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A generalized partially linear framework for variance functions

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Abstract When model the heteroscedasticity in a broad class of partially linear models, we allow the variance function to be a partial linear model as well and the parameters in the variance function to be different from those in the mean function. We develop a two-step estimation procedure, where in the first step some initial estimates of the parameters in both the mean and variance functions are obtained and then in the second step the estimates are updated using the weights calculated based on the initial estimates. The resulting weighted estimators of the linear coefficients in both the mean and variance functions are shown to be asymptotically normal, more efficient than the initial un-weighted estimators, and most efficient in the sense of semiparametric efficiency for some special cases. Simulation experiments are conducted to examine the

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numerical performance of the proposed procedure, which is also applied to data from an air pollution study in Mexico City.

Keywords Efficiency · Generalized least squares · Generalized partially linear model · Kernel regression · Profiling · Variance function