Ann Inst Stat Math (2018) 70:969–982
https://doi.org/10.1007/s10463-017-0614-0

# Smoothed nonparametric tests and approximations of $p$-values

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Received: 21 December 2015 / Revised: 11 July 2017 / Published online: 23 August 2017


**Abstract** We propose new smoothed sign and Wilcoxon's signed rank tests that are based on kernel estimators of the underlying distribution function of the data. We discuss the approximations of the $p$-values and asymptotic properties of these tests. The new smoothed tests are equivalent to the ordinary sign and Wilcoxon's tests in the sense of Pitman's asymptotic relative efficiency, and the differences between the ordinary and new tests converge to zero in probability. Under the null hypothesis, the main terms of the asymptotic expectations and variances of the tests do not depend on the underlying distribution. Although the smoothed tests are not distribution-free, making use of the specific kernel enables us to obtain the Edgeworth expansions, being free of the underlying distribution.

**Keywords** Edgeworth expansion · Kernel estimator · Sign test · Significance probability · Wilcoxon's signed rank test

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