

# Hybrid schemes for exact conditional inference in discrete exponential families

David Kahle<sup>1</sup> · Ruriko Yoshida<sup>2</sup> · Luis Garcia-Puente<sup>3</sup>

Received: 11 November 2015 / Revised: 20 April 2017 / Published online: 4 September 2017  
© The Institute of Statistical Mathematics, Tokyo 2017

**Abstract** Exact conditional goodness-of-fit tests for discrete exponential family models can be conducted via Monte Carlo estimation of  $p$  values by sampling from the conditional distribution of multiway contingency tables. The two most popular methods for such sampling are Markov chain Monte Carlo (MCMC) and sequential importance sampling (SIS). In this work we consider various ways to hybridize the two schemes and propose one standout strategy as a good general purpose method for conducting inference. The proposed method runs many parallel chains initialized at SIS samples across the fiber. When a Markov basis is unavailable, the proposed scheme uses a lattice basis with intermittent SIS proposals to guarantee irreducibility and asymptotic unbiasedness. The scheme alleviates many of the challenges faced by the MCMC and SIS schemes individually while largely retaining their strengths. It also provides diagnostics that guide and lend credibility to the procedure. Simulations demonstrate the viability of the approach.

---

✉ David Kahle  
david.kahle@gmail.com

Ruriko Yoshida  
ryoshida@nps.edu

Luis Garcia-Puente  
lgarcia@shsu.edu

- <sup>1</sup> Department of Statistical Science, Baylor University, One Bear Place #97140, Waco, TX 76798, USA
- <sup>2</sup> Department of Operations Research, Naval Postgraduate School, 1411 Cunningham Road, Monterey, CA 93943, USA
- <sup>3</sup> Department of Mathematics and Statistics, Sam Houston State University, Huntsville, TX 77341, USA

---

**Keywords** Contingency tables · Exact inference · Markov chain Monte Carlo · Sequential importance sampling · Algebraic statistics