# AN APPROXIMATION METHOD IN NUMERICAL COMPUTATION OF THE LEONTIEF'S OPEN INPUT-OUTPUT MODEL

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## 1. Introduction

In the Leontief's open input-output model, the numerical value in each industry associated with a given amount of final demand is usually computed by means of the inverse matrix. It, however, is not a easy job to obtain the inverse of given matrix. Concerning this, it seems to have been considered as necessary to use a large scale computer when the number of industries is large. In this article we shall give simple ways to compute approximate values, in which use is made only of a table-computer.

## 2. Methods of approximation

Let $Y_i$ denote the given amount of final demand in the $i$-th industry, $X_i$ that of out-put to be obtained in the $i$-th industry associated with these final demands $Y_i$'s, and $a_{ij}$ technical coefficients. Then the Leontief's system of linear equations is written as follows:

$$(1)\qquad \begin{cases} (1-a_{11})X_1-a_{12}X_2 \quad -a_{13}X_3-\cdots-a_{1n}X_n = Y_1 \\ -a_{21}X_1 \quad +(1-a_{22})X_2-a_{23}X_3-\cdots-a_{2n}X_n = Y_2 \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ -a_{n1}X_1 \quad -a_{n2}X_2 \quad -a_{n3}X_3-\cdots+(1-a_{nn})X_n=Y_n \end{cases}$$

These linear equations can also be written as

$$(2)\qquad X_i=\frac{1}{b_{ii}}\left\{Y_i+\sum_{j\neq i}a_{ij}X_j\right\}, \qquad i=1,\cdots,n$$

where $b_{ii}=1-a_{ii}$.

If the amounts of the final demands and those of outputs used in calculating these technical coefficients $a_{ij}$ are represented by $Y_i'$'s and $X_i'$'s, these values must also satisfy the following equations.

$$(3)\qquad X_i'=\frac{1}{b_{ii}}\left\{Y_i'+\sum_{j\neq i}a_{ij}X_j'\right\}, \qquad (i=1, 2, \cdots n)$$

Then we put $X_i^{(1)}$

$$X_i^{(1)}=X_i'+\frac{1}{b_{ii}}(Y_i-Y_i') \tag{4}$$

which is taken as an approximation to $X_i$. These values can be easily calculated, for $X_i'$'s, $Y_i'$'s are known and $Y_i$ are given.

For the purpose of getting a better approximation, we estimate the error of that approximation. From (3) and (4), we obtain

$$X_i^{(1)}=\frac{1}{b_{ii}}\left\{Y_i'+(Y_i-Y_i')+\sum_{j\neq i}a_{ij}X_j'\right\}=\frac{1}{b_{ii}}\left\{Y_i+\sum_{j\neq i}a_{ij}X_j'\right\}. \tag{5}$$

The relative error of the approximation of $X_i$ is then represented as

$$\frac{X_i-X_i^{(1)}}{X_i}=\frac{1}{X_i}\frac{1}{b_{ii}}\sum_{j\neq i}a_{ij}(X_j-X_j')=\frac{1}{X_i}\frac{1}{b_{ii}}\sum_{j\neq i}a_{ij}X_j\frac{(X_j-X_j')}{X_j} \tag{6}$$

$$=\frac{1}{X_i}\frac{\sum_{j\neq i}a_{ij}X_j}{b_{ii}}\cdot\frac{\sum_{j\neq i}a_{ij}X_j\frac{(X_j-X_j')}{X_j}}{\sum_{j\neq i}a_{ij}X_j}.$$

Since

$$\sum_{j\neq i}a_{ij}X_j=b_{ii}X_i-Y_i, \tag{7}$$

from (6) and (7), we have

$$\frac{X_i-X_i^{(1)}}{X_i}=\frac{1}{X_i}\cdot\frac{b_{ii}X_i-Y_i}{b_{ii}}\cdot\frac{\sum_{j\neq i}a_{ij}X_j\frac{(X_j-X_j')}{X_j}}{\sum_{j\neq i}a_{ij}X_j} \tag{8}$$

$$=\frac{1}{X_i}\cdot\left(X_i-\frac{Y_i}{b_{ii}}\right)\frac{\sum_{j\neq i}a_{ij}X_j\frac{(X_j-X_j')}{X_j}}{\sum_{j\neq i}a_{ij}X_j}.$$

Subtracting (3) from (2) side by side, we get

$$X_i-X_i'=\frac{1}{b_{ii}}\left\{(Y_i-Y_i')+\sum_{j\neq i}a_{ij}(X_j-X_j')\right\}. \tag{9}$$

From (9) and (8) it follows that

$$\frac{X_i-X_i^{(1)}}{X_i}=\frac{X_i-\frac{Y_i}{b_{ii}}}{X_i}\cdot\left\{\frac{\sum_{j\neq i}a_{ij}X_j\frac{(Y_j-Y_j')/b_{jj}}{X_j}}{\sum_{j\neq i}a_{ij}X_j}+\frac{\sum_{j\neq i}a_{ij}\frac{1}{b_{jj}}\sum_{k\neq j}a_{jk}(X_k-X_k')}{\sum_{j\neq i}a_{ij}X_j}\right\}. \tag{10}$$

As $X_i$ are unknown, we estimate these values by replacing $X_i$'s, $Y_i$'s by $X_i''$s, $Y_i''$s and by neglecting the second term in the braces of (10), that is, we estimate the values from

$$\frac{X_i - X_i^{(1)}}{X_i} \doteqdot \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \frac{\sum_{j \neq i} a_{ij} X_j' \cdot \frac{(Y_j - Y_j')/b_{jj}}{X_j'}}{\sum_{j \neq i} a_{ij} X_j'} \tag{11}$$

Now, $a_{ij}X_j'$ is the input of the $i$-th industry into the $j$-th industry, and may be positively correlated with $X_i'$ for fixed $i$. Therefore, replacing $a_{ij}X_j'$ by $X_j'$, we have

$$\frac{X_i - X_i^{(1)}}{X_i} \doteqdot \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \frac{\sum_{j \neq i} X_j' \cdot \frac{(Y_j - Y_j')/b_{jj}}{X_j'}}{\sum_{j \neq i} X_j'} \tag{12}$$

$$= \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \frac{\sum_{j \neq i} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j \neq i} X_j'} .$$

Using these values, we obtain a better approximation of $X_i$

$$X_i^{(2)} = X_i^{(1)} \left( 1 + \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i} \cdot \frac{\sum_{j=1}^{n} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j=1}^{n} X_j'} \right) \tag{13}$$

If $\frac{(Y_i - Y_i')/b_{ii}}{X_i}$ has an extra-ordinary value for large $X_i'$, we should use $\frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \frac{\sum_{j \neq i} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j \neq i} X_j'}$ for the second term in the parentheses of (13) intead of $\frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \frac{\sum_{j=1}^{n} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j=1}^{n} X_j'}$.

From the equations (2) and (3) we have

$$X_i = \frac{1}{b_{ii}} \left\{ Y_i + \sum_{j \neq i} a_{ij} \frac{Y_j}{b_{jj}} + \sum_{j \neq i} a_{ij} \frac{1}{b_{jj}} \sum_{k \neq j} a_{jk} X_k \right\} \tag{14}$$

$$X_i' = \frac{1}{b_{ii}} \left\{ Y_i' + \sum_{j \neq i} a_{ij} \frac{Y_j'}{b_{jj}} + \sum_{j \neq i} a_{ij} \frac{1}{b_{jj}} \sum_{k \neq j} a_{jk} X_k' \right\} \tag{15}$$

Subtracting (15) from (14) side by side and neglecting the third term, we have

(16) $$X_i - X_i' \doteqdot \frac{Y_i - Y_i'}{b_{ii}} + \frac{1}{b_{ii}} \sum_{j \neq i} a_{ij} \frac{(Y_j - Y_j')}{b_{jj}}$$

The second term of the right hand side of (16) can be estimated in the similar way as in obtaining (12), that is,

(17) $$\frac{1}{b_{ii}} \sum_{j \neq i} a_{ij} \frac{Y_j - Y_j'}{b_{jj}} = \frac{1}{b_{ii}} \sum_{j \neq i} a_{ij} X_j' \frac{(Y_j - Y_j')/b_{jj}}{X_j'}$$

$$= \left( X_i' - \frac{Y_i'}{b_{ii}} \right) \cdot \frac{\sum_{j \neq i} a_{ij} X_j' \frac{(Y_j - Y_j')/b_{jj}}{X_j'}}{\sum_{j \neq i} a_{ij} X_j'}$$

$$= \left( X_i' - \frac{Y_i'}{b_{ii}} \right) \cdot \frac{\sum_{j \neq i} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j \neq i} X_j'}$$

Then substituting (17) into (16), we get an approximation $X_i^{(3)}$ of $X_i$

(18) $$X_i^{(3)} = X_i' + \frac{Y_i - Y_i'}{b_{ii}} + \left( X_i' - \frac{Y_i'}{b_{ii}} \right) \cdot \frac{\sum_{j=1}^{n} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j=1}^{n} X_j'}$$

$$= X_i^{(1)} + X_i' \cdot \frac{\left( X_i' - \frac{X_i'}{b_{ii}} \right)}{X_i'} \cdot \frac{\sum_{j=1}^{n} \frac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j=1}^{n} X_j'} .$$

For large $X_i'$ the same consideration must be taken into account as in obtaining $X_i^{(2)}$.

## 3. Errors and remark

The relative error of the approximation $X_i^{(1)}$ is of nearly the same as the value of the second term of the approximation $X_i^{(2)}$. For the short term forecasting, even that approximation may be satisfactory.

For the long range forecasting it is better to use the approximation $X_i^{(2)}$ or $X_i^{(3)}$. In these two approximations, the second term multiplier of $X_i^{(1)}$ in $X_i^{(2)}$ and that of $X_i'$ in $X_i^{(3)}$ are of the same. And the error of this term comes from the discrepancy between the weights used in computation, $X_i'$, and the true weight $a_{ij}X_j$, and from the replacement of $X_i$'s, $Y_i$'s by $X_i'$'s, $Y_i'$'s. The error caused by these replacements is fairly small. It is the error due to the discrepancy between the weights that must be taken up. For the evaluation of this error, we estimate the difference between weights $d_{ij}$ by the difference between $\frac{X'_j}{\sum X'_j}$ and $\frac{a_{ij}X'_j}{X'_i-\frac{Y'_i}{b_{ii}}}$ for such a $j$ that $\frac{(Y_j-Y'_j)/b_{jj}}{X'_j}=p_j$ has an extraordinary value. It can be seen that $\frac{X'_i-\frac{Y'_i}{b_{ii}}}{X'_i}\cdot\sum_j d_{ij}p_j$ may give the rough estimate of the error in consideration.

Next, we estimate the neglected term by the formula

$$
(19)\qquad \frac{1}{X_i}\frac{1}{b_{ii}}\sum_{j\neq i}a_{ij}\frac{1}{b_{jj}}\sum_{k\neq j}a_{jk}(X_k-X'_k)
$$

$$
\doteqdot\frac{1}{X'_i}\frac{1}{b_{ii}}\sum_{j\neq i}a_{ij}\frac{1}{b_{jj}}\sum_{k\neq j}a_{jk}\frac{Y_k-Y'_k}{b_{kk}}
$$

$$
=\frac{1}{X'_i}\frac{1}{b_{ii}}\sum_{j\neq i}a_{ij}\frac{\sum_{k\neq j}a_{jk}X'_k}{b_{jj}}\cdot\frac{\sum_{k\neq j}a_{jk}X'_k\frac{(Y_k-Y'_k)/b_{kk}}{X'_k}}{\sum_{k\neq j}a_{jk}X'_k}
$$

$$
\doteqdot\frac{1}{X'_i}\cdot\frac{\sum_{j\neq i}a_{ij}X'_j}{b_{ii}}\cdot\frac{\sum_{j\neq i}a_{ij}X'_j\frac{\left(X'_j-\frac{Y'_j}{b_{jj}}\right)}{X'_j}}{\sum_{j\neq i}a_{ij}X'_j}\cdot\frac{\sum_{k=1}^{n}a_{jk}X'\frac{(Y_k-Y'_k)/b_{kk}}{X'_k}}{\sum_{k=1}^{n}a_{jk}X'_k}
$$

$$
\doteqdot\frac{\left(X'_i-\frac{Y'_i}{b_{ii}}\right)}{X'_i}\cdot\frac{\sum_{j\neq i}\left(X'_j-\frac{Y'_j}{b_{jj}}\right)}{\sum_{j\neq i}X'_j}\cdot\frac{\sum_{k=1}^{n}\frac{(Y_k-Y'_k)}{b_{kk}}}{\sum_{k=1}^{n}X'_k}
$$

$$
=\frac{\left(X'_i-\frac{Y'_i}{b_{ii}}\right)}{X'_i}\cdot\frac{\sum_{j=1}^{n}\left(X'_j-\frac{Y'_j}{b_{jj}}\right)}{\sum_{j=1}^{n}X'_j}\cdot\frac{\sum_{k=1}^{n}\frac{(Y_k-Y'_k)}{b_{kk}}}{\sum_{k=1}^{n}X'_k}
$$

From two values estimated above, the roughly estimated error can be obtained for $X_i^{(2)}$ and $X_i^{(3)}$. When this relative error is negative, we use $X_i^{(2)}$. When it is positive, we use $X_i^{(3)}$.

When some of technical coefficients are changed from those obtained from the survey by taking into account technological change, we should use the values of the final demands computed from the changed technical coefficients and the output data of the survey. These values of the final demands can easily be computed.

If a further better approximation is wanted, we can get it by the successive approximation method, in which $X_i^{(2)}$ or $X_i^{(3)}$ is to be used the first approximation. In this case, stratifying $X_i'$'s, we can use in place of each $X_i'$ the mean of the stratum in which $X_i'$ lies. In this way the computation is made much simpler.

## 4. Example

This table of technical coefficients is concerned with the Japan Economy in 1951 and is presented by Japan Economic Council Board. $X_i$'s in 1952 are forecast while $Y_i$'s are given.

Industry 1. Agriculture, Forestry and Fisheries
2. Mining 3. Construction
4. Manufacturing 5. Whole sale and Retail Trade
6. Transportation and Communication
7. Public Utilities 8. Service
9. Unknown.

Technical Coefficients

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.128667 | 0.028314 | 0.017017 | 0.094223 | 0.002212 |
| 2 | 0.000633 | 0.019320 | 0.027813 | 0.030733 | 0.000466 |
| 3 | 0.003796 | 0.005330 | 0.005124 | 0.004937 | 0.008963 |
| 4 | 0.071494 | 0.124251 | 0.436780 | 0.413359 | 0.028635 |
| 5 | 0.031059 | 0.018654 | 0.043001 | 0.042147 | 0.006053 |
| 6 | 0.018866 | 0.048301 | 0.067521 | 0.038398 | 0.053661 |
| 7 | 0.000633 | 0.030646 | 0.000549 | 0.016016 | 0.010825 |
| 8 | 0.018751 | 0.026649 | 0.029643 | 0.018262 | 0.044695 |
| 9 | 0.008570 | 0.098268 | 0.068070 | 0.033573 | 0.022349 |

| | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| 1 | 0.004733 | 0. | 0.028374 | 0.073596 |
| 2 | 0.043543 | 0.312999 | 0.001587 | 0.007885 |
| 3 | 0.017850 | 0.037058 | 0.010360 | 0. |
| 4 | 0.151589 | 0.045610 | 0.089322 | 0.210514 |
| 5 | 0.013387 | 0.015393 | 0.017547 | 0.030108 |
| 6 | 0.037593 | 0.130559 | 0.046108 | 0.226284 |
| 7 | 0.008655 | 0.012543 | 0.010267 | 0.014098 |
| 8 | 0.077214 | 0.026226 | 0.090069 | 0.236798 |
| 9 | 0.011765 | 0.062714 | 0.002240 | 0. |

| | $X_i'$ | $Y_i'$ | $\frac{1}{b_{ii}}$ | $\frac{Y_i'}{b_{ii}}$ | $X_i' - \frac{Y_i'}{b_{ii}}$ | $\frac{X_i' - Y_i'/b_{ii}}{X_i'}$ | $Y_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 17,386 | 9,228 | 1.147667 | 10,591 | 6,795 | 0.39 | 9,926 |
| 2 | 3,002 | 200 | 1.019701 | 204 | 2,798 | 0.93 | 259 |
| 3 | 5,465 | 4,704 | 1.005150 | 4,728 | 737 | 0.16 | 5,119 |
| 4 | 53,883 | 24,322 | 1.704620 | 41,460 | 12,423 | 0.23 | 24,351 |
| 5 | 8,591 | 4,997 | 1.006090 | 5,027 | 3,564 | 0.41 | 5,349 |
| 6 | 7,395 | 2,075 | 1.039061 | 2,502 | 4,893 | 0.66 | 2,408 |
| 7 | 1,754 | 432 | 1.012702 | 437 | 1,317 | 0.75 | 467 |
| 8 | 10,714 | 6,205 | 1.098984 | 6,819 | 3,895 | 0.36 | 7,955 |
| 9 | 4,185 | 1,147 | 1.0 | 1,147 | 3,038 | 0.72 | 657 |
| Total | 112,375 | | | 702,915 | 39,460 | | |

| | $Y_i - Y_i'$ | $\frac{Y_i - Y_i'}{b_{ii}}$ | $\frac{\sum(Y_j - Y_j')/b_{jk}}{\sum X_j'}$ | $\frac{X_i' - Y_i'/b_{ii}}{X_i'} \cdot \frac{\sum(Y_j - Y_j')/b_{jj}}{\sum X_j'}$ |
|---|---|---|---|---|
| 1 | 698 | 801 | 0.028* | 0.012 |
| 2 | 59 | 60 | 0.031 | 0.029 |
| 3 | 415 | 417 | 0.031 | 0.005 |
| 4 | 29 | 49 | 0.059* | 0.016 |
| 5 | 352 | 354 | 0.031 | 0.017 |
| 6 | 333 | 346 | 0.031 | 0.020 |
| 7 | 35 | 35 | 0.031 | 0.023 |
| 8 | 1750 | 1923 | 0.017* | 0.006 |
| 9 | − 490 | − 490 | 0.031 | 0.022 |

| | $X_i^{(1)}$ | Error percent | $X_i^{(2)}$ | Error percent | $X_i' - \frac{Y_i'}{b_{ii}} \cdot \frac{\sum(Y_j - Y_j')/b_{jj}}{\sum X_j'}$ |
|---|---|---|---|---|---|
| 1 | 18,187 | 0.8% | 18,405 | −0.4% | 190* |
| 2 | 3,062 | 2.7 | 3,151 | −0.1 | 88 |
| 3 | 5,882 | 0.8 | 5,911 | 0.3 | 23 |
| 4 | 53,932 | 1.5 | 54,795 | −0.1 | 733* |
| 5 | 8,945 | 1.3 | 9,061 | 0.03 | 110 |
| 6 | 7,741 | 1.7 | 7,896 | −0.3 | 152 |
| 7 | 1,789 | 2.1 | 1,830 | −0.1 | 41 |
| 8 | 12,637 | 0.2 | 2,713 | −0.4 | 66* |
| 9 | 3,695 | 2.9 | 3,776 | 0.8 | 94 |

| | $X_i^{(3)}$ | Error percent | $X_i$ calculated by means of inverse matrix |
|---|---|---|---|
| 1 | 18,377 | −0.3% | 18,328 |
| 2 | 3,150 | −0.1 | 3,148 |
| 3 | 5,905 | 0.4 | 5,927 |
| 4 | 54,665 | 0.1 | 54,734 |
| 5 | 9,055 | 0.1 | 9,064 |
| 6 | 7,893 | −0.3 | 7,873 |
| 7 | 1,830 | −0.1 | 1,828 |
| 8 | 12,703 | −0.3 | 12,660 |
| 9 | 3,789 | 0.4 | 3,805 |

* In the calculation of values with star mark, *, $\dfrac{X_i' - \dfrac{Y_i'}{b_{ii}}}{X_i'} \cdot \dfrac{\sum_{j \neq i} \dfrac{(Y_j - Y_j')}{b_{jj}}}{\sum_{j \neq i} X_j'}$ is used as the second term of (13) or (18).

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## ERRATA

These Annals, Vol. VII.

P. 118, 15th line from bottom: read

$$\text{“}\cdots\frac{\left(X_i'-\dfrac{Y_i'}{b_{ij}}\right)}{X_i'}\cdots\text{”} \quad \text{instead of} \quad \text{“}\cdots\frac{\left(X_i'-\dfrac{X_i'}{b_{ii}}\right)}{X_i'}\cdots\text{”}$$

P. 119, 9th line: read

$$\cdot\frac{\sum_{k=1}^{n} a_{jk}X_k'\,\dfrac{(Y_k-Y_k')/b_{kk}}{X_k'}}{\sum_{k=1}^{n} a_{jk}X_k'} \quad \text{instead of}$$

$$\cdot\frac{\sum_{k=1}^{n} a_{jk}X'\,\dfrac{(Y_k-Y_k')/b_{kk}}{X_k'}}{\sum_{k=1}^{n} a_{jk}X_k'}$$

P. 119, 11th line from bottom: read “$\doteqdot$” instead of “$=$”

P. 121, 12th line: read

$$\text{“}\frac{\sum(Y_i-Y_i')/b_{jj}}{\sum X_j'}\text{”} \quad \text{instead of} \quad \text{“}\frac{\sum(Y_i-Y_j')b_{jk}}{\sum X_j'}\text{“}.$$

P. 121, 10th line from bottom: read

$$\text{“}\left(X_i'-\frac{Y_i'}{b_{ii}}\right)\cdot\frac{\sum(Y_j-Y_j')/b_{jj}}{\sum X_j'}\text{”} \quad \text{instead of}$$

$$\text{“}X_i'-\frac{Y_i'}{b_{ii}}\cdot\frac{\sum(Y_j-Y_j')/b_{jj}}{\sum X_j'}\text{”}.$$

## ERRATA

These Annals Vol. V, No. 2

Page 63, line 9,

read "$\leqq \frac{27}{n^4} \sum_{i=1}^{l} n \cdots$" instead of "$\leqq \frac{8}{n^4} \sum_{i=1}^{l} \cdots$"

Page 63, line 10, 13, 14, 19, 23,

read "108" instead of "32"

Vol. VII, No. 2

Page 117, line 10,

read "$X_i^{(2)} = X_i^{(1)} \left( 1 + \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i'} \cdot \cdots \right.$"

instead of "$X_i^{(2)} = X_i^{(1)} \left( 1 + \frac{X_i' - \frac{Y_i'}{b_{ii}}}{X_i} \cdot \cdots \right.$"

Page 121, line 2 from bottom,

read "12.713", instead of "2.713"

Vol. VII, No. 3

Page 147, line 2 & 3,

read "$\eta_u = \frac{\sinh^{-1}\left[\frac{1}{2} \cdots\right.}{\alpha'(n+1/\sigma^2)}$" instead of "$-\eta_L = \frac{\sin h^{-1}\left[\frac{1}{2}(\cdots\right.}{\alpha'(n+1/\sigma^2)}$"

Page 151, line 6 from bottom,

read "$P_{\theta'}\{S_n \text{ covers } \theta\}$ is$\cdots$"

instead of "$P_{\theta}\{S_n \text{ covers } \theta'\}$ is$\cdots$"

Vol. VIII, No. 1

Page 59, line 9 from bottom,

read "0.01234, 0.04344, 0.12803, 0.28807, 0.52812"

instead of "0.52812, 0.28807, 0.12803, 0.04344, 0.01234"