

## Hazard rate estimation for left truncated and right censored data

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**Abstract** Left truncation and right censoring (LTRC) presents a unique challenge for nonparametric estimation of the hazard rate of a continuous lifetime because consistent estimation over the support of the lifetime is impossible. To understand the problem and make practical recommendations, the paper explores how the LTRC affects a minimal (called sharp) constant of a minimax MISE convergence over a fixed interval. The corresponding theory of sharp minimax estimation of the hazard rate is presented, and it shows how right censoring, left truncation and interval of estimation affect the MISE. Obtained results are also new for classical cases of censoring or truncation and some even for the case of direct observations of the lifetime of interest. The theory allows us to propose a relatively simple data-driven estimator for small samples as well as the methodology of choosing an interval of estimation. The estimation methodology is tested numerically and on real data.

Keywords Adaptation  $\cdot$  Breast cancer  $\cdot$  Longevity  $\cdot$  MISE  $\cdot$  Nonparametric estimation  $\cdot$  Sharp minimax  $\cdot$  Survival analysis

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