accident of 4th kind with that of the crossings with accident of the same kind

Table 3.1. Main factors.

Factors	$z=\sqrt{2\chi^2}-\sqrt{2\nu}$ *	Correlation ratio***
1. Reduced traffic volume	25.09 (1)**	0.2711
2. Coefficient of congestion	20.70	0.2171
3. Width of crossing	13.36 (2)	0.1674
4. Number of rails	11.57 (4)	0.1457
5. Circumferences	11.05 (3)	0.1636
6. Number of passing trains	10.67 (7)	0.1369
7. Grade of railway	9.22 (5)	0.1086
8. Coefficient of regional trait	7.58	0.0944
9. Distance of visibility	4.24 (6)	0.0572
10. Angle of intersection	3.05 (8)	
11. Gradient of road	2.52 (7)	

* ν is the degrees of freedom.
** ( ) is the rank of z for the data in 1955.
*** maximized correlation ratio.

by the chi-square test. By these tests we could select the main foctors as in Table. 3.1.

The coefficient of congestion is highly correlated to the reduced traffic volume ($r=0.8104$) and is not so much effective to the crossing of the 4th kind as to that of the 1st kind. Therefore we have excluded that from the selected factors. So we have adopted eight factors, that is 1, 3, 4, 5, 6, 7, 8, and 9 in Table 3.1.

For the categories of these eight factors we define the numerical value by the maximization of each correlation ratio.

For example, let the disitribution be that in Table 3.2.

Table 3.2.

Factor A	$A_1\ A_2\ \dots\ A_l$	Total	Sampling ratio
With no accident	$f_{11}\ f_{12}\ \dots\ f_{1l}$	$f_{1\cdot}$	k_1
With accident	$f_{21}\ f_{22}\ \dots\ f_{2l}$	$f_{2\cdot}$	k_2

If we can define the numerical value x_i for the category A_i, then for the correlation ratio η we have

$$\eta^2=\frac{\sigma_b^2}{\sigma^2}=\frac{k_1k_2f_{1\cdot}f_{2\cdot}(\bar{x}_1-\bar{x}_2)^2}{(k_1f_{1\cdot}+k_2f_{2\cdot})^2\sigma^2}\ , \tag{3.1}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the mean value of population of crossings with no accident and with accident respectively. Then making $\partial\eta/\partial x_i=0$ assuming $\bar{x}=0$ without loss of generality, we get

$$x_i=c\left(\frac{f_{1i}}{f_{1\cdot}}-\frac{f_{2i}}{f_{2\cdot}}\right)\Big/(k_1f_{1i}+k_2f_{2i}) \tag{3.2}$$

where c is an arbitrary constant.

In this way we have numerical values of the categories of each factor and the correlation coefficient among factors (Table 3.3, 3.4).

Then we must define *the risk index* of the crossing. There are many ways to define the risk index, but we have adopted the method of maximization of correlation ratio between the risk index y of crossings with no accident and that of crossings with accident, where y is defined by the equation with unknown weights w_i

$$y=w_1x_1+w_2x_2+\cdots+w_8x_8\ . \tag{3.3}$$

Thus we get w_i from the simultaneous equations

Table 3.3. Numerical value given to the categories.

x_1	Number of rails	1	2	3	more than 4			
		−0.746	3.159	3.647	7.980			
x_2	Grade of railways	special	1st	2nd	3rd	Simple and private		
		5.549	2.300	0.293	−0.486	−1.752		
x_3*	Number of passig trains	~50	~100	~150	~350	~550	~750	~950
		−0.820	0.282	3.787	4.459	5.535	6.611	7.687
x_4*	Width of crossing	~2m	~4	~6	~8	~22	~36	~50
		−1.632	0.619	4.419	3.126	2.997	2.770	2.544
x_5	Distance of visibility	~100 m	~250	~400	~600	more than 600		
		0.766	−0.067	−0.356	−1.122	−1.181		
x_6	Circumferences	Factory area	Business area	Residential area	Farm, Fishing, Mountain area	School area	Harbour area	
		6.796	8.236	4.178	−0.742	2.786	−0.068	
x_7	Coefficient of regional trait	~2	~3	~4	~6	~8	~10	
		−1.730	−0.988	−0.837	0.124	1.722	3.246	
x_8*	Reduced traffic volume	~250	~500	~1,000	~3,000	~5,000	~20,000	
		−2.434	−1.684	−0.850	2.457	8.816	8.916	
		~40,000	~60,000	~80,000	~100,000	~140,000	~180,000	
		9.123	9.359	9.596	9.832	10.186	10.659	
		~220,000	~260,000	~300,000	~400,000			
		11.131	11.604	12.076	12.549			

* These values are partly estimated be the interpolation and the extrapolation.

Table 3.4. Correlation coefficient among the factors.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	1	0.5098	0.6250	0.0178	0.0588	0.1942	0.2406	0.1629
x_2		1	0.7260	−0.0535	−0.0116	0.0854	0.1495	0.0648
x_3			1	−0.0364	0.0064	0.1832	0.2896	0.1218
x_4				1	0.0350	0.1909	−0.0392	0.5478
x_5					1	0.1452	0.1139	0.0689
x_6						1	0.1815	0.3745
x_7							1	0.1429
x_8								1

$$\sum_{k=1}^{8} c_{ik} w_k = \lambda(\bar{x}_{iB} - \bar{x}_{iA}), \quad i=1, 2, \cdots, 8 \tag{3.4}$$

where $c_{ik} = \sum_j x_{ij} x_{kj}$, λ is arbitrary constant and $\bar{x}_{iB}, \bar{x}_{iA}$ are the mean value of the crossings with accident and that of the crossings with no accident in regard to i-th factor.

The distribution of y of crossings with no accident and that of crossings with accident are shown in Table 3.6.

Table 3.5. Weights of the main factors.

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
1	0.0606	0.4701	0.2215	0.3575	0.3396	0.3001	0.6254

Thus we can define the risk index $y_{(j)}$ to a crossing (j) by the equation

$$y_{(j)} = X_{1j} + X_{2j} + X_{3j} + \cdots + X_{8j} \tag{3.5}$$

where $X_{ij} = W_i x_{ij}$ and x_{ij} is the numerical value of category to which the crossing (j) belongs in regard to i-th factor.

Table 3.6. Distribution of the risk index.

y	−26∼	−16∼	−6∼	4∼	14∼	24∼	34∼	44∼	54∼	64∼	74∼	Total
With no accident	19.9	39.6	12.0	14.9	3.7	2.7	3·5	2.1	1.0	0.6	—	100.0
With accident	1.6	10.7	7.7	21.1	8.8	8.0	19.9	11.5	6.1	3.5	1.1	100.0

Using this risk index we may think the larger the value of y is, the higher the degree of danger is. The relation between the risk index y and *the coefficient of accident* P, which is defined by the percentage of the number of crossings with accident to that of all crossings having the same value of risk index, is indicated by Table 3.7.

Table 3.7. Relation between the risk index and the coefficient of accident.

y	−26∼	−16∼	−6∼	4∼	14∼	24∼	34∼	44∼	54∼	64∼	74∼
P	0.5	1.5	3.4	7.2	11.6	14.0	23.8	23.1	25.1	24.3	100.0

This relation is presented by the equation

$$P = 0.9960\left(\frac{y+31}{10}\right)^{1.57839} - 1\,. \tag{3.6}$$

4. Application of the risk index to the crossings of the 1st and the 3rd kind

If we have to make good separation between the crossing with no accident and that with accident, we must apply the method of the quantification to the ones of each kind respectively. However we need to simplify the method of the alternation among the kinds of crossings, so that we use also the same value of categories for the 1st and the 3rd kind. The result is shown in Tables 4.1～4.4.

Table 4.1. Distribution of y of crossings of the 1st kind.

y	4～	14～	24～	34～	44～	54～	64～	74～	84～	Total
With no accident	1.5	1.0	—	21.5	24.5	21.5	16.0	13.0	1.0	100.0
With accident	—	—	0.9	12.5	12.5	23.1	25.0	18.3	7.7	100.0

Table 4.2. Distribution of y of crossings of the 3rd kind.

y	−16～	−6～	4～	14～	24～	34～	44～	54～	64～	74～	Total
With no accident	0.3	0.7	6.7	3.7	5.7	60.3	12.3	7.3	3.0	—	100.0
With accident	—	0.4	2.1	2.1	5.9	40.1	18.6	21.6	8.4	0.8	100.0

Table 4.3. Relation between y and P of crossings of the 1st kind.

y	14～	34～	44～	54～	64～	74～	84～
P''	1.4	2.1	1.8	3.9	5.6	5.1	21.6

Table 4.4. Relation between y and P of crossings of the 3rd kind.

y	−16～	−6～	4～	14～	24～	34～	44～	54～	64～	74～
P'	0	8.3	5.3	9.2	15.7	10.5	20.8	33.9	32.3	100.0

The relation between y and P is as follows.

$$P'=0.3877\left(\frac{y+31}{10}\right)^{2.05011}-1 \quad \text{(3rd kind)} \tag{4.1}$$

$$P''=0.05606\left(\frac{y+31}{10}\right)^{2.10477}-1 \quad \text{(1st kind)}\,. \tag{4.2}$$

The curves (3.6), (4.1) and (4.2) are shown in Fig. 4.1.

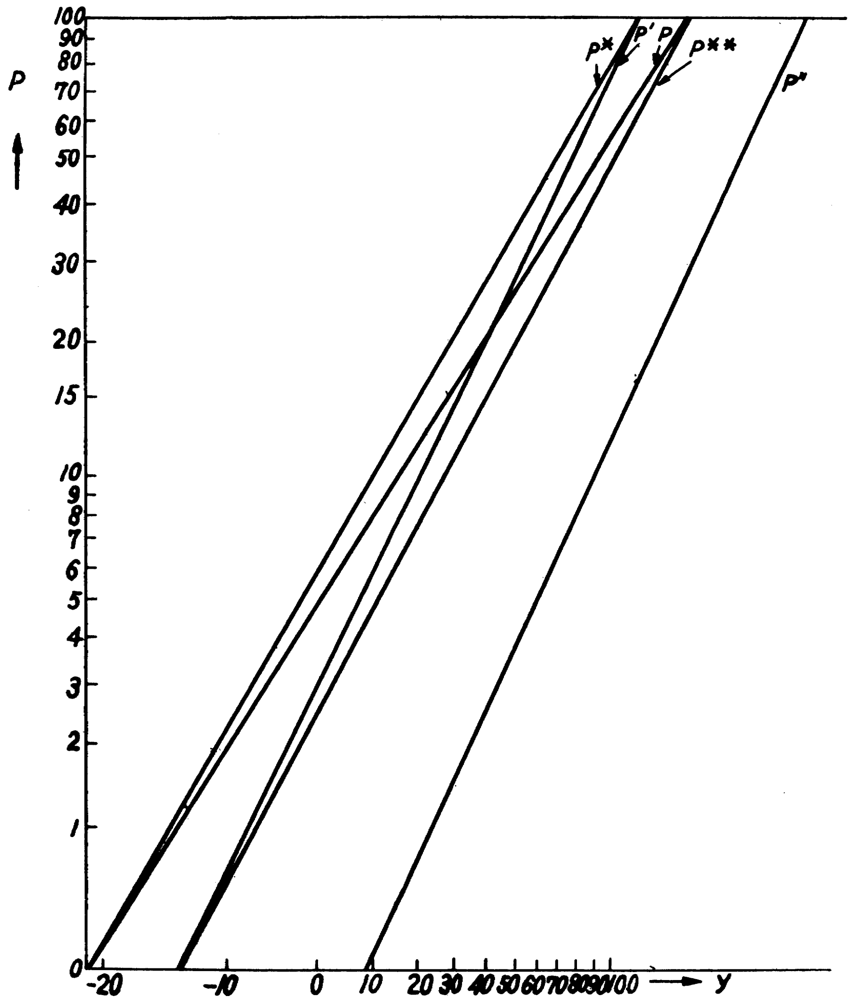

Fig. 4.1.

On the other hand if we express the effectiveness of selected factors by z value, we can make clear the difference of the effectiveness among three kinds of crossings. The rank correlation ρ_{13}, ρ_{14} and ρ_{34} are 0.58, 0.10 and 0.08 respectively. Therefore the effectiveness of the selected factors is different from those of the 3rd and the 1st kinds. This result indicates the effect of the alternation of the kinds and we can see the effect of the reduced traffic volume is depleted by adding the equipments.

Table 4.5. Rank of z value.

Factors	1st kind	3rd kind	4th kind
Reduced traffic value	4	5	1
Coefficient of congestion	2	6*	2
Width of crossing	8*	9*	3
Number of rails	6	1	4
Circumferences	9*	7*	5
Number of passing trains	1	2	6
Grade of railway	3	3	7
Coefficient of regional trait	5	4	8
Distance of visibility	7*	8*	9

* No significance between the crossings with no accident and those with accident.

5. Effect of the crossing equipments and the alternation of kinds

In the preceding section we have analysed the risk index of the crossings of three kinds. These crossings are themselves already equipped by a certain criterion which is constructed under the consideration of the traffic volume, number of passing trains and the distance of visibility. Therefore we can not know the true effect of the equipment, nevertheless we can use the risk index for the criterion to alternate the kind of crossing.

The coefficient of accident (P') of the crossings of the 3rd kind is greater than that (P) of the crossings of the 4th kind at $y \geqq 43$, as can be seen by Fig. 4.1. However it does not mean that the effect of the equipment of the 3rd kind does not exist, for the crossings of the 3rd kind are already altered by a certain criterion from the 4th kind because of higher danger. If we let these crossings be as it was, the reduced coefficient of accident P^0 of these crossings will be greater than P'.

Now let the distribution of the risk index of crossings of three kinds be that in the following table.

Table 5.1.

4th kind	$y_1 \cdots y_i \cdots$	Total
With no accident	$\cdots\cdots N_{Ai} \cdots$	N_A
With accident	$\cdots\cdots N_{Bi} \cdots$	N_B
Total	$\cdots\cdots N_{\cdot i} \cdots$	N

Table 5.2.

3rd kind	$y_1 \cdots y_i \cdots$	Total
With no accident	$\cdots N'_{Ai} \cdots$	N_A'
With accident	$\cdots N'_{Bi} \cdots$	N_B'
Total	$\cdots N'_{\cdot i} \cdots$	N'

Table 5.3.

1st kind	$y_1 \cdots y_i \cdots$	Total
With no accident	$\cdots N''_{Ai} \cdots$	N''_A
With accident	$\cdots N''_{Bi} \cdots$	N''_B
Total	$\cdots N''_{\cdot i} \cdots$	N''

If we altered all crossings of the 1st and the 3rd kinds to the 4th kind, then the true coefficient of accident P_i^* of the 4th kind having the same y_i value is equal to

$$P_i^* = \frac{P_i N_{\cdot i} + P^0_{3\to4,i} N'_{\cdot i} + P^{00}_{1\to4,i} N''_{\cdot i}}{N_{\cdot i} + N'_{\cdot i} + N''_{\cdot i}} \tag{5.1}$$

where $P_i = N_{Bi}/N_{\cdot i}$ and $P^0_{3\to4,i}$, $P^{00}_{1\to4,i}$ are the reduced coefficient of accident of the crossings of the 3rd and the 1st kind respectively when they are altered to those of the 4th kind.

For the simplicity we omit the suffix i. If we can assume the crossings of the 3rd and the 1st kind without the equipments are more dangerous than those of the 4th kind with same y_i, then we have

$$P^{00}_{1\to4} > P^0_{3\to4} > P \, . \tag{5.2}$$

On the other hand we have

$$P' > P > P'' \quad \text{for} \quad y > 43 \tag{5.3}$$

and

$$P > P' > P'' \quad \text{for} \quad y < 43 \, , \tag{5.4}$$

also we can easily find $\quad P^{00}_{1\to4} > P^* > P \, . \quad (5.5)$

and

$$P^* - P^0_{3\to4} = \frac{1}{N + N' + N''} \{N(P - P^0_{3\to4}) + N''(P^{00}_{1\to4} - P^0)\} \, . \tag{5.6}$$

The last form has not definite sign, but N'' is large for the large y, therfore

$$P^* > P^0_{3\to4} \, . \tag{5.7}$$

Meanwhile if we assume the effect of the equipment of the 3rd kind we have necessarily

$$P^{0}_{3\to4} > P' . \qquad (5.8)$$

Thus for $y>43$ we have

$$P^{00}_{1\to4} > P^* > P^{0}_{3\to4} > P' > P > P'' . \qquad (5.9)$$

Similarly we get for $y<43$

$$P^{00}_{1\to4} > P^{0}_{3\to4} > P^* > P > P' > P'' . \qquad (5.10)$$

Therefore the true coefficient of accident of the crossings of the 4th kind P^* is greater than P' for $y>43$ and greater than P for $y<43$, so that if we can assume the equation of P^* has the form $a\left(\frac{y+31}{10}\right)^b-1$,

we have

$$P^* \geqq 0.9957\left(\frac{y+31}{10}\right)^{1.70249} - 1 . \qquad (5.11)$$

Quite similarly let $P^{00}_{1\to3}$, $P^{0}_{4\to3}$ be the reduced coefficient of accident of crossings of the 1st and the 4th kind respectively when they are altered to those of the 3rd kind. Then the true coefficient of accident of crossings of the 3rd kind P^{**} is equal to

$$P^{**} = \frac{P^{0}_{4\to3}N + P'N' + P^{00}_{1\to3}N''}{N+N'+N''} . \qquad (5.12)$$

Then the following inequalities hold

$$P^{00}_{1\to3} > P' > P^{0}_{4\to3} \qquad (5.13)$$

$$P > P^{0}_{4\to3} \qquad (5.14)$$

$$P^{00}_{1\to3} > P^{**} > P^{0}_{4\to3} . \qquad (5.15)$$

But the signs of $P^{**}-P'$, $P^{**}-P$ and $P^{**}-P''$ are not definite.

For $y>43$ we have following three possibilities

(A)
$$\begin{cases} P^{00}_{1\to3} > P' > P > P^{**} > P'' > P^{0}_{4\to3} & (5.16) \\ P^{00}_{1\to3} > P' > P > P^{**} > P^{0}_{4\to3} > P'' & (5.16)' \end{cases}$$

(B)
$$\begin{cases} P^{00}_{1\to3} > P' > P^{**} > P > P'' > P^{0}_{4\to3} & (5.17) \\ P^{00}_{1\to3} > P' > P^{**} > P > P^{0}_{4\to3} > P'' & (5.17)' \end{cases}$$

(C)
$$\begin{cases} P^{00}_{1\to3} > P^{**} > P' > P > P'' > P^{0}_{4\to3} & (5.18) \\ P^{00}_{1\to3} > P^{**} > P' > P > P^{0}_{4\to3} > P'' . & (5.18)' \end{cases}$$

For $y<43$ the possibility that $P^{**}<P'$, $P^{**}>P''$ is so great that we shall have

$$\begin{cases} P>P^{00}_{1\to3}>P'>P^{**}>P''>P^{0}_{4\to3} & (5.19) \\ P>P^{00}_{1\to3}>P'>P^{**}>P^{0}_{4\to3}>P'' & (5.19)' \end{cases}$$

$$\begin{cases} P^{00}_{1\to3}>P>P'>P^{**}>P''>P^{0}_{4\to3} & (5.20) \\ P^{00}_{1\to3}>P>P'>P^{**}>P^{0}_{4\to3}>P'' . & (5.20)' \end{cases}$$

Hence we have $P'>P^{**}$ and $P^{*}>P^{**}$ in any case. We cannot determine the exact curves unless we can have more data of the alternation among the crossings of three kinds. So we adopt the case (A) illustrated by Fig. 5.1.

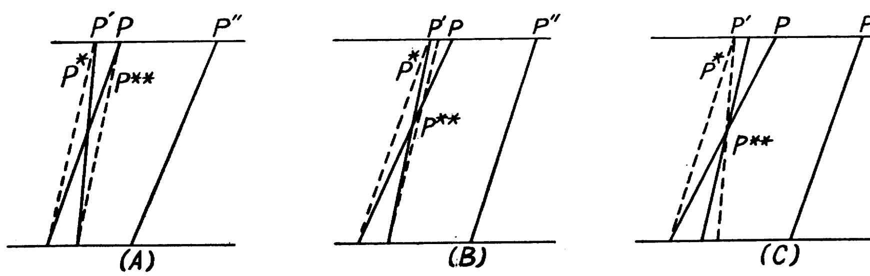

Fig. 5.1.

Similarly for the true coefficient of accident of the 1st kind P^{***} we have for $y>43$

$$\begin{cases} P'>P>P''>P^{***}>P^{00}_{3\to1}>P^{0}_{4\to1} & (5.21) \\ P'>P>P''>P^{00}_{3\to1}>P^{***}>P^{0}_{4\to1} & (5.21)' \end{cases}$$

and for $y<43$

$$\begin{cases} P>P'>P''>P^{***}>P^{00}_{3\to1}>P^{0}_{4\to1} & (5.22) \\ P>P'>P''>P^{00}_{3\to1}>P^{***}>P^{0}_{4\to1} & (5.22)' \end{cases}$$

hence $P''>P^{***}$, $P^{**}>P^{***}$.

Thus we have

$$P^{**} \sim 0.4154\left(\frac{y+31}{10}\right)^{1.70249}-1 \qquad (5.23)$$

$$P^{***} \leqq 0.05606\left(\frac{y+31}{10}\right)^{2.10477}-1 . \qquad (5.24)$$

Using equations (5.11), (5.23) and (5.24) we have following table of relation between y and P^{*}, P^{**} and P^{***}.

Table 5.4. Relation among the risk index and the coefficients of accident.

y	−26∼	−16∼	−6∼	4∼	14∼	24∼	34∼	44∼	54∼	64∼	74∼	84∼
$P_i^* \geqq$	0.4	2.2	5.5	9.6	14.4	20.0	26.4	33.3	40.9	49.2	58.0	67.5
P_i	0.4	2.0	4.6	7.9	11.6	15.8	20.5	25.5	31.0	36.7	42.9	50.5
P_i'	0	0.6	2.7	5.6	9.5	14.3	19.9	26.5	34.1	42.5	51.9	60.0
$P_i^{**}\sim$	0	0.6	2.4	4.8	7.9	11.5	15.8	20.6	26.1	32.1	38.6	45.8
P_i'' $P_i^{***} \leqq$	0	0	0	0	0.7	1.4	2.4	3.5	4.7	6.1	7.7	9.5

For example, when we alter the crossing of the 3rd kind having y to that of the 4th kind, the coefficient of accident P' increases, to P^* having the same index y.

6. Cutting points of the distribution of the risk index to alter the kinds of crossings

Let the number of crossings of i-th kind be N_i and the density function having the risk index y be $f_i(y)$, then

$$\int_{-\infty}^{\infty} f_i(y)dy=1 \quad (i=1, 3, 4) \tag{6.1}$$

Let also the cutting point for the alternation from the 4th to the 3rd kind be y_4 and the one from the 3rd to the 1st kind be y_3 and the costs of equipments of the 3rd kind and the 1st kind be w_3 and w_1 respectively.

Then the total coefficient of accident by the elevation of the kinds (P) is given by the following equation:

$$\begin{aligned} NP=N_4\left[\int_{-\infty}^{y_4} f_4(y)P_4(y)dy+\int_{y_4}^{y_3} f_4(y)P_3^{**}(y)dy+\int_{y_3}^{\infty} f_4(y)(P)_1^{***}(y)dy\right] \\ +N_3\left[\int_{-\infty}^{y_3} f_3(y)P_3(y)dy+\int_{y_3}^{\infty} f_3(y)P_1^{***}(y)dy\right] \end{aligned} \tag{6.2}$$

where $P_3(y)$, $P_4(y)$, $P_3^{**}(y)$ and $P_1^{***}(y)$ are the coefficient of accident of the crossing of the 3rd kind, that of the 4th kind, true coefficient of accident of the crossing of the 3rd kind and that of the 1st kind respectively and $N=N_1+N_3+N_4$.

On the other hand the total cost of alternation C is given by

$$C=N_4\left[w_3\int_{y_4}^{y_3} f_4(y)dy+w_1\int_{y_3}^{\infty} f_4(y)dy\right]+N_3 w_1\int_{y_3}^{\infty} f_3(y)dy\ . \tag{6.3}$$

Taking minimum of NP under the fixed total cost, we have y_3 and y_4

from (6.3) and

$$\frac{P_4(y_4)-P_3^{**}(y_4)}{w_3}=\frac{N_4f_4(y_3)\{P_3^{**}(y_3)-P_1^{***}(y_3)\}+N_3f_3(y_3)\{P_3(y_3)-P_1^{***}(y_3)\}}{N_4(w_1-w_3)f_4(y_3)+N_3w_1f_3(y_3)}. \tag{6.4}$$

If we put $C=55{,}000$, $w_1=500$, $w_3=100$ using the Table 6.1, we get approximately $y_4=59$ and $y_3=81$. Then we ought to alter 514 crossings of the 4th kind to the 3rd kind and alter six of the 4th kind and one of the 3rd kind to the 1st kind.

Table 6.1. Population distribution.

y	−26~	−16~	−6~	4~	14~	24~	34~	44~	54~	64~	74~	84~	Total
$N_4f_4(y)\Delta y$	6,859	13,792	4,263	5,512	1,436	1,078	1,576	938	458	272	21		36,205
$N_3f_3(y)\Delta y$		5	12	114	65	108	1,081	249	178	72	2		1,886
$N_1f_1(y)\Delta y$				43	29	1	630	716	642	486	393	37	2,977

7. Other factors influencing to the danger at crossings

We have investigated the other factors which were studied only for the crossings with accident but not adopted in the above mentioned analysis. They are the age, the experience and the psychological condition of the drivers or the pedestrians, the weather, the time, and the day of the week, the month and so forth. These factors are changing continuously so that the adoption of these factors means that the changeable risk index with time, weather and so on is given to a crossing. On the other hand the above-mentioned data were not always complete and the fundamental data of the traffiic volume for these different factors, which are necessary to analyse the data of crossings with accident, didn't exist. Hence we have been obliged to abandon these factors, but we should like to give some results.

Table 7.1. Psychological condition at the time when the accident happened.

Psychological condition	Crossing of the 1st kind	Crossing of the 3rd kind	Crossing of the 4th kind
Indifferent to the train	21.6	49.3	52.5
Thinking to be safety	6.2	20.0	15.0
Thinking to be not safety	3.1	1.9	2.7
Not to notice the opposite side train	7.2	7.4	4.1
Carelessness of protectors	2.1	2.6	5.6
Others	59.8	18.8	20.1
Total	100.0	100.0	100.0

Fig. 7.1. The coefficient of accident and the mean traffic volume in time bands (Sample of size 1000)—Passenger cars.

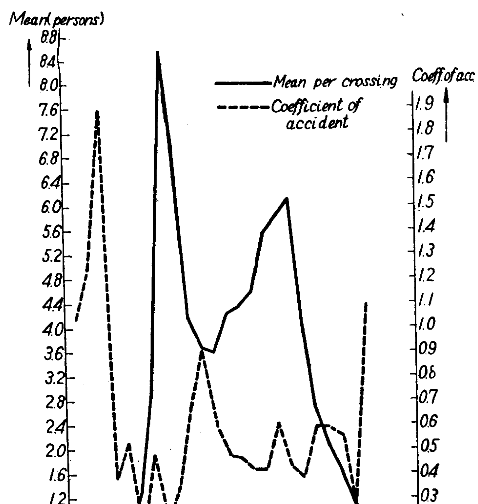

Fig. 7.2. The coefficient of accident and the mean traffic volume in time bands (Sample of size 1000)—Pedestrians.

Table 7.2. Cause of accidents.

Cause	Crossing of the 1st kind	Crossing of the 3rd kind	Crossing of the 4th kind
Doze	3.1	0.4	0.2
Engine stop	3.1	2.9	2.1
Drop out of the crossing	3.1	1.4	3.1
Trouble of the brake	8.2	1.9	2.0
Across the front of the train	22.7	56.3	54.9
Misdriving	15.4	8.1	7.3
Not to stop to confirm the safety	19.6	22.2	21.8
Drunk	2.1	1.9	0.2
Others	22.7	4.9	8.4
Total	100.0	100.0	100.0

Table 7.3 Effect of the weather.

Weather	Fine	Cloudy	Rain	Slightly rain	Snow	Total
Tokyo-to	53.7	13.2	23.0	9.3	0.8	100.0
At the crossing with accident in Tokyo Railway Controlling Department	55.3	31.0	9.5	3.7	0.5	100.0

The effect of the bad weather to the accident does exist of course, nevertheless we had above-mentioned data which seemed to indicate the existence of the compensatory action of drivers or pedestrians to the bad weather. But we had no data of traffic volume in the different weathers, so we couldn't get any conclusion. The variation of the coefficient of accident in different time bands also does exist. Fig. 7.1～2 are the examples of the variable coefficients of accident in different time bands.

Acknowledgement.

This study was carried out under the auspices of Signal Society in Japan and Japan Railway Engineering Association both sponsored by Train Operation Department, Japanese National Railways. The author expresses his heartful thanks to Miss Sadako Tanaka for tha execution of calculation and analysis of these enormous works and also to the members of the committee for the encouragement to carry out this study.

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