

# A constructive hypothesis test for the single-index models with two groups

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**Abstract** Comparison of two-sample heteroscedastic single-index models, where both the scale and location functions are modeled as single-index models, is studied in this paper. We propose a test for checking the equality of single-index parameters when dimensions of covariates of the two samples are equal. Further, we propose two test statistics based on Kolmogorov–Smirnov and Cramér–von Mises type functionals. These statistics evaluate the difference of the empirical residual processes to test the equality of mean functions of two single-index models. Asymptotic distributions of estimators and test statistics are derived. The Kolmogorov–Smirnov and Cramér–von Mises test statistics can detect local alternatives that converge to the null hypothesis at a parametric convergence rate. To calculate the critical values of Kolmogorov–Smirnov and Cramér–von Mises test statistics, a bootstrap procedure is proposed. Simulation studies and an empirical study demonstrate the performance of the proposed procedures.

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