

## Note on the Moments of the Transformed Correlation

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Let  $r$  be the correlation coefficient in the sample of size  $n$  taken out of the bivariate normal population with the parent correlation  $\rho$ . R. A. Fisher\*<sup>2</sup> has pointed out that the correlation may be transformed by the formulae,

$$z = \frac{1}{2} \log \frac{1+r}{1-r}, \quad \zeta = \frac{1}{2} \log \frac{1+\rho}{1-\rho},$$

with some advantages, and has given the sampling moments for  $x$ , putting  $z = \zeta + x$ . But we are doubtful of some of his formulae for the sampling moments. Therefore, in the following we shall give the results of our calculation for these.

$$\begin{aligned} \mu_1' &= \frac{\rho}{2(n-1)} \left\{ 1 + \frac{5 + \rho^2}{4(n-1)} + \dots \right\}, \\ \mu_2' &= \frac{1}{n-1} \left\{ 1 + \frac{8 - \rho^2}{4(n-1)} + \frac{88 - 9\rho^2 - 9\rho^4}{24(n-1)^2} + \dots \right\}, \\ \mu_3' &= \frac{3\rho}{2(n-1)^2} \left\{ 1 + \frac{13 + 2\rho^2}{4(n-1)} + \dots \right\}, \\ \mu_4' &= \frac{3}{(n-1)^2} \left\{ 1 + \frac{28 - 3\rho^2}{6(n-1)} + \frac{736 - 84\rho^2 - 51\rho^4}{48(n-1)^2} + \dots \right\}; \\ \mu_2 &= \frac{1}{n-1} \left\{ 1 + \frac{4 - \rho^2}{2(n-1)} + \frac{22 - 6\rho^2 - 3\rho^4}{6(n-1)^2} + \dots \right\}, \\ \mu_3 &= \frac{\rho^3}{(n-1)^3} + \dots, \\ \mu_4 &= \frac{3}{(n-1)^2} \left\{ 1 + \frac{14 - 3\rho^2}{3(n-1)} + \frac{184 - 48\rho^2 - 21\rho^4}{12(n-1)^2} + \dots \right\}; \\ \beta_1 &= \frac{\rho^6}{(n-1)^3} + \dots, \\ \beta_2 &= 3 + \frac{2}{n-1} + \frac{4 + 2\rho^2 - 3\rho^4}{(n-1)^2} + \dots. \end{aligned}$$

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### REFERENCE

R. A. Fisher: On the "probable error" of a coefficient of correlation deduced from a small sample. *Metron* 1 (1921).