TABLES FOR TESTING THE HOMOGENEITY OF k INDEPENDENT BINOMIAL EXPERIMENTS ON A CERTAIN EVENT BASED ON THE RANGE

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Let $\nu_{(i)}(i=1, 2, \cdots, k)$ and $p_i(i=1, 2, \cdots, k)$ be the observed number of occurrences and the probability, of a certain event in the ith binomial experiment of N trials, respectively. In [1], one of the present authors has discussed the test of the homogeneity of k experiments, that is, the test of the hypothesis that

$$H: p_1=p_2=\cdots=p_k\equiv p \text{ (say)}$$

based on the range defined by

$$R_k(N, p)=\nu_1-\nu_2 ,$$

where ν_1 and ν_2 are the largest and the smallest values, respectively, in a set of k numbers $\nu_{(1)}, \nu_{(2)}, \cdots, \nu_{(k)}$. In this case we are considering the situation that N is not so large enough to apply the normal approximation to the single succession of trials, but, under the null hypothesis H, Nk is large enough to regard the estimate

$$\hat{p}=\sum_{i=1}^{k}\nu_{(i)}/Nk$$

as the true value p.

The present tables are related to the probabilities, $Pr\{R_k(N, p)\geqq r_k\}$, calculated by the automatic relay computor, FACOM-128 in our institute, for $N=10(1)20$, 22, 25, 27, 30 ; $k=2(1)15$; $p=0.50$, 0.40, 0.30, 0.20, 0.10 and for values of r_k such that $Pr\{R_k(N, p)\geqq r_k\}$ does not exceed 0.10. However, for the sake of saving the number of pages, each entry in the tables is not the value of $Pr\{R_k(N, p)\geqq r_k\}$ itself but the greatest value of r_k such that $Pr\{R_k(N, p)\geqq r_k\}$ does not exceed the assigned value $\alpha(=0.001, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10)$ for fixed N, p and k. But there is one exception denoted by '+' sign attached to the right upper corner of the r_k corresponding to some α. For fixed α, the r_k^+ means that $Pr\{R_k(N, p)\geqq r_k^+\}$ is slightly larger than α, but the rounded value, obtained by dropping the digit less than 5 at the

fourth decimal place, is equal to α. For example, in the case that $N=15$, $p=0.40$ and $k=6$, we have the exact value (up to the five decimal places) for $r_6=8$,

$$Pr\{R_6(15,\ 0.40)\geqq 8\}=0.05018\ (>0.05)$$

and in this case we have written 8^+ instead of 9 in the table though the general rule of representation is violated. On the other hand, the entry with '$-$' sign, i.e. r_k^-, for a fixed α indicates that $Pr\{R_k(N, p)\geqq r_k^-\}$ lies between α and $\alpha-0.005$.

For the numerical illustration, let us consider the following data*) which were used in [1].

$N=17$, $k=8$,

$\nu_{(1)}=14$, $\nu_{(2)}=12$, $\nu_{(3)}=6$, $\nu_{(4)}=13$, $\nu_{(5)}=13$, $\nu_{(6)}=16$, $\nu_{(7)}=8$, $\nu_{(8)}=13$,

$r_8=15-6=9$, $\hat{p}=(14+12+\cdots+13)/(8\times 7)=0.69\approx p$

From the table for $N=17$, $k=8$ and $p=0.30$, we see that

$$0.03>Pr\{R_8(17,\ 0.30)\geqq 9\}>0.02\ .$$

Noting that $Pr\{R_8(17,\ 0.69)\geqq 9\}=Pr\{R_8(17,\ 0.31)\geqq 9\}$, we can say that the required value is near the value for $p=0.30$ and hence is between 0.02 and 0.03, or at most near 0.03. In fact the exact value up to the five decimal places is 0.02793.

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REFERRENCE

[1] Siotani, M., "Order statistics for discrete case with a numerical application to the binomial distribution." *Annals of the Institute of Statistical Mathematics*, Vol. 8. (1956), pp. 95-104.

*) The example with these data in [1] is good only if the concordance between judges is guaranteed and each trial is carried out independently. This is difficult to realize completely in practice and the example there is only for the numerical illustration.

TABLES of the greatest r_k such that $P_r\{R_k(N.\ p) \geqq r_k\} \leqq \alpha$

$N=10$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	8	7	7	6	6	6	5	5	5	5	8	7	6+	6	6	5	5	5	5	5
3	8+	8	7	7	7	6	6	6	6	6	8-	8	7	7	6	6	6	6	6	5
4	9	8	8	7	7	7	7	6	6	6	9	8	8	7	7	7	6+	6	6	6
5	9	8	8	8	7	7	7	7	7	6	9	8	8	7	7	7	7	7	6	6
6	9-	9	8	8	8	7	7	7	7	7	9	8	8	8	7	7	7	7	7	7
7	9-	9	8	8	8	8	7	7	7	7	9-	9	8	8	8	7	7	7	7	7
8	9+	9	8-	8	8	8	8	7	7	7	9-	9	8	8	8	8	7	7	7	7
9	9+	9	9	8	8	8	8	8	7	7	9-	9	8-	8	8	8	8	7	7	7
10	10	9	9	8	8	8	8	8	7+	7	9+	9	9	8	8	8	8	8	7	7
11	10	9	9	8	8	8	8	8	8	7	9+	9	9	8	8	8	8	8	7	7
12	10	9	9	9	8	8	8	8	8	8	10	9	9	8	8	8	8	8	8	7
13	10	9	9	9	8	8	8	8	8	8	10	9	9	8	8	8	8	8	8	8
14	10	9	9	9	8	8	8	8	8	8	10	9	9	9	8	8	8	8	8	8
15	10	9	9	9	9	8	8	8	8	8	10	9	9	9	8	8	8	8	8	8
	$p=0.30$										$p=0.20$									
2	7+	7	6	6	5	5	5	5	5	4	7	6	5+	5	5	5	4	4	4	4
3	8	7	7	6	6	6	6	6	5	5	7-	6+	6	6	5	5	5	5	5	5
4	8-	8	7	7	6+	6	6	6	6	6	7+	7	6	6	6	6	5	5	5	5
5	8+	8	7	7	7	7	6	6	6	6	8	7	7	6	6	6	6	6	5	5
6	9	8	8	7	7	7	7	7	6	6	8	7	7	6	6	6	6	6	6	5
7	9	8	8	7	7	7	7	7	7	6	8	7	7	7	6	6	6	6	6	6
8	9	8	8	8	7	7	7	7	7	7	8	7	7	7	6+	6	6	6	6	6
9	9	8	8	8	7	7	7	7	7	7	8	7-	7	7	7	6	6	6	6	6
10	9	8	8	8	8	7	7	7	7	7	8-	8	7	7	7	7	6	6	6	6
11	9	8+	8	8	8	7	7	7	7	7	8-	8	7	7	7	7	6	6	6	6
12	9-	9	8	8	8	8	7	7	7	7	8-	8	7	7	7	7	7	6	6	6
13	9-	9	8	8	8	8	7-	7	7	7	8-	8	7	7	7	7	7	7	6	6
14	9-	9	8	8	8	8	8	7	7	7	8-	8	7-	7	7	7	7	7	6	6
15	9-	9	8	8	8	8	8	8	7	7	8-	8	7+	7	7	7	7	7	6	6
	$p=0.10$																			
2	5+	5	4+	4	4	4	4	3-	3	3										
3	6	5	5	5	4	4	4	4	4	4										
4	6	5	5	5	5	4	4	4	4	4										
5	6-	6	5	5	5	5	5	4	4	4										
6	6-	6	5	5	5	5	5	5	4	4										
7	6-	6	5+	5	5	5	5	5	4+	4										
8	6+	6	6	5	5	5	5	5	5	4										
9	6+	6	6	5	5	5	5	5	5	5										
10	6+	6	6	5	5	5	5	5	5	5										
11	7	6	6	5	5	5	5	5	5	5										
12	7	6	6	5	5	5	5	5	5	5										
13	7	6	6	6	5	5	5	5	5	5										
14	7	6	6	6	5	5	5	5	5	5										
15	7	6	6	6	5	5	5	5	5	5										

$N=11$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	8⁻	7	7	6	6	6	6	5	5	5	8⁻	7	7	6	6	6	5	5	5	5
3	9	8	8	7	7	7	6	6	6	6	9	8	7⁺	7	7	6⁻	6	6	6	6
4	9⁻	8⁻	8	8	7	7	7	7	6⁺	6	9⁻	8	8	7	7	7	7	7	6	6
5	9⁺	9	8	8	8	7	7	7	7	7	9⁻	9	8	8	7⁻	7	7	7	7	7
6	10	9	9	8	8	8	8	7	7	7	9⁺	9	8	8	8	8	7	7	7	7
7	10	9	9	8	8	8	8	8	7	7	10	9	9	8	8	8	8	7	7	7
8	10	9	9	8⁺	8	8	8	8	8	7	10	9	9	8	8	8	8	8	7	7
9	10	9	9	9	8	8	8	8	8	8	10	9	9	8⁻	8	8	8	8	8	7
10	10	9⁻	9	9	8⁺	8	8	8	8	8	10	9	9	9	8	8	8	8	8	8
11	10⁻	10	9	9	9	8	8	8	8	8	10	9	9	9	8	8	8	8	8	8
12	10⁻	10	9	9	9	9	8	8	8	8	10	9⁺	9	9	9	8	8	8	8	8
13	10⁻	10	9	9	9	9	8	8	8	8	10⁻	10	9	9	9	8	8	8	8	8
14	10⁻	10	9⁻	9	9	9	9	8	8	8	10⁻	10	9	9	9	9	8	8	8	8
15	10⁺	10	10	9	9	9	9	9	8	8	10⁻	10	9	9	9	9	8	8	8	8
	$p=0.30$										$p=0.20$									
2	8	7	6	6	6	5	5	5	5	4⁺	7	6	6	6	5	5	5	4	4	4
3	8⁻	7⁺	7	7	6	6	6	6	6	5	7⁺	7	6	6	6	5	5	5	5	5
4	9	8	7⁺	7	7	7	6	6	6	6	8	7	7	6	6	6	6	6	5	5
5	9	8	8	7	7	7	7	7	6	6	8	7	7	7	6	6	6	6	6	5⁻
6	9	8	8	8	7	7	7	7	7	6	8⁻	7⁻	7	7	6⁺	6	6	6	6	6
7	9⁻	8⁺	8	8	7⁻	7	7	7	7	7	8⁻	8	7	7	7	6⁻	6	6	6	6
8	9⁻	9	8	8	8	7	7	7	7	7	8⁻	8	7	7	7	7	6⁻	6	6	6
9	9⁻	9	8	8	8	8	7	7	7	7	8⁺	8	7⁻	7	7	7	7	6⁻	6	6
10	9⁺	9	8⁻	8	8	8	8	7	7	7	8⁺	8	8	7	7	7	7	7	6	6
11	9⁺	9	9	8	8	8	8	8	7	7	9	8	8	7	7	7	7	7	6⁺	6
12	10	9	9	8	8	8	8	8	7	7	9	8	8	7	7	7	7	7	7	6
13	10	9	9	8	8	8	8	8	8	7	9	8	8	7	7	7	7	7	7	7
14	10	9	9	8	8	8	8	8	8	7	9	8	8	7⁻	7	7	7	7	7	7
15	10	9	9	8⁺	8	8	8	8	8	8	9	8	8	8	7	7	7	7	7	7
	$p=0.10$																			
2	6	5	5	4	4	4	4	4	3	3										
3	6⁻	5⁻	5	5	5	4	4	4	4	4										
4	6⁻	6	5	5	5	5	5	4	4	4										
5	6⁺	6	6	5	5	5	5	5	4	4										
6	7	6	6	5	5	5	5	5	5	4										
7	7	6	6	5	5	5	5	5	5	5										
8	7	6	6	5⁺	5	5	5	5	5	5										
9	7	6	6	6	5	5	5	5	5	5										
10	7	6	6	6	5	5	5	5	5	5										
11	7	6	6	6	5	5	5	5	5	5										
12	7	6	6	6	6	5	5	5	5	5										
13	7	6	6	6	6	5	5	5	5	5										
14	7	6	6	6	6	5	5	5	5	5										
15	7	6	6	6	6	5⁺	5	5	5	5										

$N=12$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	9	8	7	7	6	6	6	6	5	5	8+	8	7	6-	6	6	6	5	5	5
3	9-	8	8	7	7	7	7	6-	6	6	9-	8	8	7	7	7	7	6	6	6
4	10	9	8	8	8	7	7	7	7	7	9+	9	8	8	7	7	7	7	7	6
5	10	9	9	8	8	8	8	7	7	7	10	9	9	8	8	8	7	7	7	7
6	10-	9	9	8-	8	8	8	8	7	7	10	9	9	8	8	8	8	8	7	7
7	10-	9+	9	9	8	8	8	8	8	7	10-	9	9	9	8	8	8	8	8	7
8	10-	10	9	9	9	8	8	8	8	8	10-	9+	9	9	8	8	8	8	8	8
9	10+	10	9	9	9	9	8	8	8	8	10-	10	9	9	9	8	8	8	8	8
10	11	10	9+	9	9	9	9	8	8	8	10+	10	9	9	9	9	8	8	8	8
11	11	10	10	9	9	9	9	8+	8	8	10+	10	9-	9	9	9	8	8	8	8
12	11	10	10	9	9	9	9	9	8	8	11	10	10	9	9	9	9	8	8	8
13	11	10	10	9	9	9	9	9	8+	8	11	10	10	9	9	9	9	9	8	8
14	11	10	10	9+	9	9	9	9	9	8	11	10	10	9	9	9	9	9	8	8
15	11	10	10	10	9	9	9	9	9	9	11	10	10	9	9	9	9	9	9	8
	$p=0.30$										$p=0.20$									
2	8-	7	7	6	6	6	5	5	5	5	7-	6-	6	5+	5	5	5	5	4	4
3	9	8	7	7	7	6	6	6	6	6	8	7	7	6	6	6	6	5	5	5
4	9	8	8	7	7	7	7	6	6	6	8-	7	7	7	6	6	6	6	6	5
5	9-	8+	8	8	7	7	7	7	7	6	8-	8	7	7	7	6	6	6	6	6
6	9+	9	8	8	8	7	7	7	7	7	8+	8	7	7	7	7	6	6	6	6
7	9+	9	8-	8	8	8	7	7	7	7	9	8	8	7	7	7	7	6	6	6
8	10	9	9	8	8	8	8	7	7	7	9	8	8	7	7	7	7	7	6	6
9	10	9	9	8	8	8	8	8	7	7	9	8	8	7	7	7	7	7	7	6
10	10	9	9	8	8	8	8	8	8	7	9	8	8	8	7	7	7	7	7	7
11	10	9	9	9	8	8	8	8	8	7	9	8	8	8	7	7	7	7	7	7
12	10	9	9	9	8	8	8	8	8	8	9	8	8	8	7	7	7	7	7	7
13	10-	9-	9	9	8-	8	8	8	8	8	9	8-	8	8	8	7	7	7	7	7
14	10-	9+	9	9	9	8	8	8	8	8	9-	8+	8	8	8	7	7	7	7	7
15	10-	10	9	9	9	8	8	8	8	8	9-	9	8	8	8	7	7	7	7	7
	$p=0.10$																			
2	6	5	5	4	4	4	4	4	4	3										
3	6-	6	5	5	5	5	4	4	4	4										
4	6+	6	6	5	5	5	5	5	4	4										
5	7	6	6	5	5	5	5	5	5	4										
6	7	6	6	6	5	5	5	5	5	5										
7	7	6	6	6	5	5	5	5	5	5										
8	7	6	6	6	6	5	5	5	5	5										
9	7	6-	6	6	6	5	5	5	5	5										
10	7	6+	6	6	6	5+	5	5	5	5										
11	7-	7	6	6	6	6	5	5	5	5										
12	7-	7	6	6	6	6	5-	5	5	5										
13	7-	7	6	6	6	6	6	5	5	5										
14	7-	7	6	6	6	6	6	5	5	5										
15	7-	7	6	6	6	6	6	6	5	5										

$N=13$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	9-	8	7	7	6	6	6	6	5	5	9	8	7	7	6	6	6	6	5	5
3	10	9	8	8	7	7	7	7	6	6	9+	9	8	8	7	7	7	7	6	6
4	10-	9	9	8	8	8	7	7	7	7	10	9	9	8	8	7+	7	7	7	7
5	10-	9-	9	9	8	8	8	8	7	7	10-	9	9	8	8	8	8	8	7	7
6	10+	10	9	9	8+	8	8	8	8	7	10-	10	9	9	8	8	8	8	8	7
7	11	10	9-	9	9	9	8	8	8	8	10+	10	9	9	9	8	8	8	8	8
8	11	10	10	9	9	9	9	8	8	8	11	10	9-	9	9	9	8	8	8	8
9	11	10	10	9	9	9	9	9	8	8	11	10	10	9	9	9	9	8	8	8
10	11	10	10	9-	9	9	9	9	8	8	11	10	10	9	9	9	9	9	8	8
11	11-	10	10	10	9	9	9	9	9	8	11	10	10	9	9	9	9	9	8	8
12	11-	10+	10	10	9	9	9	9	9	9	11	10	10	10	9	9	9	9	9	8
13	11-	11	10	10	10	9	9	9	9	9	11-	10	10	10	9	9	9	9	9	8
14	11-	11	10	10	10	9	9	9	9	9	11-	10+	10	10	9	9	9	9	9	8
15	11+	11	10	10	10	10	9	9	9	9	11-	11	10	10	10	9	9	9	9	9
	$p=0.30$										$p=0.20$									
2	8+	7-	7	6	6	6	6	5	5	5	7+	7	6	6	5	5	5	5	5	4
3	9	8	8	7	7	7	6	6	6	6	8-	7	7	6	6	6	6	6	5	5
4	9-	8+	8	8	7	7	7	7	6	6	8+	8	7	7	7	6	6	6	6	6
5	10	9	8	8	8	7	7	7	7	7	9	8	7+	7	7	7	6	6	6	6
6	10	9	9	8	8	8	7+	7	7	7	9	8	8	7	7	7	7	7	6	6
7	10	9	9	8	8	8	8	8	7	7	9	8	8	7	7	7	7	7	7	6
8	10-	9	9	9	8	8	8	8	8	7	9-	8	8	8	7	7	7	7	7	7
9	10-	9-	9	9	8	8	8	8	8	8	9-	8-	8	8	7	7	7	7	7	7
10	10-	10	9	9	9	8	8	8	8	8	9-	9	8	8	8	7	7	7	7	7
11	10+	10	9	9	9	8	8	8	8	8	9-	9	8	8	8	7	7	7	7	7
12	10+	10	9	9	9	9	8	8	8	8	9+	9	8	8	8	8	7	7	7	7
13	10+	10	9-	9	9	9	8-	8	8	8	9+	9	8	8	8	8	7+	7	7	7
14	11	10	10	9	9	9	9	8	8	8	9+	9	8+	8	8	8	8	7	7	7
15	11	10	10	9	9	9	9	9	8	8	9+	9	9	8	8	8	8	8	7	7
	$p=0.10$																			
2	6-	5	5	5	4	4	4	4	4	3										
3	6+	6	5-	4	4	4	4	4	4	4										
4	7	6	6	5	5	5	5	5	4-	4										
5	7	6	6	6	5	5	5	5	5	5										
6	7	6	6	6	5+	5	5	5	5	5										
7	7-	6+	6	6	6	5	5	5	5	5										
8	7-	7	6	6	6	6	5	5	5	5										
9	7-	7	6	6	6	6	6	5	5	5										
10	7-	7	6	6	6	6	6	5	5	5										
11	7+	7	6-	6	6	6	6	6	5	5										
12	7+	7	6+	6	6	6	6	6	5	5										
13	7+	7	7	6	6	6	6	6	5	5										
14	7+	7	7	6	6	6	6	6	6	5										
15	7+	7	7	6	6	6	6	6	6	5										

$N=14$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	9⁻	8	8	7	7	6	6	6	6	5	9⁻	8	8	7	7	6	6	6	5⁺	5
3	10⁻	9	8⁺	8	8	7	7	7	7	6	10	9	8	8	7	7	7	7	7	6
4	10⁺	9⁺	9	8⁻	8	8	8	8	7	7	10⁻	9	9	8	8	8	8	7	7	7
5	11	10	9	9	9	8	8	8	8	7	10⁺	10	9	9	8	8	8	8	8	7
6	11	10	10	9	9	9	8	8	8	8	11	10	9	9	9	8	8	8	8	8
7	11⁻	10	10	9	9	9	9	8	8	8	11	10	10	9	9	9	8	8	8	8
8	11⁻	10⁻	10	10	9	9	9	9	8	8	11⁻	10	10	9	9	9	9	9	8	8
9	11⁻	11	10	10	9	9	9	9	9	8	11⁻	10	10	9⁺	9	9	9	9	8	8
10	11⁺	11	10	10	10	9	9	9	9	9	11⁻	10⁺	10	10	9	9	9	9	9	8
11	11⁺	11	10	10	10	9	9	9	9	9	11⁻	11	10	10	9⁺	9	9	9	9	9
12	12	11	10⁺	10	10	10	9	9	9	9	11⁺	11	10	10	10	9	9	9	9	9
13	12	11	11	10	10	10	10	9	9	9	11⁺	11	10	10	10	9⁺	9	9	9	9
14	12	11	11	10	10	10	10	9	9	9	12	11	10⁺	10	10	10	9	9	9	9
15	12	11	11	10	10	10	10	10	9	9	12	11	11	10	10	10	10	9	9	9
	$p=0.30$										$p=0.20$									
2	9	8	7	7	6	6	6	6	5	5	8	7	6	6	6	5	5	5	5	4
3	9⁻	8⁻	8	7	7	7	7	6	6	6	8⁻	7⁺	7	7	6	6	6	6	5	5
4	10	9	8	8	8	7	7	7	7	6	9	8	7⁻	7	7	7	6	6	6	6
5	10	9	9	8	8	8	7⁻	7	7	7	9	8	8	7	7	7	7	7	6	6
6	10⁻	9	9	8	8	8	8	8	7	7	9⁻	8	8	8	7	7	7	7	7	6
7	10⁻	9⁺	9	9	8	8	8	8	8	7	9⁻	8⁺	8	8	7	7	7	7	7	7
8	10⁺	10	9	9	9	8	8	8	8	8	9⁺	9	8	8	8	7	7	7	7	7
9	10⁺	10	9	9	9	8⁻	8	8	8	8	9⁺	9	8	8	8	8	7	7	7	7
10	11	10	10	9	9	9	8	8	8	8	10	9	8⁻	8	8	8	7⁻	7	7	7
11	11	10	10	9	9	9	9	8	8	8	10	9	9	8	8	8	8	7	7	7
12	11	10	10	9	9	9	9	9	8	8	10	9	9	8	8	8	8	8	7	7
13	11	10	10	9	9	9	9	9	8	8	10	9	9	8	8	8	8	8	7	7
14	11	10	10	9⁻	9	9	9	9	9	8	10	9	9	8	8	8	8	8	8	7
15	11⁻	10	10	10	9	9	9	9	9	8	10	9	9	8	8	8	8	8	8	7
	$p=0.10$																			
2	6⁻	5⁺	5	5	4	4	4	4	4	4										
3	7	6	6	5	5	5	5	5	4	4										
4	7	6	6	6	5	5	5	5	5	5										
5	7⁻	6⁺	6	6	6	5	5	5	5	5										
6	7⁻	7	6	6	6	6	5	5	5	5										
7	7⁺	7	6	6	6	6	6	5	5	5										
8	7⁺	7	6⁺	6	6	6	6	6	5	5										
9	7⁺	7	7	6	6	6	6	6	5	5										
10	8	7	7	6	6	6	6	6	6	5										
11	8	7	7	6	6	6	6	6	6	5										
12	8	7	7	6	6	6	6	6	6	5⁺										
13	8	7	7	6	6	6	6	6	6	6										
14	8	7	7	6⁻	6	6	6	6	6	6										
15	8	7	7	7	6	6	6	6	6	6										

$N=15$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	9+	8+	8	7	7	7	6	6	6	5	9+	8	8	7	7	6	6	6	6	5
3	10−	9	9	8	8	8	7	7	7	7	10−	9	9	8	8	7	7	7	7	6
4	11	10	9+	9	8	8	8	8	7	7	10+	10	9	9	8	8	8	8	7	7
5	11−	10	10	9	9	9	8	8	8	8	11	10	9+	9	9	8	8	8	8	8
6	11−	10	10	9	9	9	9	8−	8	8	11−	10	10	9	9	9	8+	8	8	8
7	11+	11	10	10	9	9	9	9	8	8	11−	10−	10	9−	9	9	9	9	8	8
8	12	11	10	10	10	9	9	9	9	8	11+	11	10	10	9	9	9	9	9	8
9	12	11	10+	10	10	9−	9	9	9	9	11+	11	10	10	10	9	9	9	9	9
10	12	11	11	10	10	10	9	9	9	9	12	11	10	10	10	9	9	9	9	9
11	12	11	11	10	10	10	10	9	9	9	12	11	11	10	10	10	9	9	9	9
12	12−	11	11	10	10	10	10	10	9	9	12	11	11	10	10	10	10	9	9	9
13	12−	11	11	10+	10	10	10	10	9	9	12	11	11	10	10	10	10	9+	9	9
14	12−	11−	11	11	10	10	10	10	10	9	12	11	11	10	10	10	10	10	9	9
15	12−	12	11	11	10	10	10	10	10	9	12−	11	11	10+	10	10	10	10	9	9
	$p=0.30$										$p=0.20$									
2	9	8	7	7	6	6	6	6	5	5	8−	7	6+	6	6	5	5	5	5	5
3	10	9	8	8	7	7	7	7	6	6	9	8	7	7	6−	6	6	6	6	5
4	10−	9	9	8	8	8	7	7	7	7	9	8	8	7	7	7	7	6	6	6
5	10−	9−	9	8	8	8	8	8	7	7	9−	8−	8	8	7	7	7	7	6	6
6	10+	10	9	9	8	8	8	8	8	7	9+	9	8	8	7−	7	7	7	7	7
7	11	10	9	9	9	8	8	8	8	8	9+	9	8	8	8	7	7	7	7	7
8	11	10	10	9	9	9	8	8	8	8	10	9	8+	8	8	8	7	7	7	7
9	11	10	10	9	9	9	9	8	8	8	10	9	9	8	8	8	8	7	7	7
10	11−	10	10	9	9	9	9	9	8	8	10	9	9	8	8	8	8	8	7	7
11	11−	10	10	10	9	9	9	9	8	8	10	9	9	8	8	8	8	8	8	7
12	11−	10−	10	10	9	9	9	9	9	8	10−	9	9	9	8	8	8	8	8	7
13	11−	11	10	10	9	9	9	9	9	9	10−	9	9	9	8	8	8	8	8	8
14	11+	11	10	10	10	9	9	9	9	9	10−	9−	9	9	8	8	8	8	8	8
15	11+	11	10	10	10	9	9	9	9	9	10−	10	9	9	8+	8	8	8	8	8
	$p=0.10$																			
2	6+	6	5	5	5	4	4	4	4	4										
3	7	6	6	5	5	5	5	5	4	4										
4	7−	6+	6	6	5	5	5	5	5	5										
5	7+	7	6	6	6	6	5	5	5	5										
6	7+	7	6−	6	6	6	6	5	5	5										
7	8	7	7	6	6	6	6	6	5	5										
8	8	7	7	6	6	6	6	6	6	5										
9	8	7	7	6	6	6	6	6	6	5										
10	8	7	7	6−	6	6	6	6	6	6										
11	8	7	7	7	6	6	6	6	6	6										
12	8	7	7	7	6	6	6	6	6	6										
13	8	7	7	7	6	6	6	6	6	6										
14	8	7	7	7	6−	6	6	6	6	6										
15	8	7−	7	7	7	6	6	6	6	6										

$N=16$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	10⁻	9	8	7⁺	7	7	6⁺	6	6	6	10	9	8	7	7	7	6	6	6	6
3	11	10	9	8	8	8	8	7	7	7	10⁺	9⁻	9	8	8	8	7	7	7	7
4	11⁻	10	10	9	9	8	8	8	8	7	11	10	9	9	8⁺	8	8	8	8	7
5	11	10⁻	10	9	9	9	9	8	8	8	11⁻	10	10	9	9	9	8	8	8	8
6	12	11	10	10	9	9	9	9	9	8	11⁺	10⁺	10	10	9	9	9	9	8	8
7	12⁻	11	10⁻	10	10	9	9	9	9	8⁺	12	11	10	10	9	9	9	9	9	8
8	12⁻	11	11	10	10	10	9	9	9	9	12	11	10⁻	10	10	9	9	9	9	9
9	12⁻	11	11	10	10	10	10	9	9	9	12	11	11	10	10	10	9	9	9	9
10	12⁻	11⁻	11	10⁻	10	10	10	10	9	9	12⁻	11	11	10	10	10	10	9	9	9
11	12⁺	12	11	11	10	10	10	10	9	9	12⁻	11	11	10	10	10	10	10	9	9
12	12⁺	12	11	11	10	10	10	10	10	9	12⁻	11⁻	11	11	10	10	10	10	9	9
13	12⁺	12	11	11	11	10	10	10	10	10	12⁻	12	11	11	10	10	10	10	10	9
14	13	12	11	11	11	10	10	10	10	10	12⁺	12	11	11	10	10	10	10	10	9
15	13	12	11⁺	11	11	11	10	10	10	10	12⁺	12	11	11	11	10	10	10	10	10
	$p=0.30$										$p=0.20$									
2	9⁻	8	8	7	7	6	6	6	5⁺	5	8	7	7	6	6	6	5	5	5	5
3	10	9	8	8	7⁻	7	7	7	7	6	9	8	7⁺	7	7	6	6	6	6	6
4	10⁻	9	9	8	8	8	8	7	7	7	9⁻	8	8	7⁻	7	7	7	7	6	6
5	11	10	9	9	8	8	8	8	8	7	9⁺	9	8	8	7	7	7	7	7	6
6	11	10	9⁺	9	9	8	8	8	8	8	10	9	8⁻	8	8	7⁻	7	7	7	7
7	11⁻	10	10	9	9	9	8⁻	8	8	8	10	9	9	8	8	8	8	7	7	7
8	11⁻	10	10	9	9	9	9	9	8	8	10	9	9	8	8	8	8	8	7	7
9	11⁻	10⁻	10	10	9	9	9	9	9	8	10⁻	9	9	8⁻	8	8	8	8	8	7
10	11⁺	11	10	10	10	9	9	9	9	8	10⁻	9⁻	9	9	8	8	8	8	8	7
11	11⁺	11	10	10	10	9	9	9	9	9	10⁻	10	9	9	8	8	8	8	8	8
12	12	11	10	10	10	9	9	9	9	9	10⁺	10	9	9	9	8	8	8	8	8
13	12	11	10⁻	10	10	10	9	9	9	9	10⁺	10	9	9	9	8	8	8	8	8
14	12	11	11	10	10	10	9	9	9	9	10⁺	10	9	9	9	8⁺	8	8	8	8
15	12	11	11	10	10	10	10	9	9	9	11	10	9	9	9	9	8	8	8	8
	$p=0.10$																			
2	5⁻	5	4	4	4	4	3	3	3	3										
3	6	5	5	4	4	4	4	4	4	3										
4	6	5	5	5	4	4	4	4	4	4										
5	6	5	5	5	5	4	4	4	4	4										
6	6	5⁻	5	5	5	4	4	4	4	4										
7	6⁻	6	5	5	5	5	4	4	4	4										
8	6⁻	6	5	5	5	5	5	4	4	4										
9	6⁻	6	5	5	5	5	5	5	4	4										
10	6⁻	6	5	5	5	5	5	5	4	4										
11	6⁻	6	5	5	5	5	5	5	4	4										
12	6⁻	6	5⁺	5	5	5	5	5	5	4										
13	6⁻	6	6	5	5	5	5	5	5	4										
14	6⁻	6	6	5	5	5	5	5	5	4										
15	6⁻	6	6	5	5	5	5	5	5	4⁺										

$N=17$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	10−	9	8	8	7	7	7	6	6	6	10−	9	8	8	7	7	7	6	6	6
3	10−	10	9	9	8	8	8	8	7	7	11	10	9	9	8	8	8	7	7	7
4	11+	10−	10	9	9	9	8	8	8	8	11−	10	10	9	9	8	8	8	8	7
5	12	11	10	10	9	9	9	9	8	8	11+	11	10	9+	9	9	9	8−	8	8
6	12−	11	11	10	10	9	9	9	9	8	12	11	10	10	9	9	9	9	9	8
7	12−	11	11	10	10	10	9	9	9	9	12−	11	11	10	10	9	9	9	9	9
8	12+	11−	11	10	10	10	10	9+	9	9	12−	11	11	10	10	10	9−	9	9	9
9	12+	12	11	11	10	10	10	10	9	9	12−	11−	11	10	10	10	10	10	9	9
10	13	12	11	11	10	10	10	10	10	9	12+	12	11	11	10	10	10	10	9	9
11	13	12	11	11	11	10	10	10	10	10	12+	12	11	11	10	10	10	10	9	9
12	13	12	12	11	11	10+	10	10	10	10	13	12	11	11	11	10	10	10	10	9
13	13	12	12	11	11	11	10	10	10	10	13	12	11	11	11	10	10	10	10	10
14	13−	12	12	11	11	11	11	10	10	10	13	12	12	11	11	11	10	10	10	10
15	13−	12	12	11	11	11	11	10−	10	10	13	12	12	11	11	11	10	10	10	10
	$p=0.30$										$p=0.20$									
2	9+	8	8	7	7	6	6	6	6	5	8+	7+	7	6	6	6	6	5	5	5
3	10−	9	9	8	8	7	7	7	7	6	9−	8	8	7	7	7	6	6	6	6
4	11	10	9	9	8	8	8	8	7	7	9+	9	8	8	7	7	7	7	6	6
5	11	10	9+	9	9	8	8	8	8	7	10	9	8	8	8	7	7	7	7	7
6	11−	10	10	9	9	9	8	8	8	8	10−	9	9	8	8	8	8	7	7	7
7	11−	10−	10	9−	9	9	9	9	8	8	10−	9	9	8	8	8	8	8	7	7
8	11+	11	10	10	9	9	9	9	9	8	10−	9	9	9	8	8	8	8	8	7
9	12	11	10	10	10	9	9	9	9	8	10+	9+	9	9	8	8	8	8	8	7+
10	12	11	10−	10	10	9	9	9	9	9	10+	10	9	9	9	8	8	8	8	8
11	12	11	11	10	10	10	9	9	9	9	11	10	9	9	9	8	8	8	8	8
12	12	11	11	10	10	10	10	9	9	9	11	10	9	9	9	9	8	8	8	8
13	12−	11	11	10	10	10	10	9	9	9	11	10	9−	9	9	9	8	8	8	8
14	12−	11	11	10	10	10	10	10	9	9	11	10	10	9	9	9	9	8	8	8
15	12−	11	11	10+	10	10	10	10	9	9	11	10	10	9	9	9	9	9	8	8
	$p=0.10$																			
2	7	6	5+	5	5	5	4	4	4	4										
3	7−	6+	6	6	5	5	5	5	5	5										
4	8	7	6	6	6	6	5	5	5	5										
5	8	7	7	6	6	6	6	6	5	5										
6	8	7	7	6	6	6	6	6	6	5										
7	8−	7	7	7	6	6	6	6	6	6										
8	8−	7−	7	7	6	6	6	6	6	6										
9	8−	8	7	7	7	6	6	6	6	6										
10	8−	8	7	7	7	6	6	6	6	6										
11	8−	8	7	7	7	7	6	6	6	6										
12	8+	8	7	7	7	7	6	6	6	6										
13	8+	8	7	7	7	7	7	6	6	6										
14	8+	8	7+	7	7	7	7	6	6	6										
15	8+	8	8	7	7	7	7	7	6	6										

$N=18$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	10+	9	9	8	7	7	7	7	6	6	10-	9	8-	8	7	7	7	6-	6	6
3	11-	10	10	9	8+	8	8	8	7	7	11-	10	9	9	8	8	8	8	7	7
4	12	11	10	9+	9	9	9	8	8	8	11+	10+	10	9	9	9	8	8	8	8
5	12-	11	10+	10	10	9	9	9	9	8	12	11	10	10	9	9	9	9	8	8
6	12+	11	11	10	10	10	9	9	9	9	12-	11	11	10	10	9	9	9	9	8+
7	12+	12	11	11	10	10	10	9	9	9	12-	11	11	10	10	10	10	9	9	9
8	13	12	11	11	10	10	10	10	9	9	12+	12	11	11	10	10	10	10	9	9
9	13	12	11-	11	11	10	10	10	10	9	13	12	11	11	10	10	10	10	9-	9
10	13-	12	12	11	11	10+	10	10	10	10	13	12	11	11	11	10	10	10	10	9
11	13-	12	12	11	11	11	10	10	10	10	13	12	12	11	11	10	10	10	10	10
12	13-	12	12	11	11	11	11	10	10	10	13-	12	12	11	11	11	10	10	10	10
13	13-	12-	12	11-	11	11	11	11	10	10	13-	12	12	11	11	11	11	10	10	10
14	13+	13	12	12	11	11	11	11	10	10	13-	12	12	11	11	11	11	10	10	10
15	13+	13	12	12	11	11	11	11	11	10	13-	12-	12	11	11	11	11	11	10	10
	$p=0.30$										$p=0.20$									
2	10	9	8	7	7	7	6	6	6	5-	9	8	7	7	6	6	6	5	5	5
3	10+	9-	9	8	8	8	7	7	7	7	9+	8-	8	7	7	7	7	6	6	6
4	11-	10	9	9	8	8	8	8	7	7	10	9	8	8	7+	7	7	7	7	6
5	11-	10	10	9	9	9	8	8	8	8	10-	9	9	8	8	8	7	7	7	7
6	11+	10+	10	9+	9	9	9	9	8	8	10-	9	9	8	8	8	8	8	7	7
7	12	11	10	10	9	9	9	9	9	8	10+	10	9	9	8	8	8	8	8	7
8	12	11	10-	10	10	9	9	9	9	9	10+	10	9	9	9	8	8	8	8	8
9	12-	11	11	10	10	10	9	9	9	9	11	10	9	9	9	8	8	8	8	8
10	12-	11	11	10	10	10	10	9	9	9	11	10	10	9	9	9	8	8	8	8
11	12-	11	11	10	10	10	10	10	9	9	11	10	10	9	9	9	9	8	8	8
12	12-	11-	11	10+	10	10	10	10	9	9	11	10	10	9	9	9	9	9	8	8
13	12+	12	11	11	10	10	10	10	9-	9	11-	10	10	9	9	9	9	9	8	8
14	12+	12	11	11	10	10	10	10	10	9	11-	10	10	9-	9	9	9	9	8	8
15	12+	12	11	11	10-	10	10	10	10	9-	11-	10	10	10	9	9	9	9	9	8
	$p=0.10$																			
2	7	6	6	5	5	5	4+	4	4	4										
3	7+	7	6	6	6	5	5	5	5	5										
4	8	7	7	6	6	6	6	5	5	5										
5	8	7	7	6	6	6	6	6	5	5										
6	8-	7	7	7	6	6	6	6	6	6										
7	8-	8	7	7	6+	6	6	6	6	6										
8	8-	8	7	7	7	6	6	6	6	6										
9	8+	8	7	7	7	7	6	6	6	6										
10	8+	8	7	7	7	7	6+	6	6	6										
11	9	8	7+	7	7	7	7	6	6	6										
12	9	8	8	7	7	7	7	7	6	6										
13	9	8	8	7	7	7	7	7	6	6										
14	9	8	8	7	7	7	7	7	6	6										
15	9	8	8	7	7	7	7	7	7	6										

$N=19$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	11	9+	9	8	8	7	7	7	6	6	10+	9	9	8	7+	7	7	7	6	6
3	11+	10-	10	9	9	8	8	8	8	7	11+	10	10	9	9	8	8	8	7	7
4	12-	11	10	10	9	9	9	9	8	8	12	11	10	10	9	9	9	8	8	8
5	12+	11	11	10	10	10	9	9	9	8+	12-	11	11	10	10	9	9	9	9	8
6	13	12	11	10+	10	10	10	9	9	9	12+	11-	11	10	10	10	9	9	9	9
7	13	12	11	11	10	10	10	10	9	9	13	12	11	11	10	10	10	10	9	9
8	13-	12	12	11	11	10	10	10	10	9	13	12	11	11	10	10	10	10	10	9
9	13-	12	12	11	11	11	10	10	10	10	13-	12	12	11	11	10	10	10	10	9-
10	13+	12-	12	11	11	11	11	10	10	10	13-	12	12	11	11	11	10	10	10	10
11	13+	13	12	12	11	11	11	11	10	10	13-	12	12	11	11	11	11	10	10	10
12	14	13	12	12	11	11	11	11	10	10	13+	12-	12	11	11	11	11	10+	10	10
13	14	13	12	12	11	11	11	11	11	10	13+	13	12	12	11	11	11	11	10	10
14	14	13	12	12	12	11	11	11	11	10	13+	13	12	12	11	11	11	11	10	10
15	14	13	12+	12	12	11	11	11	11	11	14	13	12	12	11	11	11	11	11	10
	$p=0.30$										$p=0.20$									
2	10-	9	8	7+	7	7	6+	6	6	6	9	8	7	7	6	6	6	6	5	5
3	11	10	9	8-	8	8	8	7	7	7	9+	9	8	8	7	7	7	7	6	6
4	11-	10	10	9	9	8	8	8	8	7	10-	9	9	8	8	7	7	7	7	7
5	11+	10+	10	9	9	9	9	8	8	8	10-	9	9	8	8	8	8	7	7	7
6	12	11	10	10	9	9	9	9	8	8	10+	10	9	9	8	8	8	8	7-	7
7	12-	11	10+	10	10	9	9	9	9	8+	11	10	9	9	9	8	8	8	8	8
8	12-	11	11	10	10	10	9	9	9	9	11	10	9+	9	9	9	8	8	8	8
9	12-	11	11	10	10	10	10	9	9	9	11-	10	10	9	9	9	8-	8	8	8
10	12+	11+	11	11	10	10	10	10	9	9	11-	10	10	9	9	9	9	8	8	8
11	12+	12	11	11	10	10	10	10	9	9	11-	10	10	9	9	9	9	9	8	8
12	13	12	11	11	10	10	10	10	10	9	11-	10-	10	10	9	9	9	9	8	8
13	13	12	11	11	11	10	10	10	10	10	11-	10+	10	10	9	9	9	9	9	8
14	13	12	11-	11	11	10	10	10	10	10	11+	11	10	10	9	9	9	9	9	8+
15	13	12	12	11	11	11	10	10	10	10	11+	11	10	10	10	9	9	9	9	9
	$p=0.10$																			
2	7-	6	6	5	5	5	5	4	4	4										
3	8	7	6	6	6	5	5	5	5	5										
4	8	7	7	6	6	6	6	6	5	5										
5	8-	7	7	7	6	6	6	6	6	5										
6	8-	8	7	7	6+	6	6	6	6	6										
7	8+	8	7	7	7	6	6	6	6	6										
8	8+	8	7	7	7	7	6	6	6	6										
9	9	8	8	7	7	7	7	6	6	6										
10	9	8	8	7	7	7	7	7	6	6										
11	9	8	8	7	7	7	7	7	6	6										
12	9	8	8	7	7	7	7	7	7	6										
13	9	8	8	7	7	7	7	7	7	6										
14	9	8	8	7+	7	7	7	7	7	6+										
15	9	8	8	8	7	7	7	7	7	7										

$N=20$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	11⁻	10	9	8	8	7	7	7	6	6	11⁻	9⁺	9	8	8	7	7	7	6	6
3	12⁻	11	10	9	9	9	8	8	8	7	12	10⁺	10	9	9	8	8	8	8	7
4	12⁺	11	11	10	10	9	9	9	8	8	12⁻	11	10⁻	10	9	9	9	9	8	8
5	13	12	11	10	10	10	9⁺	9	9	9	12⁺	11⁻	11	10	10	10	9	9	9	9
6	13⁻	12	11	11	10	10	10	10	9	9	13	12	11	11	10	10	10	9⁺	9	9
7	13⁻	12	12	11	11	10	10	10	10	9	13⁻	12	11⁻	11	10⁻	10	10	10	9⁻	9
8	13⁺	12⁻	12	11	11	11	10	10	10	10	13⁻	12	12	11	11	10	10	10	10	9⁺
9	13⁺	13	12	11⁻	11	11	11	10	10	10	13⁺	12	12	11	11	11	10	10	10	10
10	14	13	12	12	11	11	11	11	10	10	13⁺	12⁺	12	11	11	11	11	10	10	10
11	14	13	12	12	11	11	11	11	11	10	13⁺	13	12	12	11	11	11	11	10	10
12	14⁻	13	12⁺	12	12	11	11	11	11	10	14	13	12	12	11	11	11	11	10	10
13	14⁻	13	13	12	12	11	11	11	11	11	14	13	12	12	11⁺	11	11	11	11	10
14	14⁻	13	13	12	12	12	11	11	11	11	14	13	12⁺	12	12	11	11	11	11	10⁺
15	14⁻	13	13	12	12	12	11⁺	11	11	11	14⁻	13	13	12	12	11⁻	11	11	11	11
	$p=0.30$										$p=0.20$									
2	10⁻	9	8	8	7	7	7	6	6	6	9	8	7⁻	7	6	6	6	6	5	5
3	11⁻	10	9	9	8	8	8	8	7	7	10	9	8	8	7	7	7	7	6	6
4	11⁺	10⁻	10	9	9	9	8	8	8	8	10⁻	9	9	8	8	8	7	7	7	7
5	12	11	10	10	9	9	9	9	8	8	10⁺	10	9	9	8	8	8	8	7	7
6	12⁻	11	11	10	10	9	9	9	9	8	11	10	9	9	9	8	8	8	8	7
7	12⁻	11	11	10	10	10	9	9	9	9	11⁻	10	10	9	9	9	8	8	8	8
8	12	11⁺	11	10	10	10	10	9	9	9	11⁻	10	10	9	9	9	9	8	8	8
9	13	12	11	11	10	10	10	10	9	9	11⁻	10	10	9	9	9	9	9	8	8
10	13	12	11	11	10	10	10	10	10	9	11⁺	10⁺	10	10	9	9	9	9	8	8
11	13	12	11⁻	11	11	10	10	10	10	9	11⁺	11	10	10	9	9	9	9	9	8
12	13⁻	12	12	11	11	10	10	10	10	10	11⁺	11	10	10	9	9	9	9	9	8⁻
13	13⁻	12	12	11	11	11	10	10	10	10	12	11	10	10	10	9	9	9	9	9
14	13⁻	12	12	11	11	11	11	10	10	10	12	11	10	10	10	9	9	9	9	9
15	13⁻	12	12	11	11	11	11	10	10	10	12	11	10⁺	10	10	10	9	9	9	9
	$p=0.10$																			
2	7⁻	6	6	5	5	5	5	5	4	4										
3	8	7	6⁺	6	6	6	5	5	5	5										
4	8⁻	7	7	6	6	6	6	6	5	5										
5	8⁻	8	7	7	6	6	6	6	6	6										
6	8⁺	8	7	7	7	6	6	6	6	6										
7	9	8	7⁺	7	7	7	6	6	6	6										
8	9	8	8	7	7	7	7	6	6	6										
9	9	8	8	7	7	7	7	7	6	6										
10	9⁻	8	8	7	7	7	7	7	7	6										
11	9	8	8	7⁺	7	7	7	7	7	6										
12	9⁻	8	8	8	7	7	7	7	7	7										
13	9⁻	8	8	8	7	7	7	7	7	7										
14	9⁻	8⁻	8	8	7	7	7	7	7	7										
15	9⁻	8⁺	8	8	7⁺	7	7	7	7	7										

$N=22$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	11+	10	9−	9	8	8	7	7	7	6	11+	10	9	8−	8	8	7	7	7	6
3	12+	11	10+	10	9	9	9	8	8	8	12−	11	10	10	9	9	9	8	8	8
4	13−	12	11	10	10	10	10	9	9	9	13	12	11	10	10	10	9	9	9	8
5	13+	12	12	11	10+	10	10	10	9	9	13−	12	11	11	10	10	10	10	9	9
6	14	12+	12	11	11	11	10	10	10	9+	13+	12	12	11	11	10	10	10	10	9
7	14−	13	12	12	11	11	11	10	10	10	14	13	12	11	11	11	10	10	10	10
8	14−	13	12−	12	11	11	11	11	10	10	14	13	12	12	11	11	11	10+	10	10
9	14−	13	13	12	12	11	11	11	11	10	14−	13	12	12	11	11	11	11	10	10
10	14+	13	13	12	12	12	11	11	11	11	14−	13	12	12	12	11	11	11	11	10
11	14+	13+	13	12	12	12	11+	11	11	11	14−	13	13	12	12	11+	11	11	11	11
12	15	14	13	12+	12	12	12	11	11	11	14+	13	13	12	12	12	11	11	11	11
13	15	14	13	13	12	12	12	12	11	11	14+	13+	13	12	12	12	12	11	11	11
14	15	14	13	13	12	12	12	12	11	11	14+	14	13	13	12	12	12	12	11	11
15	15−	14	13−	13	12+	12	12	12	12	11	15	14	13	13	12	12	12	12	11	11
	$p=0.30$										$p=0.20$									
2	11	9−	9	8	8	7	7	7	6	6	9+	8−	8	7	7	6	6	6	6	5
3	11+	10	10	9	9	8	8	8	7	7	10−	9	9	8	8	7	7	7	7	6
4	12−	11	10	10	9	9	9	9	8	8	11	10	9	9	8	8	8	8	7	7
5	12+	11	11	10	10	9	9	9	9	8	11−	10	9+	9	9	8	8	8	8	7
6	13	12	11	10	10	10	10	9	9	9	11−	10	10	9	9	9	8	8	8	8
7	13	12	11	11	10	10	10	10	9	9	11+	10+	10	9−	9	9	9	9	8	8
8	13−	12	11+	11	11	10	10	10	10	9	12	11	10	10	9	9	9	9	8	8
9	13−	12	12	11	11	10−	10	10	10	10	12	11	10	10	10	9	9	9	9	8
10	13+	12	12	11	11	11	10	10	10	10	12	11	10+	10	10	9	9	9	9	9
11	13+	12−	12	11	11	11	11	10	10	10	12−	11	11	10	10	10	9	9	9	9
12	13+	13	12	12	11	11	11	11	10	10	12−	11	11	10	10	10	9−	9	9	9
13	14	13	12	12	11	11	11	11	10	10	12−	11	11	10	10	10	10	9	9	9
14	14	13	12	12	11	11	11	11	11	10	12−	11	11	10	10	10	10	10	9	9
15	14	13	12	12	12	11	11	11	11	10	12−	11−	11	10+	10	10	10	10	9	9
	$p=0.10$																			
2	7+	7	6	6	5	5	5	5	4	4										
3	8−	7	7	6	6	6	6	5	5	5										
4	8+	8	7	7	6	6	6	6	6	5										
5	9	8	7−	7	7	7	6	6	6	6										
6	9	8	8	7	7	7	7	6	6	6										
7	9−	8	8	7	7	7	7	7	6	6										
8	9−	8	8	8	7	7	7	7	7	6										
9	9−	8−	8	8	7	7	7	7	7	7										
10	9−	9	8	8	7−	7	7	7	7	7										
11	9+	9	8	8	8	7	7	7	7	7										
12	9+	9	8	8	8	7	7	7	7	7										
13	9+	9	8	8	8	8	7	7	7	7										
14	10	9	8−	8	8	8	7	7	7	7										
15	10	9	8+	8	8	8	8	7	7	7										

$N=25$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	12−	11	10	9	9	8	8	8	7	7	12−	11	10	9	8	8	8	7−	7	7
3	13−	12	11	10	10	10	9	9	9	8	13−	12	11	10	10	9	9	9	8	8
4	14−	12+	12	11	11	10	10	10	9	9	13+	12	12	11	10	10	10	10	9	9
5	14−	13	12	12	11	11	11	10	10	10	14−	13	12	11	11	11	10	10	10	9
6	14+	13	13	12	12	11	11	11	10	10	14−	13	12−	12	11	11	11	11	10	10
7	15	14	13	12	12	12	11	11	11	10	14+	13	13	12	12	11	11	11	11	10
8	15−	14	13	13	12	12	12	11	11	11	15	14	13	12	12	12	11	11	11	11
9	15−	14	13−	13	12	12	12	12	11	11	15−	14	13	13	12	12	12	11	11	11
10	15+	14	14	13	13	12	12	12	11	11	15−	14	13	13	12	12	12	12	11	11
11	15+	14	14	13	13	12	12	12	12	11	15−	14	13+	13	12+	12	12	12	11	11
12	15+	14+	14	13	13	13	12	12	12	12	15+	14	14	13	13	12	12	12	12	11
13	16	15	14	13	13	13	13	12	12	12	15+	14	14	13	13	13	12	12	12	12
14	16	15	14	14	13	13	13	12	12	12	15+	14+	14	13	13	13	12	12	12	12
15	16−	15	14	14	13	13	13	13	12	12	16	15	14	13	13	13	13	12	12	12
	$p=0.30$										$p=0.20$									
2	11+	10	9	8−	8	8	7	7	7	6	10−	9	8	7+	7	7	6−	6	6	6
3	12−	11	10	10	9	9	9	8	8	8	11−	10	9	8+	8	8	8	7	7	7
4	13	12	11	10	10	9+	9	9	9	8	11+	10	10	9	9	8	8	8	8	7
5	13−	12	11	11	10	10	10	10	9	9	12	11	10	9+	9	9	9	8	8	8
6	13+	12	12	11	11	10	10	10	10	9	12−	11	10	10	9	9	9	9	8	8
7	14	13	12	11	11	11	10	10	10	10	12−	11	11	10	10	9	9	9	9	9
8	14−	13	12	12	11	11	11	10−	10	10	12+	11	11	10	10	10	9	9	9	9
9	14−	13	12	12	11	11	11	11	10	10	12+	11+	11	10	10	10	10	9	9	9
10	14−	13	13	12	12	11	11	11	11	10	13	12	11	11	10	10	10	10	9	9
11	14+	13	13	12	12	11−	11	11	11	11	13	12	11	11	10	10	10	10	9+	9
12	14+	13−	13	12	12	12	11	11	11	11	13	12	11	11	11	10	10	10	10	9
13	14+	13+	13	12	12	12	12	11	11	11	13−	12	11+	11	11	10	10	10	10	10
14	15	14	13	12+	12	12	12	11+	11	11	13−	12	12	11	11	10+	10	10	10	10
15	15	14	13	13	12	12	12	12	11	11	13−	12	12	11	11	11	10	10	10	10
	$p=0.10$									
2	8−	7	6+	6	6	5	5	5	5	4
3	8+	8	7	7	6	6	6	6	6	5
4	9−	8	8	7	7	7	6	6	6	6
5	9−	8	8	7	7	7	7	7	6	6
6	9+	9	8	8	7	7	7	7	7	6
7	9+	9	8	8	8	7	7	7	7	7
8	10	9	8	8	8	7	7	7	7	7
9	10	9	9	8	8	8	7	7	7	7
10	10	9	9	8	8	8	8	7	7	7
11	10−	9	9	8	8	8	8	8	7	7
12	10−	9	9	8	8	8	8	8	7	7
13	10−	9	9	8−	8	8	8	8	7+	7
14	10−	9	9	9	8	8	8	8	8	7
15	10−	9−	9	9	8	8	8	8	8	7

$N=27$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	13−	11	10	9−	9	8+	8	8	7	7	12+	11	10	9	9	8	8	8	7	7
3	14−	12	12	11	10	10	10	9	9	8+	13+	12	11	11	10	10	9	9	9	8
4	14+	13	12	11−	11	11	10	10	10	9	14−	13	12	11	11	10	10	10	10	9
5	15−	13−	13	12	12	11	11	11	10	10	14+	13	12+	12	11	11	11	10−	10	10
6	15−	14	13	12	12	12	11	11	11	10	15−	14	13	12	12	11	11	11	11	10
7	15+	14	13−	13	12	12	12	11	11	11	15−	14	13	12+	12	12	11−	11	11	11
8	15+	14	14	13	13	12	12	12	11	11	15−	14	13−	13	12	12	12	12	11	11
9	16	15	14	13	13	13	12	12	12	11	15+	14	14	13	13	12	12	12	11	11
10	16−	15	14	13	13	13	12	12	12	12	16	14+	14	13	13	12−	12	12	12	11
11	16−	15	14	14	13	13	13	12	12	12	16	15	14	13	13	13	12	12	12	12
12	16−	15	14−	14	13	13	13	13	12	12	16−	15	14	14	13	13	13	12	12	12
13	16+	15	15	14	14	13	13	13	12	12	16−	15	14	14	13	13	13	13	12	12
14	16+	15	15	14	14	13	13	13	13	12	16−	15	14−	14	13	13	13	13	12	12
15	16+	15−	15	14	14	13+	13	13	13	12	16−	15	15	14	14	13	13	13	12−	12
	$p=0.30$										$p=0.20$									
2	12−	10	10	9	8	8	8	7	7	7	10+	9	8+	8	7	7	7	6+	6	6
3	13	11	11	10	9	9	9	9	8	8	11−	10	9+	9	8	8	8	8	7	7
4	13−	12	11	11	10	10	10	9	9	9	12	11	10	9	9	9	8	8	8	8
5	14	12−	12	11	11	10	10	10	9+	9	12−	11	10−	10	9	9	9	9	8	8
6	14−	13	12	11−	11	11	10−	10	10	10	12+	11	11	10	10	9+	9	9	9	9
7	14−	13	12−	12	11	11	11	11	10	10	13	12	11	10	10	10	10	9	9	9
8	14+	13	13	12	12	11	11	11	11	10	13	12	11	11	10	10	10	10	9	9
9	15	13−	13	12	12	12	11	11	11	10+	13−	12	11	11	10	10	10	10	10	9
10	15	14	13	12	12	12	11+	11	11	11	13−	12	12	11	11	10	10	10	10	9
11	15−	14	13	13	12	12	12	11−	11	11	13−	12	12	11	11	11	10	10	10	10
12	15−	14	13	13	12	12	12	12	11	11	13−	12	12	11	11	11	10	10	10	10
13	15−	14	13−	13	12+	12	12	12	11	11	13+	12−	12	11	11	11	11	10	10	10
14	15−	14	14	13	13	12	12	12	12	11	13+	12+	12	11	11	11	11	11	10	10
15	15−	14	14	13	13	12	12	12	12	11	13+	13	12	12	11	11	11	11	10	10
	$p=0.10$																			
2	8−	7	7	6	6	5+	5	5	5	5										
3	9	8	7	7	7	6	6	6	6	5										
4	9−	8	8	7	7	7	7	6	6	6										
5	9+	9	8	8	7	7	7	7	7	6										
6	10	9	8	8	8	7	7	7	7	7										
7	10	9	9	8	8	8	7	7	7	7										
8	10−	9	9	8	8	8	8	7	7	7										
9	10−	9	9	8	8	8	8	8	7	7										
10	10−	9	9	8+	8	8	8	8	7	7										
11	10−	9+	9	9	8	8	8	8	8	7										
12	10+	10	9	9	8	8	8	8	8	7+										
13	10+	10	9	9	8+	8	8	8	8	8										
14	10+	10	9	9	9	8	8	8	8	8										
15	11	10	9	9	9	8	8	8	8	8										

$N=30$

k \ α	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.001	0.005	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	$p=0.50$										$p=0.40$									
2	13+	12	11	10	9	9	9	8	8	7	13−	12	11	10	9	9	8	8	8	7
3	14+	13	12	11	11	10	10	10	9	9	14−	13	12	11	11	10	10	10	9	9
4	15−	14	13	12	12	11	11	11	10	10	15−	13−	13	12	11	11	11	10	10	10
5	15+	14	13−	13	12	12	11	11	11	10	15−	14	13	12	12	12	11	11	11	10
6	16−	14+	14	13	13	12	12	12	11	11	15+	14	14	13	12	12	12	11	11	11
7	16−	15	14	13	13	13	12	12	12	11	16−	15	14	13	13	12	12	12	11	11
8	16+	15	14−	14	13	13	13	12	12	12	16−	15	14	13	13	13	12	12	12	11
9	16+	15	15	14	13−	13	13	13	12	12	16−	15	14	14	13	13	13	12	12	12
10	17−	15+	15	14	14	13	13	13	13	12	16+	15	15	14	13	13	13	13	12	12
11	17−	16	15	14	14	14	13	13	13	12	16+	15−	15	14	14	13	13	13	12	12
12	17−	16	15	15	14	14	14+	13	13	13	17−	16	15	14	14	13+	13	13	13	12
13	17−	16	15	15	14	14	14	13	13	13	17−	16	15	14	14	14	13	13	13	13
14	17+	16	15+	15	14	14	14	14	13	13	17−	16	15	15	14	14	14	13	13	13
15	17+	16	16	15	15	14	14	14	13	13	17−	16	15	15	14	14	14	13	13	13
	$p=0.30$										$p=0.20$									
2	12+	11	10	9	9	8	8	8	7	7	11−	10	9	8	8	7	7	7	6	6
3	13+	12	11	10	10	10	9	9	9	8	12−	11	10	9	9	8−	8	8	8	7
4	14−	13	12	11	11	10	10	10	9	9	12+	11	11	10	9	9	9	9	8	8
5	14+	13	12	12	11	11	11	10	10	10	13	12	11	10	10	10	9	9	9	9
6	15	13−	13	12	12	11	11	11	10	10	13−	12	11	11	10	10	10	10	9	9
7	15−	14	13	12	12	12	11	11	11	10	13−	12	12	11	11	10	10	10	10	9
8	15−	14	13	13	12	12	12	11	11	11	13+	12	12	11	11	10+	10	10	10	10
9	15+	14	14	13	12	12	12	12	11	11	14	12+	12	11	11	11	10−	10	10	10
10	15+	14	14	13	13	12	12	12	12	11	14	13	12	12	11	11	11	10	10	10
11	16	14+	14	13	13	13	12	12	12	11	14−	13	12	12	11	11	11	11	10	10
12	16−	15	14	13	13	13	12	12	12	12	14−	13	12	12	11	11	11	11	10	10
13	16−	15	14	14	13	13	13	12	12	12	14−	13	13	12	12	11	11	11	11	10
14	16−	15	14	14	13	13	13	13	12	12	14−	13	13	12	12	11	11	11	11	11
15	16−	15	14	14	13	13	13	13	12	12	14−	13	13	12	12	12	11	11	11	11
	$p=0.10$																			
2	8+	8	7	6	6	6	6	5	5	5										
3	9−	8	8	7	7	7	6	6	6	6										
4	10	9	8	8	7	7	7	7	6	6										
5	10−	9	9	8	8	7	7	7	7	7										
6	10−	9	9	8	8	8	8	7	7	7										
7	10+	9−	9	8−	8	8	8	8	7	7										
8	10+	10	9	9	8	8	8	8	8	7										
9	11	10	9	9	8−	8	8	8	8	7+										
10	11	10	9	9	9	8	8	8	8	8										
11	11	10	9+	9	9	9	8	8	8	8										
12	11−	10	10	9	9	9	8	8	8	8										
13	11−	10	10	9	9	9	9	8	8	8										
14	11−	10	10	9	9	9	9	8	8	8										
15	11−	10	10	9	9	9	9	9	8	8										

Errata

Annals, Vol. X, No. 1.

Page	Line	Read	Instead of
48	13	$V_{(6)}=15$	$V_{(6)}=16$
"	14	(8 17)	(8 7)

Vol. X, No. 3.

200	17—18	$\cdots \sum_{j=0}^{\infty} \frac{\Gamma(p/2+1+j)}{j!}\left(\frac{\delta}{\gamma}\right)^{2j}$ $\times\left[\left(\eta+\frac{p}{2}-j\right)\cdots\right.$	$\cdots \sum_{j=0}^{\infty} \frac{\Gamma(p/2+1+j)}{j!}$ $\times\left[\left(\eta+\frac{p}{2}-j\right)\cdots\right.$

Vol. XI, No. 2.

108	16	$[X]_n$	$[X_n]$
112	11	while X_{ii} are defined by···	X_{ij} defined by···